8 out of 8

Q1) $3 + j2 + 2e^{j\pi/2}$

$3+j2 + 2\left[\cos(\pi/2) + j\sin(\pi/2)\right]$

$= 3+j2 + 2j = 3+4j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{3^2+4^2} = 5$

no calculators allowed but I used it calculate the degree here.

$\alpha = \tan^{-1}(y/x) = \tan^{-1}(4/3) \approx 53.13°$

polar form $5\angle 53.13°$ or $5\left[\cos(53.13) + j\sin(53.12)\right]$

Q2) $\frac{d}{dt} x(t) = 2t\, x(t)$

for my clarity $\Rightarrow \frac{dy}{dx} = 2xy$

$\Rightarrow \frac{dy}{y} = 2x\,dx \Rightarrow \int \frac{dy}{y} = \int 2x\,dx$

$= \ln(y) = x^2 + C_1 \Rightarrow y = e^{x^2} e^{C_1} \Rightarrow y = Ce^{x^2}$

or $\boxed{x(t) = Ce^{t^2}}$

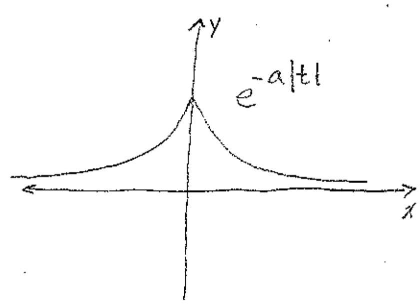
at $t=-1$ $x(t) = e \Rightarrow e = Ce \Rightarrow C = 1$

So $\boxed{x(t) = e^{t^2}}$

Q2) a)

$$\int_{-\infty}^{\infty} e^{-2|t|-1}\,dt = \int_{-\infty}^{\infty} e^{-2|t|} e^{-1}\,dt = \frac{1}{e}\int_{-\infty}^{\infty} e^{-2|t|}\,dt$$

$$= \frac{1}{e}\left[\int_{-\infty}^{0} e^{2t}\,dt + \int_{0}^{\infty} e^{-2t}\,dt\right]$$



$$= \frac{1}{e}\left[\frac{e^{2t}}{2}\Big|_{-\infty}^{0} + \frac{-e^{-2t}}{2}\Big|_{0}^{\infty}\right]$$

$$= \frac{1}{e}\left[\left(\tfrac{1}{2} - 0\right) + \left(0 - \left(-\tfrac{1}{2}\right)\right)\right] = \boxed{\frac{1}{e}}$$ not sure

---

Q3b)

$$\int_{-\infty}^{a} \frac{1}{1+\pi^2 x^2}\,dx = \int_{-\infty}^{a} \frac{1}{1+(\pi x)^2}\,dx$$

let $u = \pi x \Rightarrow \frac{du}{dx} = \pi \Rightarrow dx = \frac{du}{\pi}$

So $$\frac{1}{\pi}\int_{-\infty}^{a} \frac{1}{1+u^2}\,du = \frac{1}{\pi}\tan^{-1}(u)\Big|_{-\infty}^{a}$$

$$= \frac{1}{\pi}\tan^{-1}(\pi x)\Big|_{-\infty}^{a} = \frac{1}{\pi}\left[\tan^{-1}(\pi a) - \left(-\pi/2\right)\right]$$

$$= \boxed{\frac{1}{\pi}\left[\tan^{-1}(\pi a) + \pi/2\right]}$$

(Q4)

$$A = \int_{-\infty}^{\infty} x(t)\, e^{-j\pi t}\, dt$$

Solve in terms of A

$$\int_{-\infty}^{\infty} x(t+1)\, e^{-j\pi t}\, dt$$

$$\int_{-\infty}^{\infty} x(t)\, e^{-j\pi(t-1)}\, dt = e^{j\pi} \int_{-\infty}^{\infty} x(t)\, e^{-j\pi t} = \boxed{Ae^{j\pi}}$$

$(s+1)(s+2)$

$s^2+2s+s+2$

Q5) $\dfrac{d^2y(t)}{dt^2} + 3\dfrac{dy(t)}{dt} + 2y(t) = 4e^{-3t}u(t)$

$$\mathcal{L}\left[\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t)\right] = \mathcal{L}\left[4e^{-3t}u(t)\right]$$

$$= s^2Y(s) - sY(0) - y'(0) + 3(sY(s) - Y(0)) + 2Y(s) = \frac{4}{s+3}$$

Since no initial conditions are assumed

$$s^2Y(s) + 3sY(s) + 2Y(s) = \frac{4}{s+3}$$

$$Y(s)(s^2+3s+2) = \frac{4}{s+3} \Rightarrow Y(s) = \frac{4}{(s^2+3s+2)(s+3)}$$

$= Y(s) = \dfrac{4}{(s+1)(s+2)(s+3)}$ using partial decomposition

$$\frac{4}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

continued on next page

Q5 continued.

$$\frac{4}{(s+2)(s+3)} = A + \frac{B(s+1)}{s+2} + \frac{C(s+1)}{(s+3)}$$

let $s = -1 \Rightarrow A = 2$

following that scheme

$$\frac{4}{(s+1)(s+3)} = \frac{A(s+2)}{s+1} + B + \frac{C(s+2)}{s+3}$$

let $s = -2$

$\Rightarrow B = -4$

and $C = 2$

so $Y(s) = \frac{2}{s+1} - \frac{4}{s+2} + \frac{2}{s+3}$

then... $\boxed{y(t) = \left[2e^{-t} - 4e^{-2t} + 2e^{-3t}\right]u(t)}$ answer to Q5

Q6) Since $30 + 45 = 75$

we can say $\sin(75°) = \sin(5\pi/12)$

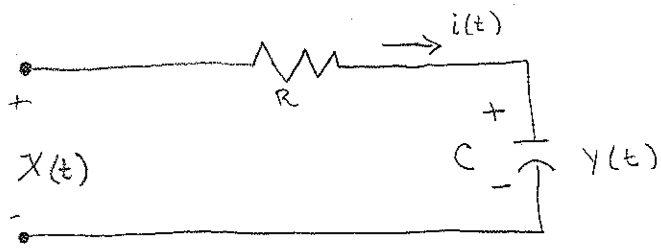
$\sin(5\pi/12) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$ using the identity $\sin(x+y) = \sin x \cos y + \cos x \sin y$

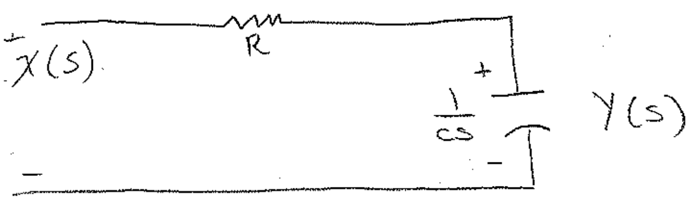
we get

$$\sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$= \frac{\sqrt{2} + \sqrt{3}\sqrt{2}}{4} = \frac{\sqrt{2}(1+\sqrt{3})}{2}$ which is exactly $\sin(75°)$

Q7)





$$Y(s) = \frac{1/cs}{R + 1/cs} \cdot X(s)$$

$$\Rightarrow \quad \frac{1}{Rcs+1} \cdot X(s) \qquad \text{since } H(s) = \frac{Y(s)}{X(s)}$$

then $$\frac{Y(s)}{X(s)} = H(s) = \frac{1/RC}{s + 1/RC}$$

$$\boxed{h(t) = \frac{1}{RC} e^{-t/RC}}$$

or do you want

$$i(t) = C \frac{dy}{dt}(t)$$

so $$X(t) = i(t)R + ci(t)$$

$$= X(t) = c\frac{dy(t)}{dt}(1+R)$$

a relationship exists in both these answers (between input & output)

ECE314: Signals and Systems

Knowledge Probe - Fall 2009

4 out of 8

Instructions:

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2. For any question you **does not completely solve**, please indicate the best explanation from the list below:

   A. I have no idea how to solve this problem.

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3. Whenever possible, please indicate in which class you learnt to solve each of the questions and when you took that class

4. Strictly no calculators are allowed.

Student 2

Q1. Write the following complex number in the polar form (Note: $j = \sqrt{-1}$):

$$3 + j2 + 2\exp\left(j\frac{\pi}{2}\right)$$

$3 + j2 + 2(\cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}))$

$3 + j2 + 2(0 + j)$

$3 + 4j = \boxed{5\angle 53.13^\circ}$

Q2. Solve the following differential equation for $t \geq -1$. The initial condition is given as $x(-1) = e$ (Note: e is the base of the natural logarithm).

$\boxed{C}$

learned in Math 316

$$\frac{d}{dt}x(t) = 2\,t\,x(t)$$

$\frac{dx}{dt}$

int Factor $= e^{\int -2t\,dt} = e^{-t^2}$

$x' - 2tx = 0$

$e^{-t^2}x' - 2tx e^{-t^2} = 0$

$\frac{d}{dt} e^{-t^2}x = 0$

$e^{-t^2}x = 0$

$x(t) = 0$

X

Q3. Solve the following integrals:

(a).

$$\int_{-\infty}^{\infty} \exp(-2|t| - 1)\,dt = \int_{-\infty}^{\infty} e^{-2(|t|+\frac{1}{2})}\,dt$$

$= -\frac{e^{-2(|t|+\frac{1}{2})}}{2}\Big|_{-\infty}^{\infty} = \frac{-e^{-2(|t|+\frac{1}{2})}}{2}\Big|_{-\infty}^{0} + \frac{-e^{-2(|t|+\frac{1}{2})}}{2}\Big|_{0}^{\infty}$

$2\left(\frac{-(e^{-1} - e^{-\infty})}{2}\right) + \cancel{\frac{-(e^{-\infty} - e^{-1})}{2}}$

$\boxed{+e^{-1}}$

(b).

$$\int_{-\infty}^{a} \frac{1}{1+\pi^2 x^2}dx = \frac{1}{\pi}\tan^{-1}(\pi x)\Big|_{-\infty}^{a}$$

$= \frac{1}{\pi}\left(\tan^{-1}(a\pi) - (\frac{\pi}{2})\right)$

Hints: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$, and $\tan^{-1}(-\infty) = -\frac{\pi}{2}$.

$\boxed{C}$

learned in Math 264

4

Q4. Suppose that, $\boxed{A}$

$$\int_{-\infty}^{\infty} x(t) \exp(-j\pi t)\, dt \;=\; A.$$

Solve the following integral in terms of A.

$$\int_{-\infty}^{\infty} x(t+1) \exp(-j\pi t)\, dt$$

X

Q5. Assuming zero initial conditions, find $y(t)$ from the following 2^{nd} order differential equation using Laplace Transform method:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) \;=\; 4\, e^{-3t}u(t)$$

where $u(t)$ is the unit step function.

Hint: $\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$, where $\mathcal{L}\{.\}$ denotes the Laplace Transform.

$\mathcal{L}\{4e^{-3t}u(t)\} = \frac{4}{s+3}$

$y'' + 3y' + 2y = \frac{4}{s+3}$

$s^2Y(s) + 3sY(s) + 2Y(s) = \frac{4}{s+3}$

$Y(s)(s^2+3s+2) = \frac{4}{s+3}$

$Y(s)(s+2)(s+1) = \frac{4}{s+3}$

$Y(s) = \frac{4}{(s+2)(s+1)(s+3)}$

$\frac{4}{(s+2)(s+1)(s+3)} = \frac{A}{s+2}(s^2+4s+3) + \frac{B}{s+1}(s^2+5s+6) + \frac{C}{s+3}(s^2+3s+2)$

$4 = As^2 + 4As + 3A + Bs^2 + 5Bs + 6B + Cs^2 + 3Cs + 2C$

$4 = s^2(A+B+C) + s(4A+5B+3C) + (3A+6B+2C)$

$A+B+C=0$ $\quad 4A+5B+3C=0$ $\quad 3A+6B+2C=4$

$A=-B-C$ $\quad 4(-B-C)+5B+3C=0$ $\quad 3(-C-B)+6B+2C=4$

$B-C=0$, $B=C$ $\quad -C+3B=4$

$-B+3B=4$

$2B=4$

$A=-4 \quad B=2 \quad C=2$

$Y(s) = -\frac{4}{s+2} + \frac{2}{s+1} + \frac{2}{s+3}$

init. conditions zero

$\boxed{y(t) = -4e^{-2t} + 2e^{-t} + 2e^{-3t}}$

Q6. If $\sin(30^o) = \frac{1}{2}$ and $\sin(45^o) = \frac{1}{\sqrt{2}}$, can you find $\sin(75^o)$ using trigonometric identities?

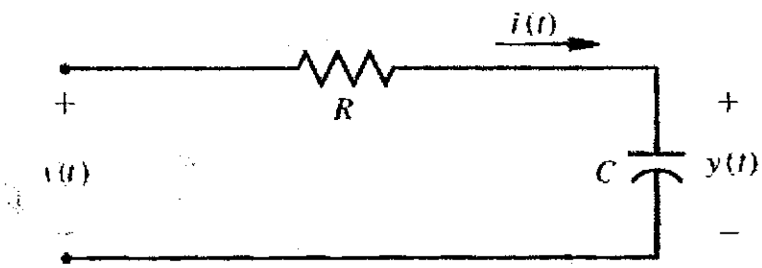
$\sin(75°) = \sin(30+45) = \sin(30)\cos(45) + \cos(30)\sin(45)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

/2

Q7. Suppose that the input and output voltages in the following circuit are $x(t)$ and $y(t)$, respectively. Write down the system equation.

(Note: You do not need to solve the system equation)



Hint: System equation is a relationship between the input and output.

$$\mathcal{L}\{x(t)\} = X(s) = V_{in}(s)$$

$$\mathcal{L}\{y(t)\} = Y(s) = V_{out}(s)$$

$$H_c(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Y(s)}{X(s)}$$

$$H_c = \frac{C}{R+C}$$

ECE314: Signals and Systems

Knowledge Probe - Fall 2009

0 out 8

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3. Whenever possible, please indicate in which class you learnt to solve each of the questions and when you took that class

4. Strictly no calculators are allowed.

Student 3

Q1. Write the following complex number in the polar form (Note: $j = \sqrt{-1}$):

I can do this, with a little review. Simple conversion.

$$3 + j2 + 2\exp\left(j\frac{\pi}{2}\right)$$

X

Q2. Solve the following differential equation for $t \geq -1$. The initial condition is given as $x(-1) = e$ (Note: e is the base of the natural logarithm).

Diff EQ
1 YR Ago

$$\frac{d}{dt}x(t) \;=\; 2\,t\,x(t)$$

X

Q3. Solve the following integrals:

(a).

Calc 2/3
1½-2 yrs Ago

$$\int_{-\infty}^{\infty} \exp\left(-2|t| - 1\right) dt$$

X

(b).

$$\int_{-\infty}^{a} \frac{1}{1+\pi^2 x^2} dx$$

X

Hints: $\frac{d}{dx}\tan^{-1}(x) \;=\; \frac{1}{1+x^2}$, and $\tan^{-1}(-\infty) \;=\; -\frac{\pi}{2}$.

Q4. Suppose that,

$$\int_{-\infty}^{\infty} x(t)\exp(-j\pi t)\,dt = A.$$

Solve the following integral in terms of A.

$$\int_{-\infty}^{\infty} x(t+1)\exp(-j\pi t)\,dt$$

X

X

Q5. Assuming zero initial conditions, find $y(t)$ from the following 2^{nd} order differential equation using Laplace Transform method:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 4\,e^{-3t}u(t)$$

where $u(t)$ is the unit step function.

Hint: $\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$, where $\mathcal{L}\{.\}$ denotes the Laplace Transform.

DiffEQ last summer → Little in circuits 2

X

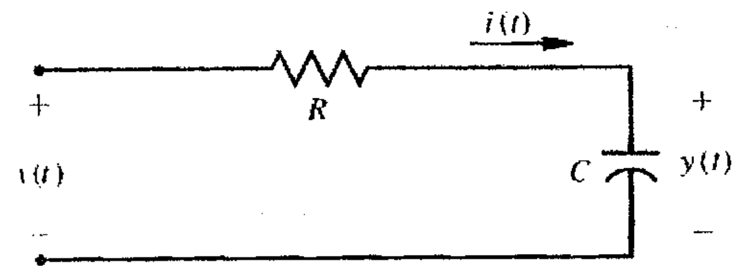
Q6. If $\sin(30^o) = \frac{1}{2}$ and $\sin(45^o) = \frac{1}{\sqrt{2}}$, can you find $\sin(75^o)$ using trigonometric identities?

Simple trig. I do not remember the Identeties But could do It with a chart.

X

Q7. Suppose that the input and output voltages in the following circuit are $x(t)$ and $y(t)$, respectively. Write down the system equation.

(Note: You do not need to solve the system equation)



Hint: System equation is a relationship between the input and output.

I HAVE SEEN ALL of these Before
and can do them with a LITTLE review
But without I do not have the ability at
this point in time to do them without my
Reference materal. I HAVE SEEN, and can
solve EACH of these Questions.