MURI-Transformational Electromagnetics

Innovative use of Metamaterials in Confining, Controlling, and Radiating Intense Microwave Pulses

University of New Mexico

August 21, 2012

Engineering Dispersive Properties of Metamaterials

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Overview

1. What range of ε^{eff} , μ^{eff} parameter space is accessible to simple metamaterial geometries? ``

2. A systematic & mathematically rigorous multi-scale ``bottom up'' methodology for linking subwavelength metamaterial geometry to effective parameters ε^{eff} , μ^{eff} ?

Overview & Perspective

3. Goal: optimal beam-wave interaction.

Design metallic sub-wavelength structures that mimic the response of a dielectric for generating slow electromagnetic waves.

D. Shiffler, J. Luginsland, D.M. French, J. Watrous, IEEE Trans. Plasma Sci. 2010.

4. Objective.

Control beam wave interaction frequencies by controlling intrinsic electrostatic resonances of metallic sub-wavelength structure.

Metamaterials & novel effective properties

- The first double negative materials made from metallic resonators and metallic posts. Smith et. al. Phys Rev. (2000). Operational at microwave frequencies.
- Several subsequent novel designs operational up to infrared.
  1. Dolling, G., Enrich, C., Wegener, M., Soukoulis, C. M., Linden, S.: Opt. Lett. Vol 31, 1800–1802(2006)
  2. Huangfu, J., Ran, L., Chen, H., Zhang, X., Chen, K., Grzegorczyk, T. M., Kong, J., A.: Appl. Phys. Lett. Vol 84, 1537(2004)
- <u>Metamaterials proposed for Higer frequencies</u>

  1. Plasmonic nano particles:

A. Alù, A. Salandrino, and N. Engheta, Opt. Expr. **14**, 1557 (2006).

  2. Optically thin metal films:

V. Lomakin†, Y. Fainman, Y. Urzhumov‡, & G. Shvets, Opt Expr. 14 11164 (2006)

  3. Large permittivity dielectric inclusions

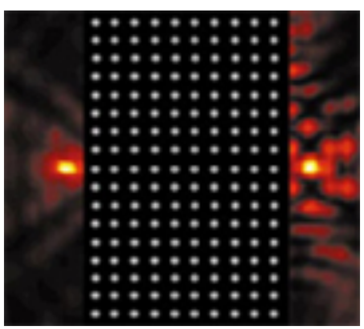
K.C. Huang, M.L. Povinelli, & J.D. Joannopoulos, J. D. 2004 Appl. Phys.Lett. 85, 543.

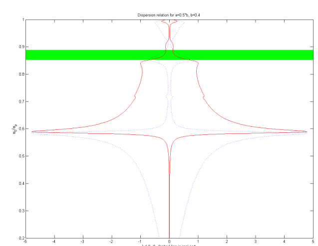
Macroscopic properties μ^{eff} , ε^{eff} from sub-wavelength structure

Objectives 1 and 2: recover an explicit multi-scale link between macroscopic properties μ^{eff} , ε^{eff} and geometry of sub-wavelength structures and understand range of properties accessible by simple sub-wavelength geometries.

With explicit relation connecting microstructure and properties of an effective medium can systematically design sub-wavelength geometries

Map sub-wavelength structure $\xrightarrow{\text{to}}$ Dispersion relations involving effective properties μ^{eff} , ε^{eff}





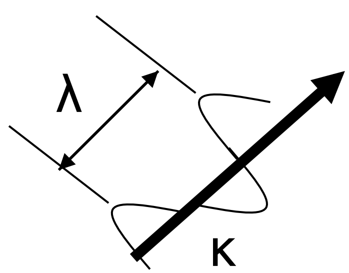
Electromagnetic Waves in Metamaterials

Sub-wavelength power series: a bottom up approach

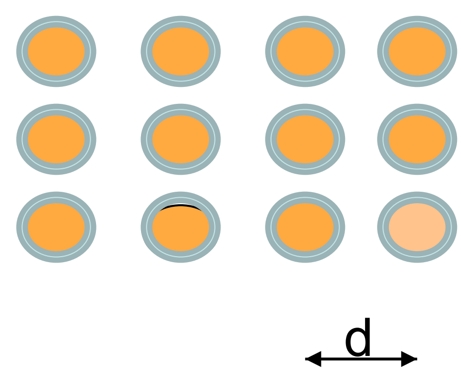
Effective properties appear as leading order terms
In an explicit sub-wavelength power series for dispersion relation
and power series solution to Maxwell Equations

Expansion parameter is length scale of structure
divided by wavelength. (Sub-wavelength expansion).

Higher order terms provide explicit corrections to
the effective theory.



$k=2\pi/\lambda$



Sub-wavelength power series

Negative index/ double negative properties from nonmagnetic materials:

1. Y. Chen & R. Lipton, Double Negative Dispersion Relations from Coated Plasmonic Rods. *(Multiscale Modeling and Simulation)* under revision*,* arXiv:1202.0602v1 [math.AP] 3 Feb 2012.

2. Y. Chen & R. Lipton, Resonance and double negative behavior in metamaterials. (Submitted) arXiv:1111.3586v1 [math.AP] 15 Nov 2011

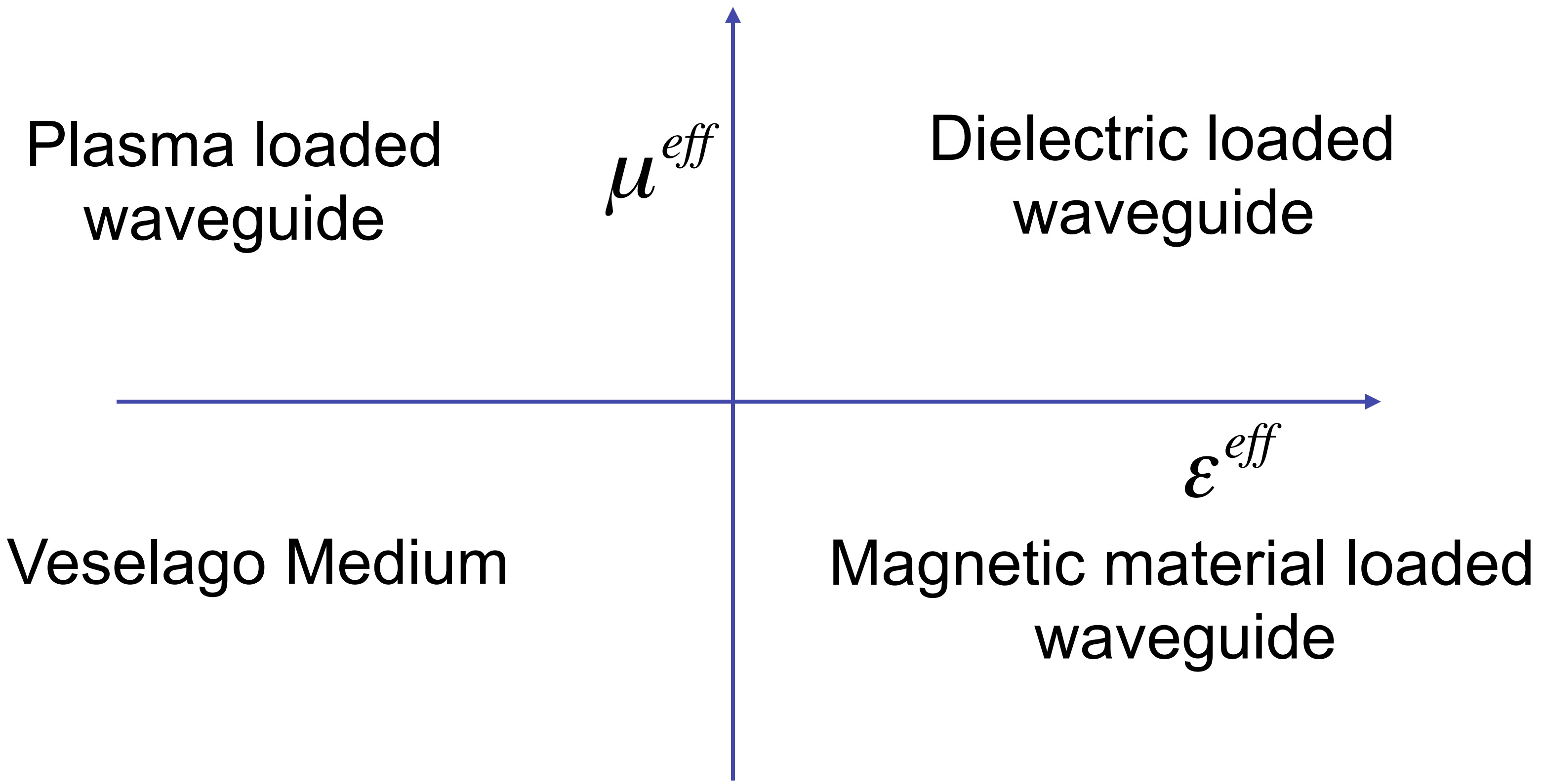
3. Y. Chen & R. Lipton Tunable double negative band structure from non-magnetic coated rods. New Journal of Physics Vol. 12 (2010) 083010

Negative magnetic permeability from non magnetic materials:

1. S. Fortes, R. Lipton, S. Shipman ``Power series for high contrast sub wavelength media.'' Proc. R. Soc. London Series A2010 466: 1993-2022

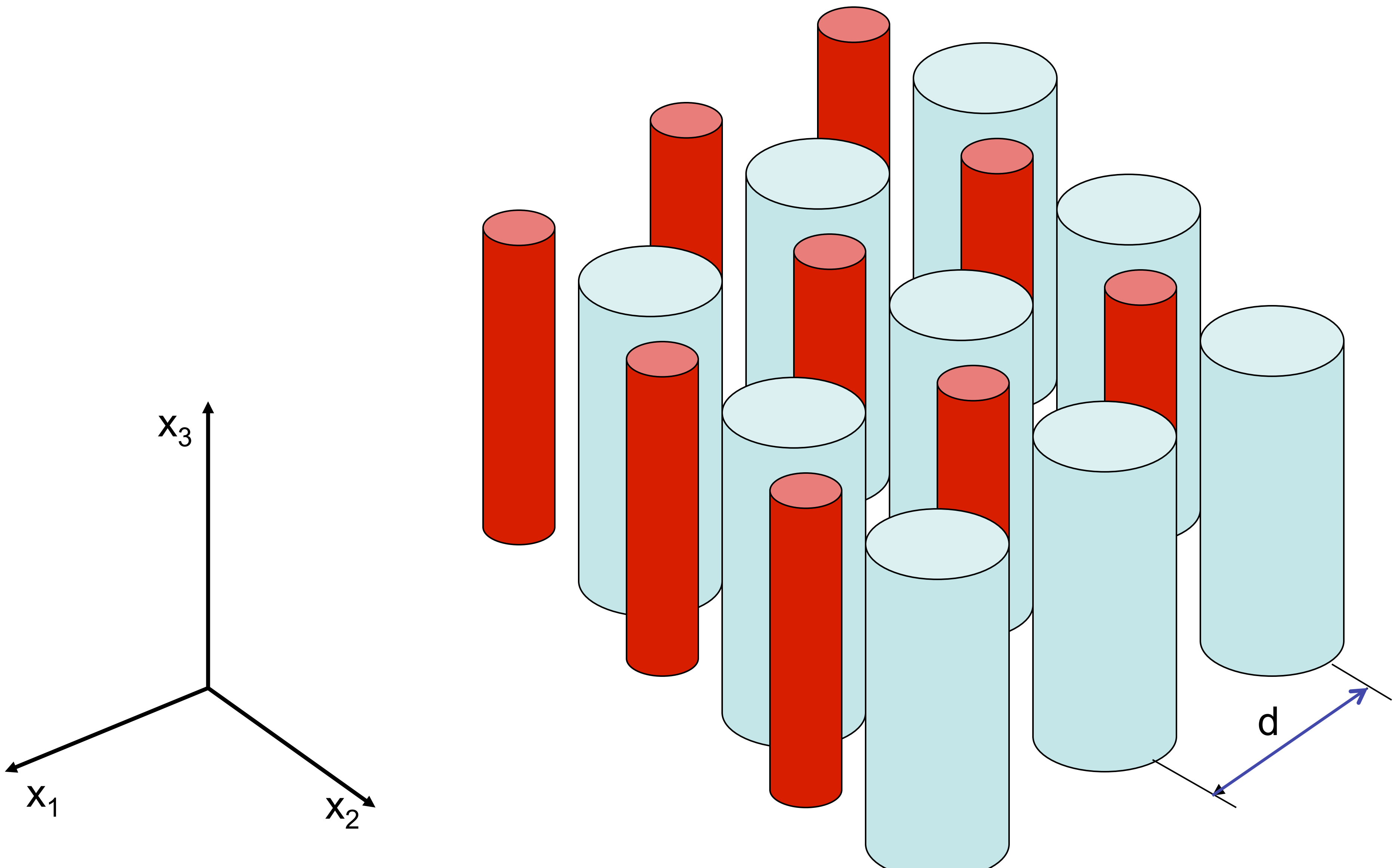
2. S. Fortes, R. Lipton, S. Shipman``Convergent power series for fields in positive or negative high-contrast periodic media.'' Communications in PDE 36, 2011, pp.1016--1043

Examples of simple sub-wavelength materials with exotic EM properties



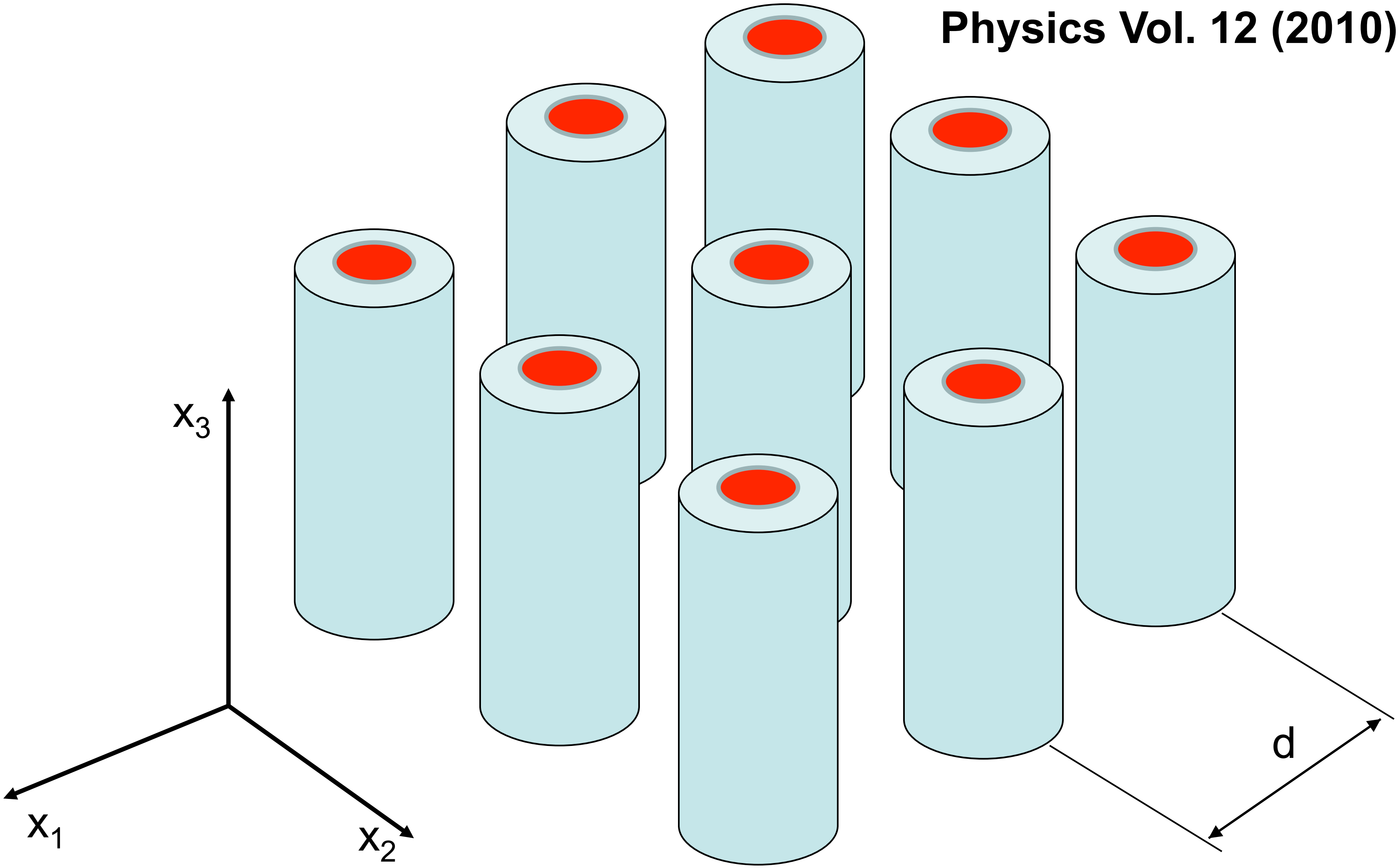
We provide examples of sub-wavelength geometries that capture the behavior in all four quadrants depending upon frequency of operation

Microstructured materials made from metallic and high dielectric rods

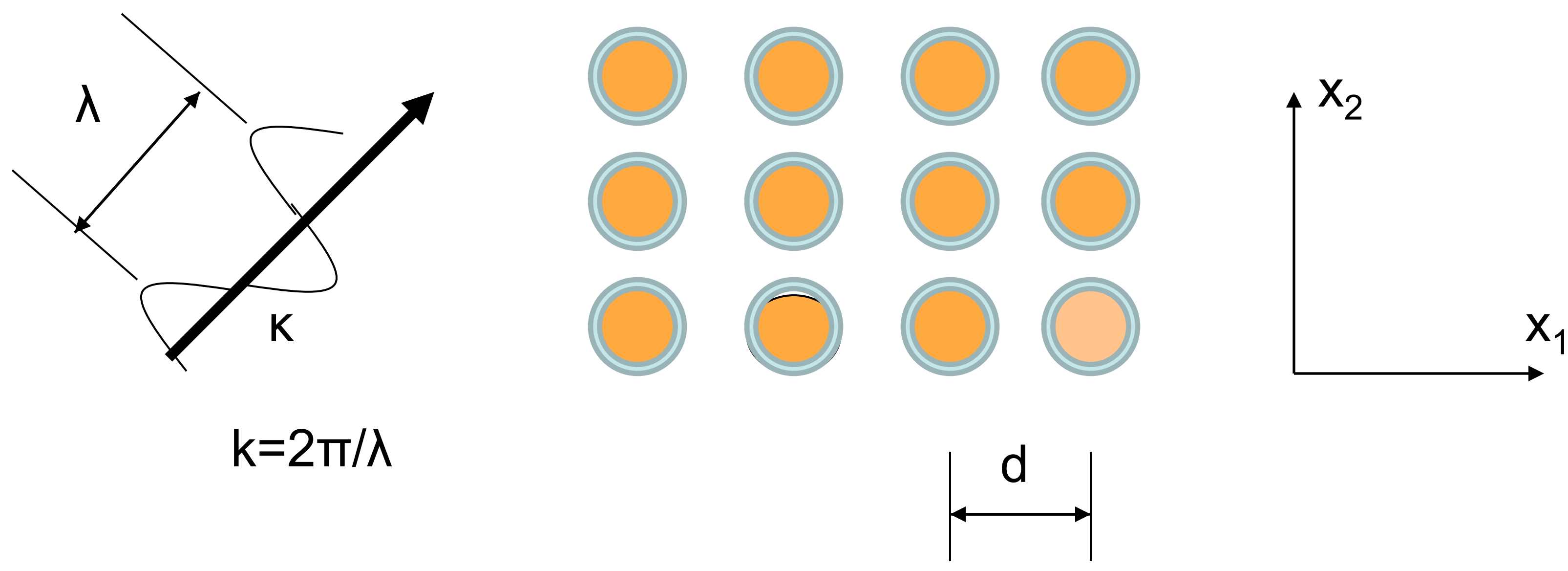


Periodic array of metal coated dielectric rods

Chen, Lipton, New Journal of Physics Vol. 12 (2010) 083010



Determine Waves Supported by the Metamaterial



Sub-wavelength power series solution for the H field

$$H = H_3(x)e^{i(k\kappa \cdot x - \omega t)}$$

Frequency dependent sub-wavelength local properties

To fix ideas consider metal coated rods

- Magnetic permeability $\mu=1$ in host, rod core & rod coating
- Crystal period is d.
- Metallic coating, Drude model: $\varepsilon_P=\varepsilon(\omega)=(1-(\omega_p/\omega)^2)$
- Host permittivity $\varepsilon_H=1$.
- Rod core permittivity $\varepsilon_R=\varepsilon_r/d^2$.
- The wave number is $k=2\pi/\lambda$.
- Investigate wave propagation when

$$\mathbf{2\pi d/\lambda=kd=\eta<1}$$

Find convergent power series solution in $kd=\eta$.

Sub-wavelength power series solution

Power series for the H field

$$H_3 = \underline{u}_0 \left(\psi_0(x/d) + \sum_{j=1}^{\infty} (dk)^j i^j \psi_j(x/d) \right) \exp\left\{ i(k\vec{\kappa} \bullet x - \omega t) \right\}$$

Power series for the dispersion relation

$$\omega = (\xi_0(k,\vec{\kappa}) + dk \sum_{m=0}^{\infty} (dk)^m \xi_m)^{1/2}$$

Leading order term. Effects of spatial dispersion

The first order term in the dispersion relation

Leading order dispersion relation for the metamaterial

$$\omega = \sqrt{\xi_0(k, \vec{\kappa})}$$

ξ_0 is a root of:

$$\mu^{eff}(\xi_0)\xi_0 = k^2 \varepsilon^{eff}(\xi_0)^{-1} \vec{\kappa} \bullet \vec{\kappa}$$

Natural Emergence of Effective Properties

The generic effective dielectric constant and magnetic permeability exhibit multiple surface modes!
These explicit formulas emerge naturally via the power series ``bottom up'' approach and modes are controlled by the shape of the metallc inclusions and their relative positions

$$\mu_{eff}(\xi_0) = \vartheta_H + \vartheta_p + \sum_{n=1}^{\infty} \frac{\mu_n \langle \phi_n \rangle_R^2}{(\mu_n - (\xi_0)}$$

$$\varepsilon_{eff}^{-1}(\xi_0)\vec{\kappa} \bullet \vec{\kappa} = \vartheta_H + \vartheta_p \frac{\xi_0}{\xi_0 - \left(\varepsilon_r \omega_p^2\right)}$$

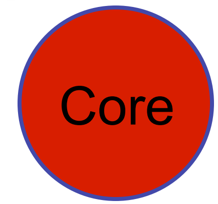
$$-\sum_{-1/2<\lambda_h<1/2} \frac{\left(\xi_0 - \varepsilon_r \omega_p^2\right)^2 |\alpha_h^1|^2 + 2\left(\xi_0 - \varepsilon_r \omega_p^2\right)\varepsilon_r \omega_p^2 \alpha_h^1 \alpha_h^2 + (\varepsilon_r \omega_p^2)^2 |\alpha_h^2|^2}{\left(\xi_0 - (\lambda_h + \frac{1}{2})\varepsilon_r \omega_p^2\right)\left(\xi_0 - \varepsilon_r \omega_p^2\right)}$$

Formulas emerge from asymptotic analysis, They are not postulated from the top down

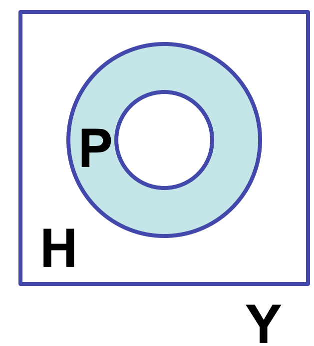
A general theory for effective metamaterial properties

The surface modes are determined by two distinct spectral problems intrinsic to the geometry & independent of materials!

Effective magnetic permeability:



The poles μ_n and weights $<\phi_n>$ are the Dirichlet Eigenvalues μ_n and averages of the Dirichlet eigenfunctions ϕ_n for associated with the core.



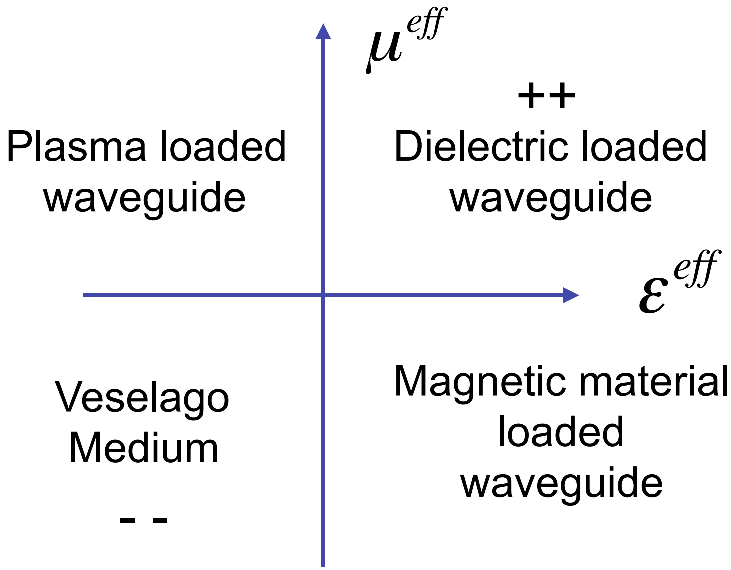
Effective Dielectric Constant:

The numbers λ_n and weights α_n are the Eigenvalues λ_n and averages of the eigenfunctions ψ_n for an electrostatic resonance problem:

$$\frac{1}{2}\int_H \nabla\psi\bullet\nabla v^* - \frac{1}{2}\int_P \nabla\psi\bullet\nabla v^* = \lambda\left(\int_H \nabla\psi\bullet\nabla v^* + \int_P \nabla\psi\bullet\nabla v^*\right)$$

Generalization of Bergman-Milton Electrostatic resonances to multiple phases

Intervals of double negative and double positive effective properties



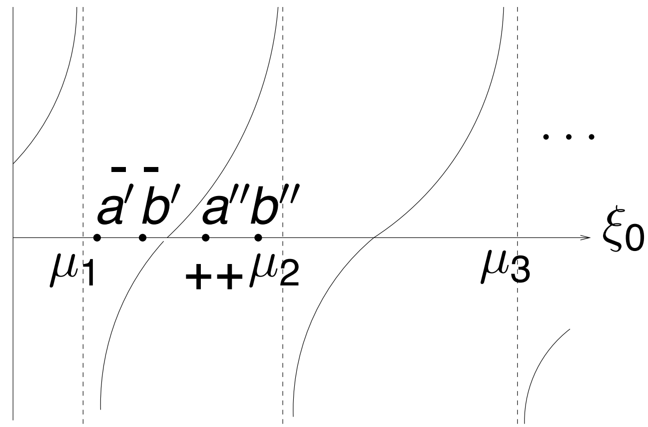


Figure: the relation between μ_{eff} and ξ_0

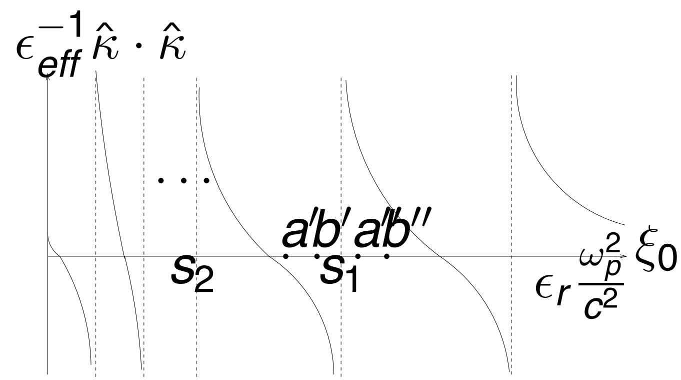
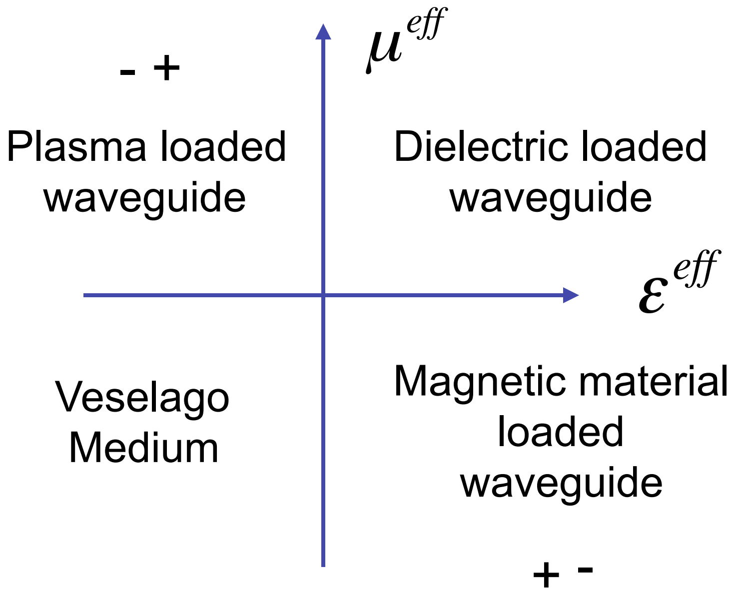
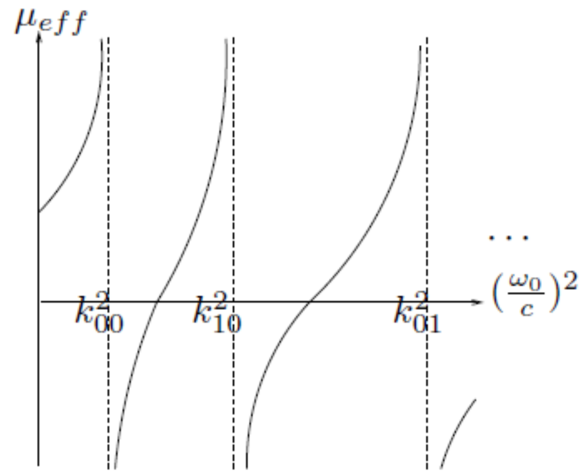
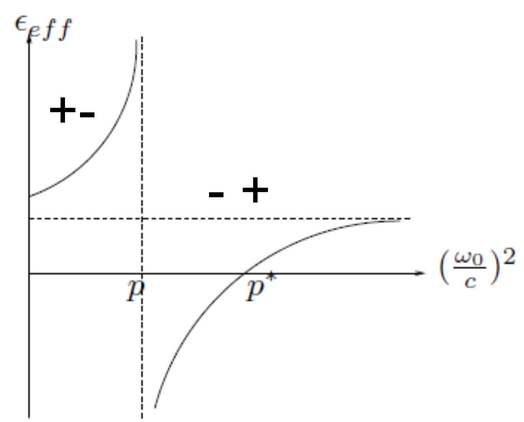


Figure: the relation between $\epsilon_{eff}^{-1}\hat{\kappa}\cdot\hat{\kappa}$ and ξ_0

Intervals of single negative and single positive effective properties







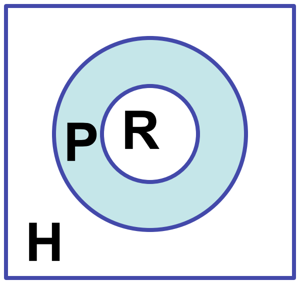
Example: Electrostatic Resonance Problem for Coated Cylinders

The electrostatic resonances λ_h are found by solving the following problem for the potential u inside a unit cell, i.e., $d = 1$:

$$\begin{cases} \Delta u = 0 & \text{in } H, \\ \Delta u = 0 & \text{in } P, \end{cases} \tag{33}$$

with the boundary conditions

$$\begin{cases} u|^{-} = u|^{-} & \text{on } \partial P, \\ \partial_r u|_{r=a} = 0 & \text{on } \partial R, \\ \lambda[\partial_r u]_{+}^{-} = -\frac{1}{2}(\partial_r u^{-} + \partial_r u^{+}) \text{ on } \partial P, \bigcap \partial H \\ u \text{ is } Y\text{-periodic} . \end{cases} \tag{34}$$



Electrostatic Resonances & Dirichlet Spectra
Tools for designing leading order dispersion relation

The first 10 electrostatic resonances λ_n explicitly control the first 10 surface modes for the metamaterial

Electro static resonances

3.508×10^{-1}

1.537×10^{-2}

9.755×10^{-4}

6.103×10^{-5}

3.814×10^{-6}

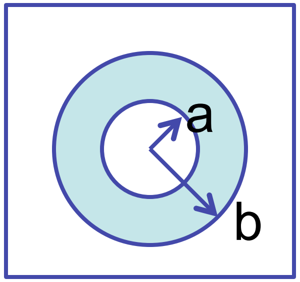
-2.028×10^{-3}

-5.533×10^{-3}

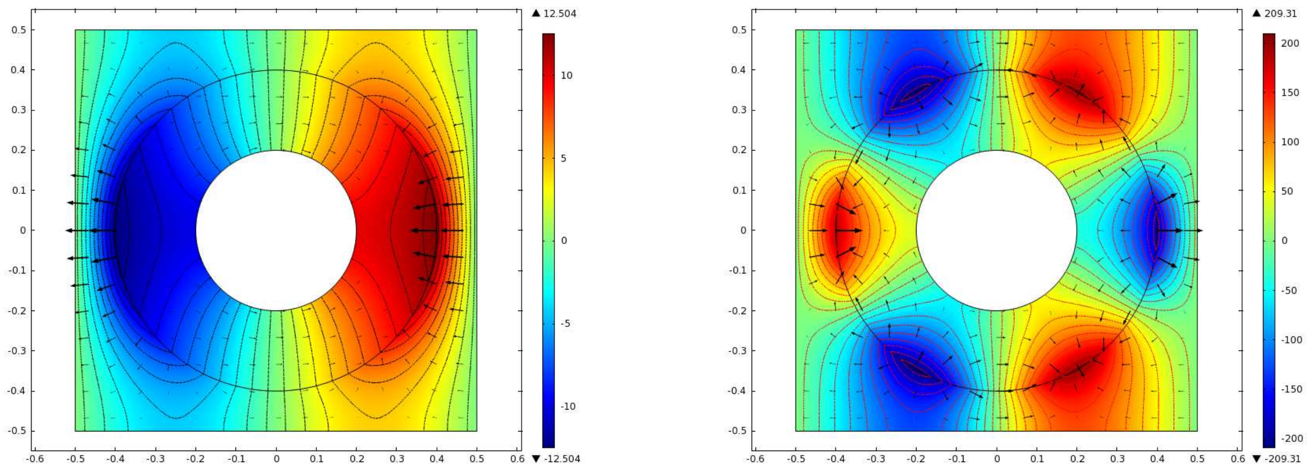
-1.501×10^{-2}

-4.453×10^{-2}

-4.497×10^{-2}



Electrostatic Resonances

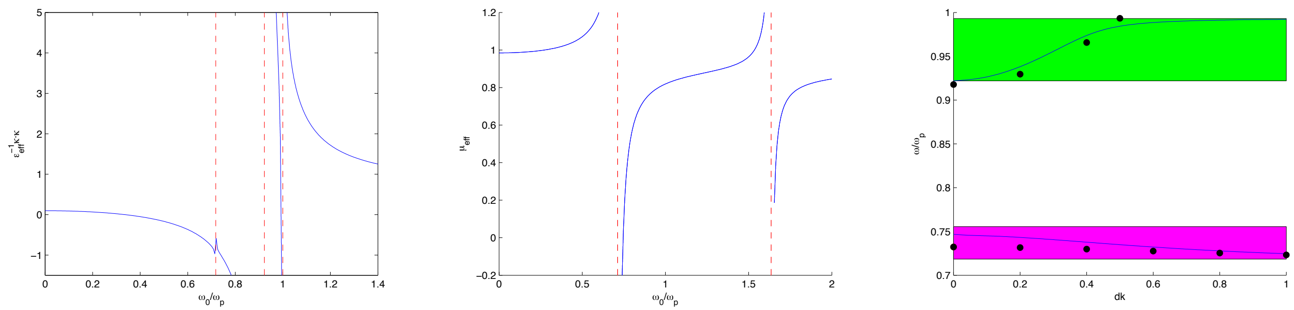


(a) (b)

Figure: (a) the solution corresponding to the eigenvalue $\lambda = 3.5080 \times 10^{-1}$; (b) the solution corresponding to the eigenvalue $\lambda = 1.5379 \times 10^{-2}$.

Comparison of leading order dispersion relations with direct numerical simulation I

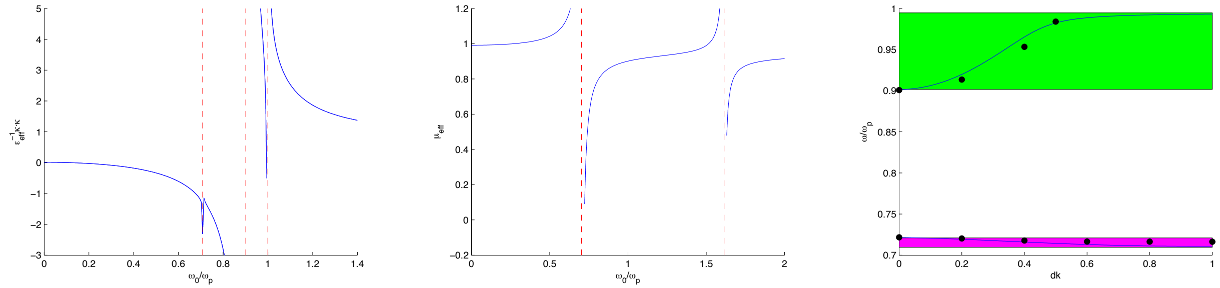
The following figures show the exact numerical solutions via COMSOL and how they compare to the leading order dispersion relation.



(a) (b) (c)

Figure: the case of $a = 0.2d, b = 0.4d$ and $\epsilon_R = 285$.

Comparison of leading order dispersion relations with direct numerical simulation II

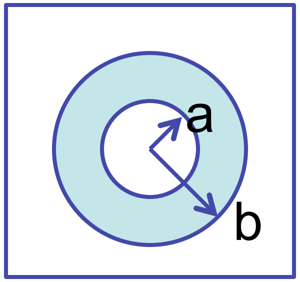


(a) (b) (c)

Figure: the case of $a = 0.15d$, $b = 0.4d$ and $\epsilon_R = 285$.

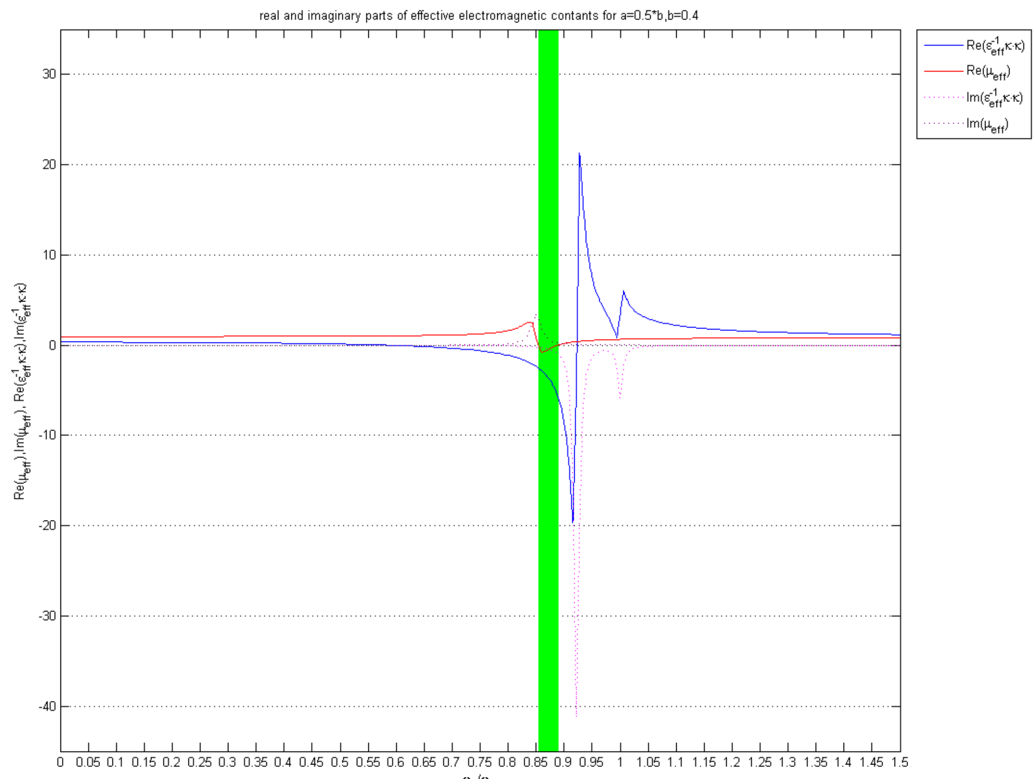
Design of Microstructure for Double Negative Properties via Electrostatic & Dirichlet resnonances

a	0.5xb	0.55xb	0.6xb	0.65xb
b				
0.3	0	0	0	0.8919 0.0332
0.35	0	0.9003 0.02733	0.8315 0.03824	0.7716 0.04425
0.4	0.8707 0.03541	0.7960 0.04204	0.7345 0.04893	0.6830 0.05579
0.45	0.7795 0.04366	0.7141 0.05143	0.6605 0.05944	0.6161 0.06801



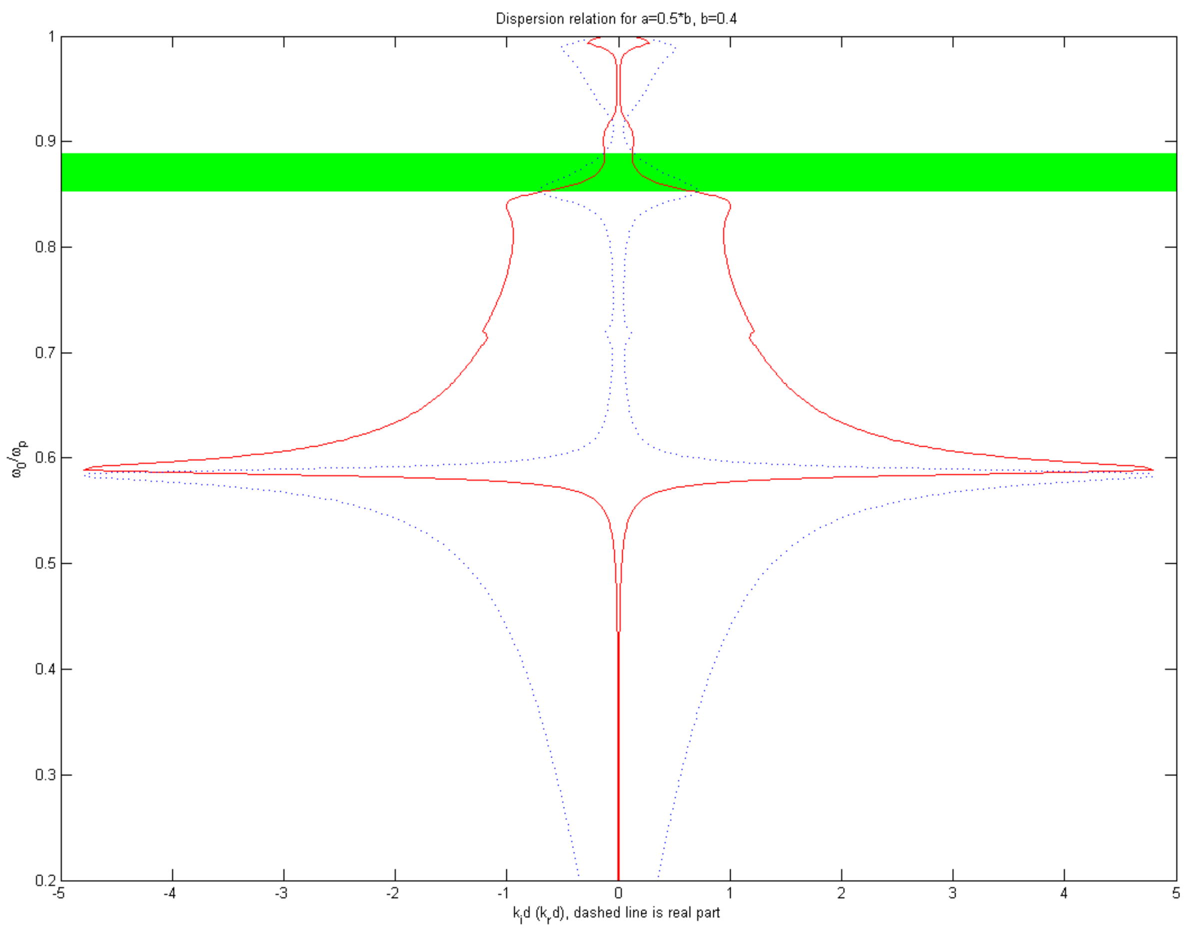
To add the additional effects of damping: just change frequency dependence of metallic phase to include damping – the geometrically intrinsic electrostatic resonances stay the same!

$$\varepsilon_p\left(\omega/\omega_p\right) = 1 - \frac{\left(\omega_0/\omega_p\right)^2}{\left(\omega/\omega_p\right)^2 - i\left(\omega_c/\omega_p\right)\left(\omega/\omega_p\right)} \qquad \omega_c/\omega_p = 0.01$$



Dispersion curves

Complex wave number $dk = dk_r + idk_i$



Dielectric behavior from sub-wavelength metallic structure

J.B. Pendry, A.J. Holden, W.J.Stewart, & I. Youngs, Phys. Rev. Lett. (76)25 4773 (1996). Use of structural geometry to generate low frequency plasmons.

Apply this phenomena to design slow wave structures for amplifiers ``A Cerenkov-like Maser Based on a Metamaterial Structure,'' D.Shiffler, J. Luginsland, D.M. French, & J. Watrous, IEEE Transactions on Plasma Science (38)6 1462 (2010).

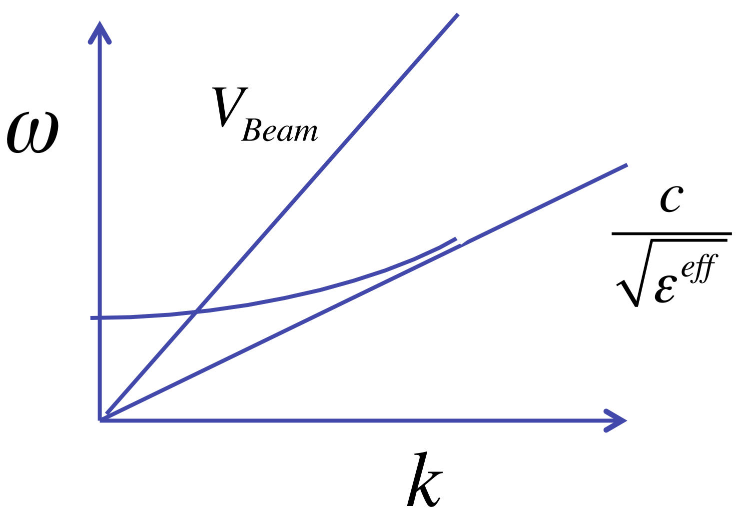
Control beam wave interaction frequencies by tailoring electrostatic resonances of metal structure

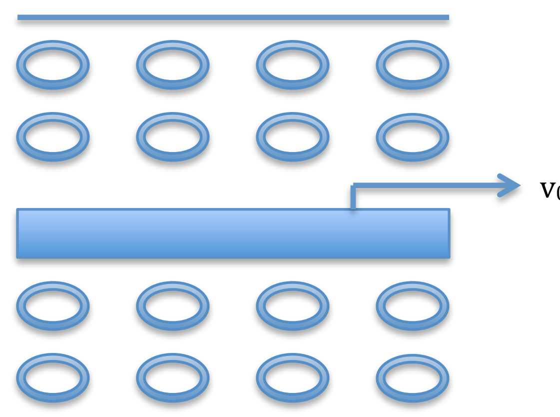
Custom design of TWT from subwavelength metal dielectric structure in a circular waveguide through:

(1) Geometric tailoring of intrinsic electrostatic resonances of the metallic structure. - This controls beam wave interaction frequencies.

(2) Provide a capability for systematic design of TWT for maximum gain over maximum band width.

(3) Corrections to dispersion relation associated with spatial dispersion can be computed explicitly with no phenomenology.





Dielectric behavior from sub-wavelength metallic structure

Goal: Engineer shape so surface modes far below $\omega_p/\sqrt{2}$.
Method: engineer electrostatic resonances

$$\varepsilon_{eff}^{-1}\left(\omega_0/\omega_p\right) = \vartheta_H + \vartheta_p \frac{\left(\omega_0/\omega_p\right)^2}{\left(\omega_0/\omega_p\right)^2 - 1}$$

$$- \sum_{-1/2<\lambda_h<1/2} \frac{\left((\omega_0/\omega_p)^2 - 1\right)^2 |\alpha_h^1|^2 + 2\left((\omega_0/\omega_p)^2 - 1\right)\alpha_h^1\alpha_h^2 + |\alpha_h^2|^2}{\left((\omega_0/\omega_p)^2 - (\lambda_h + 1/2)\right)\left((\omega_0/\omega_p)^2 - 1\right)}$$

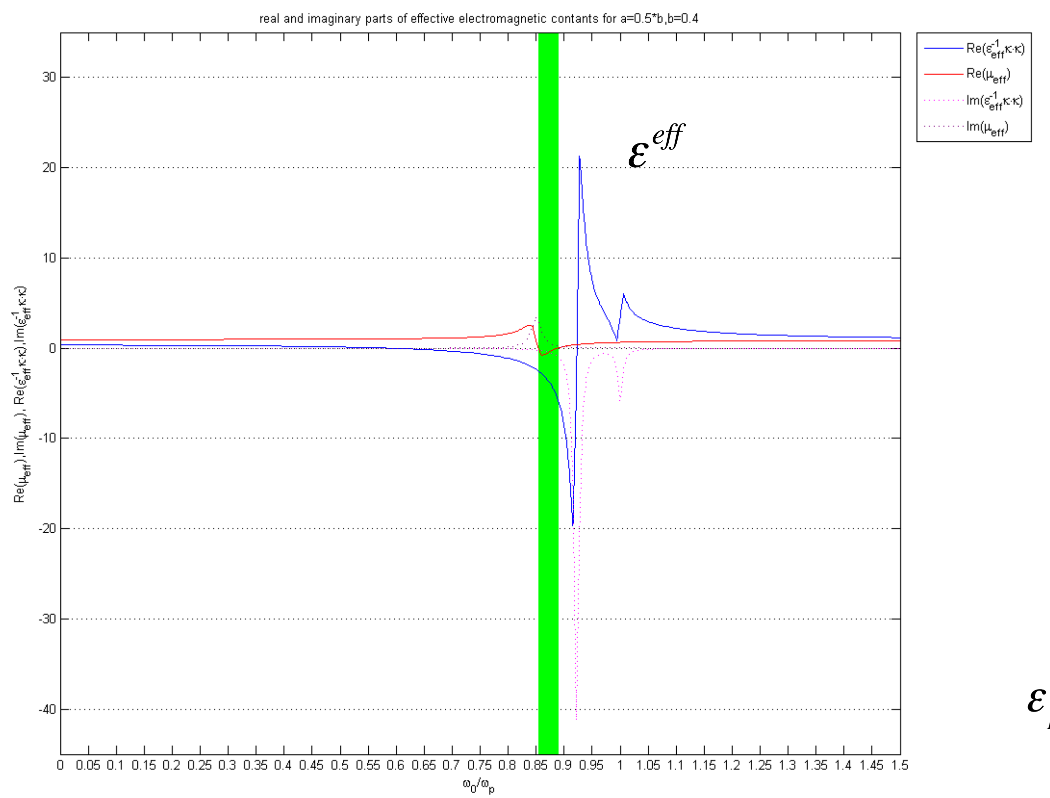
Chen, Lipton (Multiscale Modeling and Simulation) **under revision,** arXiv:1202.0602v1 [math.AP] 3 Feb 2012.

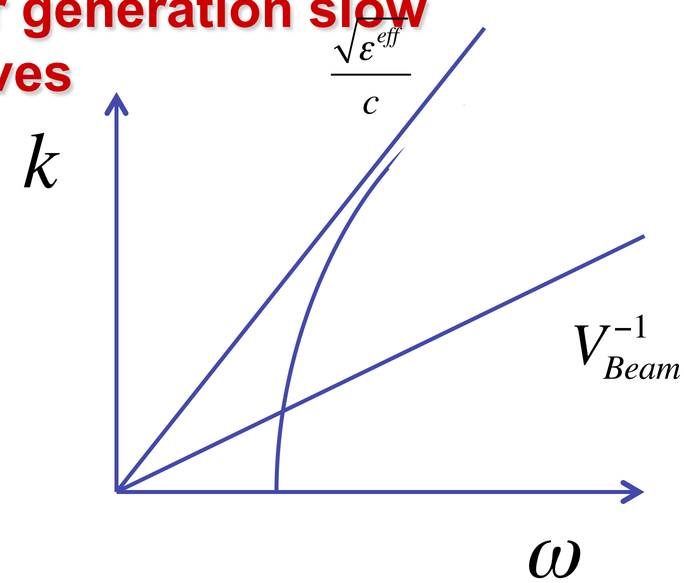
Dielectric resonances or surface modes for $\omega_0 = \omega_s = \omega_p\sqrt{\lambda_h + \frac{1}{2}}$

Analysis gives $-\frac{1}{2} < \lambda_h < \frac{1}{2}$

And $\lambda_h \approx -\frac{1}{2}$ Gives lowest surface mode ω_s near zero

Controlling λ_h provides control of surface modes and allows for the systematic design of effective dielectric functions for generation slow electromagnetic waves





$$\varepsilon_r / d = 200 + i5$$

$$\omega_c / \omega_p = 0.01$$

$$\varepsilon_p\left(\omega/\omega_p\right) = 1 - \frac{\left(\omega_0/\omega_p\right)^2}{\left(\omega/\omega_p\right)^2 - i\left(\omega_c/\omega_p\right)\left(\omega/\omega_p\right)}$$

Plans

Development of a systematic theory & methodology for design of sub-wavelength metallic structures for dielectric response for generating slow electromagnetic waves.

Method will address the control of surface modes through the tuning of sub-wave length geometry for control of electrostatic resonances. Provides a means for engineering dispersion relations for wave guides with slow waves.

Plans

The associated analysis of bandwidth for short TWT's loaded with metamaterial will be carried out following the Methods of Schachter, Nation and Kerslick J. Appl. Phys. 68 (11) 1990, 5874.

The results will be compared with direct numerical simulation for TWT's loaded with metallic sub-wavelength structures.