Lagrangian variational framework for boundary value problems

Alexander Figotin and Guillermo Reyes
*Department of Mathematics, University of California at Irvine,
Irvine, California 92697-3875, USA*

A boundary value problem is commonly associated with constraints imposed on a system at its boundary. We advance here an alternative point of view treating the system as interacting "boundary" and "interior" subsystems. This view is implemented through a Lagrangian framework that allows to account for (i) a variety of forces including dissipative acting at the boundary; (ii) a multitude of features of interactions between the boundary and the interior fields when the boundary fields may differ from the boundary limit of the interior fields; (iii) detailed pictures of the energy distribution and its flow; and (iv) linear and nonlinear effects. We provide a number of elucidating examples of the structured boundary and its interactions with the system interior. We also show that the proposed approach covers the well known boundary value problems. © *2015 AIP Publishing LLC.* [http://dx.doi.org/10.1063/1.4931135]