A NOVEL PRIMARY-SECONDARY USER POWER CONTROL GAME FOR COGNITIVE RADIOS WITH LINEAR RECEIVERS

Tianming Li*
WINLAB, Rutgers University, Piscataway, NJ 08854, USA.
Email: tmli@eden.rutgers.edu

Sudharman K. Jayaweera
Department of Electrical and Computer Engineering
University of New Mexico
Albuquerque, NM 87131-0001, USA.
Email: jayaweera@ece.unm.edu

ABSTRACT

In cognitive radio (CR) networks, proper power controlling is of importance to ensure efficient operation of both primary and secondary networks. The previous work on power control in cognitive radios has mainly focused on the interactions among a set of secondary users while ignoring the influence of primary users. In [1], we introduced a new game model for power control in cognitive radios by incorporating the primary users into the player set of the game. In this new framework, the primary users are rewarded (for example, monetarily) for sharing their licensed spectrum with secondary users. In our proposed game formulation, they achieve this by setting a reasonable interference cap (IC) for the secondary users. However, the primary users are to be severely penalized if they do not meet their transmission quality. To ensure this, an exponential pricing term is incorporated into their utility function. Simultaneously, the secondary users aim to achieve energy efficient transmissions while not introducing too much interference to the primary as well as other secondary users. In this paper, we investigate the CR system design aspects based on our proposed new game theoretic spectrum sharing framework. In particular, the performance of the system is compared under different linear detector schemes at the secondary receiver, namely, the matched filter (MF) and the linear MMSE (LMMSE) receivers.

I INTRODUCTION

Cognitive radios (CR) has been proposed as a way to improve the wasteful current static spectrum utilization by allowing secondary users to access the white spaces in spectrum already licensed to the primary users [2, 3]. Power control is an important issue in this sharing process. In [4] and [5], for example, authors proposed schemes for power control among the secondary users. However, in these models the primary users were not considered as active decision makers, i.e. they don't actively participate in the spectrum sharing process. Thus, these schemes are essentially similar to the power control schemes in traditional wireless networks [6–10].

In [1] we argued that a more reasonable formulation in cognitive radios is to consider primary users also as active decision makers in the process of spectrum sharing. Motivated by this consideration, we proposed a novel primary-secondary user power control game for cognitive networks in [1], where the primary users are also considered as active decision makers. In this new framework, the primary users are rewarded for allowing secondary users to share their licensed spectrum. Hence, they have an incentive to leave a reasonable portion of the spectrum for the secondary users when they can still meet their own minimum required Quality-of Service (QoS). Simultaneously, the secondary users aim to achieve energy efficient transmissions, while not causing excessive interference to the primary users.

The analysis in [1] was limited to the case of matched-filter based conventional receiver. In this paper, we generalize our primary-secondary user power control game for cognitive spectrum sharing to allow a linear multiuser detector at the secondary receiver. In particular, we consider the linear minimum mean squared error (LMMSE) detector. We establish the existence of a unique Nash equilibrium in the primary-secondary user power control game, even in this case. Furthermore, we investigate the effect of different linear receivers on the performance and show that the LMMSE receiver leads to a more flexible system design due to its superior performance in handling both in and out of network interference.

The remainder of this paper is organized as follows: Section II summarizes the assumed cognitive radio system model and the game model. Section III establishes the existence and the uniqueness of the Nash Equilibrium (NE) of the proposed power control game with the linear MMSE receiver. Section IV analyzes the performance of the new scheme with an LMMSE-based secondary-system receiver through numerical simulations. Performance comparison of the proposed power control game with the LMMSE and the MF receivers is also discussed in Section IV. Section V concludes the paper.

II COGNITIVE RADIO SYSTEM AND GAME MODELS

In this paper, we consider a cognitive wireless network with one primary user and K secondary users, as shown in Fig. 1. All secondary users and the primary user operate on the same frequency band. There are one primary receiver and one secondary receiver (generalization to more than one is possible). The cross correlation coefficients between the signalling waveforms of the k-th secondary user and that of a primary user is ρ_{kp}, between a primary user and the k-th secondary user is ρ_{pk} and between the k-th and the j-th secondary users is ρ_{jk}, $\forall k, j \in K$. The channel gain between the k-th secondary user and the common secondary receiver is h_{sk}, between the k-th secondary user and the primary receiver is h_{pk}, between the primary user and the primary receiver is h_{p0}, and between

*This research was completed while T. Li was at the Department of Electrical and Computer Engineering, University of New Mexico, Albuquerque, NM 87131-0001, USA.

978-1-4244-2677-5/08$25.00©2008 IEEE

the primary user and the common secondary receiver is h_{s0}.

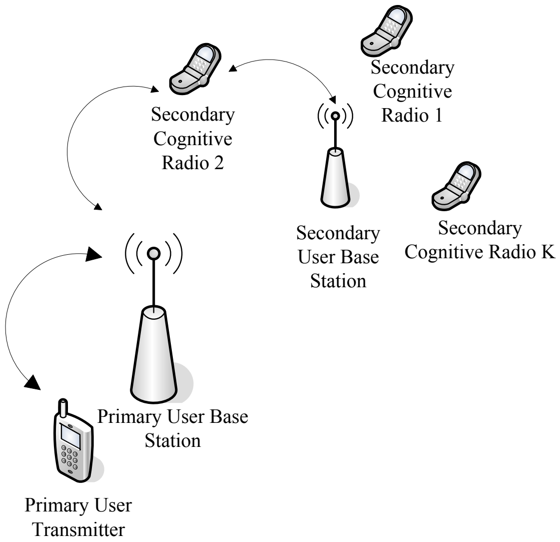

Figure 1: Cognitive radio system model.

In our proposed formulation, the primary user can adapt its Interference Cap (IC), denoted by Q_0, which is the maximum total interference the primary user is willing to tolerate from all secondary transmissions. By adjusting the IC, the primary user can control the total transmit power the secondary users impose on the channel. However, at the same time, the primary user should achieve its target SINR to ensure its required transmission quality. All secondary users should also adapt their transmission powers to achieve a certain transmission quality. However, their transmission powers must be carefully controlled in order to ensure low interference to primary user (within the IC) as well as to other secondary users. We use P_0 and p_k to represent transmission powers of primary user and that of the k-th secondary user, respectively.

In the proposed cognitive network, the primary and the secondary users interact with each other by adjusting the Interference Cap and transmit power levels, respectively. Hence, game theory provides a natural framework to analyze the behavior of this system. First, we define our game model $(\mathcal{K}, \mathcal{P}, \{u_k\})$:

1. Players: $\mathcal{K} = \{0, 1, 2, ..., K\}$, where 0-th user is the primary user and $k = 1, 2, ..., K$ represents the k-th secondary user.

2. Action space: $\mathcal{P} = \mathcal{Q} \times \mathcal{P}_1 \times \mathcal{P}_2 ... \times \mathcal{P}_K$, where $\mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, ..., K$, represents the k-th secondary user's action set. $\bar{Q}_0$ and $\bar{P}_k$ represent, respectively, the maximum IC of the primary user and the maximum transmission power of the k-th secondary user. The action vector of all users is $\mathbf{p} = [Q_0, p_1, ..., p_K]$, with $p_k \in \mathcal{P}_k$ and $Q_0 \in \mathcal{Q}$. The action vector excluding the k-th user, for $k = 0, 1, 2, ..., K$, is customarily denoted by $\mathbf{p}_{-k}$.

3. Utility function: We use $u_k(p_k, \mathbf{p}_{-k})$ $, \forall k = 1, 2, ..., K$ to represent the k-th secondary user's utility function and $u_0(Q_0, \mathbf{p}_{-0})$ to represent the primary user's utility function.

As we defined in [1], the primary user's utility function in the proposed primary-secondary user power control game is:

$$\begin{aligned} u_0(Q_0, \mathbf{p}_0) &= Q_0 - \mu_1 \left[(Q_0 - I_0)^2 u(Q_0 - I_0) \right] \\ &\quad -\mu_2 \left[\left(e^{(I_0 - Q_0)} - 1 \right) u(I_0 - Q_0) \right], \quad (1) \end{aligned}$$

where $u(.)$ is the step function with $u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$, $I_0 = \sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j$ is the total interference from secondary users to the primary user and μ_1 and μ_2 are positive pricing coefficients. The primary user's target SINR is defined as:

$$\bar{\gamma_0} = \frac{h_{p0}^2 P_0}{Q_0 + \sigma^2}, \quad (2)$$

where P_0 and Q_0 represent the primary user's transmission power and its chosen IC respectively, and σ^2 is the variance of the additive noise at the primary receiver. Since Q_0 is the maximum possible interference from secondary users the primary user is willing to tolerate, $\bar{\gamma_0}$ represents the least acceptable transmission quality of the primary user. Assuming a matched-filter (MF) detector at the primary receiver, the primary user's actual SINR is:

$$\gamma_0^{(P)} = \frac{h_{p0}^2 P_0}{\sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} = \frac{\bar{\gamma_0}(Q_0 + \sigma^2)}{I_0 + \sigma^2}$$

Since, $\gamma_0^{(P)} - \bar{\gamma_0} = (Q_0 - I_0) \frac{\bar{\gamma_0}}{I_0 + \sigma^2}$, if $Q_0 > I_0$, then $\gamma_0^{(P)} > \bar{\gamma_0}$, implying that the primary user's quality of service requirement is satisfied. The pricing functions (the second and the third terms in (1)) is introduced to ensure that the primary user's transmission quality is not be undermined at the expense of utility gain. When the primary user's instantaneous SINR is less than the target primary user SINR, i.e. $Q_0 < I_0$, the primary user is significantly penalized because it doesn't achieve its target transmission quality. On the other hand, when the instantaneous primary user's SINR is greater than its required target SINR, i.e. $Q_0 > I_0$ the primary is relatively penalized. This is because when the primary user achieves its target SINR, it doesn't need to transmit at a too high power that provides it with a too high an SINR, wasting its own power as well as causing more interference to all other users. When the primary user sets an IC, the shared spectrum should be fully utilized. i.e. the total interference from the secondary users should be as close as possible to that IC. Note that, here Q_0 is the action of primary user. By adjusting Q_0, primary user tries to increase its utility as much as possible. Hence, by providing the primary user the capability of adapting its action, i.e. the interference cap Q_0, we allow the primary user to be an active participant of the power control game, which is a more accurate and reasonable description of a cognitive radio system.

The secondary user's utility function is taken to be [9]:

$$u_k(p_k, \mathbf{p}_{-k}) = \frac{R_k f\left(\gamma_k^{(s)}\right)}{p_k}, \tag{3}$$

where R_k is the transmission rate of the k-th secondary user, $f\left(\gamma_k^{(s)}\right) = \left(1 - e^{\left(-0.5\gamma_k^{(s)}\right)}\right)^M$ is the efficiency function, $\gamma_k^{(s)}$ and p_k are the k-th secondary user's SINR and transmission power, respectively, and M is the number of bits in one packet.

III Existence and Uniqueness of the Nash Equilibrium of the Power Control Game with Linear Receivers

In [1], we proved the existence and uniqueness of the Nash Equilibrium in the above power control game with a matched-filter based secondary-system receiver. In the following, we generalize this game model to the case in which the secondary system receiver is based on the LMMSE detector and establish the existence and uniqueness of the Nash equilibrium.

A. *LMMSE-based secondary-system receiver*

We assume that the secondary system is are equipped with an LMMSE receiver, while that of the primary system is based on an MF receiver. Note that, it is also possible for the primary receiver to be equipped with an LMMSE detector, if we were to further generalize this model to include more than one primary user. In current formulation with only a single primary user, this is not necessary since the primary user and the secondary users are not in the same system and thus the primary receiver may not have information regarding secondary user signaling waveforms.

The received signal at the secondary receiver can be written as

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \Theta A_0 b_0 s_0(t) + \sigma n(t),$$

where $A_k = h_{sk}^2 p_k$, b_k and $s_k(t)$ are the k-th secondary user's received signal amplitude, transmitted symbol and the signalling waveform, respectively. Further, $A_0 = h_{s0}^2 P_0$, b_0 and $s_0(t)$ are the primary user's received signal amplitude, transmitted symbol and the signalling waveform, respectively. In (5), $n(t)$ is white Gaussian noise with unit variance. The random variable Θ, assumed to be Bernoulli with a parameter p, is introduced to denote that in an overlay CR system the primary user interferes with secondary transmissions only when secondary users make an error in detecting white spaces. Note that, in an overlay cognitive radio system, secondary users seek white spaces to transmit via spectrum sharing. However, there may be sensing errors that can lead to erroneous detection of white spaces with a probability p. Thus, p is the probability of collision of the transmissions from a secondary user with that of the primary user. On the other hand, in an underlay system, we may assume that $\Theta = 1$ with probability 1 since secondary users are assumed to be always active simultaneously with the primary user. By projecting $r(t)$ onto a set of N orthonormal signals $\{\psi_1, \psi_2, ..., \psi_N\}$ defined on $[0, T]$, where T is the symbol duration, we obtain the following discrete time model:

$$\mathbf{r} = \sum_{j=1}^{K} A_j b_j \mathbf{s_j} + \Theta A_0 b_0 \mathbf{s_0} + \sigma \mathbf{m},$$

where $\mathbf{s}_k = [s_{k1}, ..., s_{kn}]$ with $s_{kl} = \int_0^T s_k(t)\psi_l(t)dt,\ \forall k = 0, 1, 2..., K$ and $\mathbf{m}$ is an N-dimensional Gaussian vector with independent, zero-mean and unit-variance components.

For detecting the k-th secondary user, the common secondary receiver employs the following LMMSE filter:

$$\min_{\mathbf{w}_k, \mathbf{s}_0} E[(b_k - \mathbf{w_k r})^2] \quad s.t.\ \ E[\Theta]\mathbf{S}^T \mathbf{s}_0 = \underline{\rho}_p, \tag{4}$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_K]$ is an $N \times K$ matrix and $\underline{\rho}_p = [\rho_{p1}, \rho_{p2}, ..., \rho_{pK}]^T$ is the effective cross-correlation vector between the primary user and secondary users. Note that,

$$E[\mathbf{r}\mathbf{r}^T] = \sum_{j=1}^{K} A_j^2 \mathbf{s}_j \mathbf{s}_j^T + E[\Theta^2] A_0^2 \mathbf{s_0} \mathbf{s}_0^T + \sigma^2 \mathbf{I},$$

and

$$E[b_k \mathbf{r}] = A_k \mathbf{s}_k.$$

The LMMSE filter solution to (4) is given by:

$$\mathbf{w_k} = E[\mathbf{r}\mathbf{r}^T]^{-1} E[b_k \mathbf{r}] = \frac{A_k}{1 + A_k^2 \mathbf{s}_k^T \mathbf{\Sigma}_k^{-1} \mathbf{s}_k} \mathbf{\Sigma}_k^{-1} \mathbf{s}_k, \tag{5}$$

where

$$\begin{aligned}\mathbf{\Sigma}_k &= \sigma^2 \mathbf{I} + E[\Theta^2] A_0^2 \left((E[\Theta])^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p\right) \left((E[\Theta])^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p\right)^T + \sum_{j=1}^{K} A_j^2 \mathbf{s}_j \mathbf{s}_j^T \\ &= \sigma^2 \mathbf{I} + p A_0^2 \left(p^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p\right) \left(p^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p\right)^T + \sum_{j=1}^{K} A_j^2 \mathbf{s}_j \mathbf{s}_j^T\end{aligned}$$

where $\mathbf{S}^+$ is the pseudo-inverse of $\mathbf{S}$. Note that, in general, the secondary system may not know $\mathbf{s}_0$. However, it can estimate the cross-correlation ρ_{pk} between the primary user and the k-th secondary user. Suppose that the fixed waveform correlation between the primary and the k-th secondary user is $\rho'_{pk} = \mathbf{s}_0^T \mathbf{s}_k$, so that effectively $\rho_{pk} = E[\Theta]\rho'_{pk} = p\rho'_{pk}$. The k-th secondary user's SINR at the output of the secondary-system receiver can be written as:

$$\gamma_k^{MMSE} = A_k^2 \mathbf{s_k} \mathbf{\Sigma}_k^{-1} \mathbf{s_k} = h_{sk}^2 p_k \mathbf{s_k} \mathbf{\Sigma}_k^{-1} \mathbf{s_k}. \tag{6}$$

At this point, it should be pointed out that the only difference between the above power control game with the LMMSE receiver and that with the MF receiver considered in [1] is in the SINR expression of secondary users. It is well known that the linear MMSE receiver maximizes the output SINR. Thus, with the same target SINR constraints, the linear MMSE receiver may require secondary radios to transmit at a lower power than that with the MF receiver.

B. Existence of the NE with LMMSE

As discussed in [1], a Nash equilibrium exists in game $G = (K, \mathcal{P}, u_k(.))$, if for all $k = 0, 1, 2, ..., K$: The k-th user's action set, $\mathcal{P}_k$, is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^N$, and $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k. Here, $\mathcal{P}_0 = \mathcal{Q}$ and $p_0 = Q_0$ for the primary user.

The power action sets of the primary user and the secondary users are closed subsets of $\mathbb{R}$. Thus, the first condition is satisfied. Furthermore, it is easy to verify that the utility functions of the primary user and the secondary users are continuous in $\mathbf{p}$. Finally, the quasi-concavity of the utility function (4) of the secondary users with the LMMSE receiver is established in [9]. Note that, the extra interference from the primary user in our proposed game does not change the quasi-concavity of the secondary user utility function. Further, the quasi-concavity of the utility function of the primary user with the MF secondary receiver is proved in [1]. Since the common secondary receiver does not influence the behavior of the primary user utility function, the quasi-concavity of the primary user utility function, with the LMMSE secondary receiver still holds. Hence, it follows that there exists at least one NE in the above power control game with the LMMSE receiver.

C. Uniqueness of the NE

Definition A best response correspondence, $\forall k \in K$, $r_k : \mathcal{P}_{-k} \rightarrow \mathcal{P}_k$, is

$$r_k(\mathbf{p}_{-k}) = \{p_k \in \mathcal{P}_k : u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k}) \;\; \forall p'_k \in \mathcal{P}_k\} \quad (7)$$

The round robin best response decision rule is defined as that in every adaptation round, every player updates its action sequentially. In particular, the k-th player $\forall k \in K$, updates its action to its best response action.

To establish the uniqueness of the NE of the power control game with the LMMSE receiver, we first show that the best response correspondence $\mathbf{r}(\mathbf{p}) = (r_0(\mathbf{p}), r_1(\mathbf{p}), ..., r_K(\mathbf{p}))$ is a standard function, where $r_0(\mathbf{p})$ represents the primary user's best response correspondence and $r_k(\mathbf{p}),\ k = 1, 2, ..., K$ represents the $k-$th secondary user's best response correspondence. From [11], $\mathbf{r}(\mathbf{p})$ is a standard function if it satisfies the following conditions:

1. positivity: $\mathbf{r}(\mathbf{p}) > 0$
2. monotonicity: if $\mathbf{p} \geq \mathbf{p}'$, then $\mathbf{r}(\mathbf{p}) \geq \mathbf{r}(\mathbf{p}')$
3. scalability: for all $\mu > 1,\ \mu\mathbf{r}(\mathbf{p}) > \mathbf{r}(\mu\mathbf{p})$.

where, all inequalities above are to be taken component-wise, i.e. $\mathbf{r}(\mathbf{p}) \geq \mathbf{r}(\mathbf{p}') \Leftrightarrow r_k(\mathbf{p}) \geq r_k(\mathbf{p}') \;\; \forall k \in K$.

In the power control game with the LMMSE receiver, the primary user's utility function stays the same as that in [1]. Thus, the discussion regarding the primary user utility function in [1] are still valid. In particular, the best response correspondence of the primary user utility function was shown to be a standard function in [1]. Hence, in the following, we only need to show that the best response correspondence of the secondary user utility function is a standard function.

Before proving the best-response of the secondary users is a standard function, we first state the the following Proposition that was proved in [12]. It will be needed in the sequel.

Proposition If two $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ are both real, symmetric and positive definite, such that $\mathbf{B} - \mathbf{A} \geq \mathbf{0}$ (i.e. $\mathbf{B} - \mathbf{A}$ is positive semi-definite), then $\mathbf{A}^{-1} - \mathbf{B}^{-1} \geq \mathbf{0}$. In particular, when $\mathbf{B} - \mathbf{A} > \mathbf{0}$, than $\mathbf{A}^{-1} - \mathbf{B}^{-1} > \mathbf{0}$.

From the discussion in [1], the best response correspondence of the k-th secondary user is the transmit power which provides it with the optimum SINR γ^*, where $f'(\gamma^*)\gamma^* = f(\gamma^*)$ and $f(.)$ is the efficiency function defined earlier. Then, from [12], the best response correspondence of the k-th secondary user is

$$r_k^*(\mathbf{p}) = \frac{\gamma^* I_k}{h_{sk}^2},$$

where $I_k = \left(\mathbf{s}_k^T \mathbf{\Sigma}_k^{-1} \mathbf{s}_k\right)^{-1}$.

1. positivity: Since $\gamma^* > 0, I_k > 0$, the best response correspondence of the k-th secondary user satisfies
$$r_k^*(\mathbf{p}) = \frac{\gamma^* I_k}{h_{sk}^2} > 0.$$
2. monotonicity: By following a proof similar to that in [12], if $\mathbf{p} \geq \mathbf{p}'$, then $p_k > p_{k'}$, for $\forall k = 0, 1, ..., K$. Hence $\mathbf{\Sigma_k}(\mathbf{p}) - \mathbf{\Sigma_k}(\mathbf{p}') \geq 0$. From Proposition 1, we then have
$$\begin{aligned} &\mathbf{\Sigma_k}(\mathbf{p})^{-1} - \mathbf{\Sigma_k}(\mathbf{p}')^{-1} \leq 0, \\ &\Leftrightarrow \mathbf{s}_k^T \mathbf{\Sigma_k}(\mathbf{p})^{-1} \mathbf{s}_k - \mathbf{s}_k^T \mathbf{\Sigma_k}(\mathbf{p}')^{-1} \mathbf{s}_k \leq 0, \\ &\Leftrightarrow I_k(\mathbf{p}') \leq I_k(\mathbf{p}). \end{aligned}$$
Thus
$$\begin{aligned} r_k^*(\mathbf{p}) &= \frac{\gamma^* I_k(\mathbf{p})}{h_{sk}^2} \\ &\geq \frac{\gamma^* I_k(\mathbf{p}')}{h_{sk}^2} = r_k^*(\mathbf{p}'). \end{aligned}$$
3. scalability: For $\mu > 1$,
$$\mu r_k^*(\mathbf{p}) = \frac{\mu \gamma^* I_k(\mathbf{p})}{h_{sk}^2},$$
and
$$r_k^*(\mu\mathbf{p}) = \frac{\gamma^* I_k(\mu\mathbf{p})}{h_{sk}^2}.$$
From [12], by using Proposition 1, we have that $\mu I_k(\mathbf{p}) > I_k(\mu\mathbf{p})$. Hence,
$$\mu r_k^*(\mathbf{p}) > r_k^*(\mu\mathbf{p}).$$

Hence, the power control game with an LMMSE-based secondary-system receiver also has a unique Nash equilibrium. In particular, the game converges to this unique NE via the round robin best response decision rule.

In the above discussion, for the simplicity of exposition , we have assumed that $\bar{P}_k = +\infty$ and $\bar{Q}_0 = +\infty$. However, in practice, $\bar{P}_k$ and $\bar{Q}_0$ are finite. In this case, the best response correspondence of the k-th secondary user is $\min\left(\bar{P}_k, p_k^*\right)$ where p_k^* is the transmit power which provides the k-th secondary user the optimum SINR γ^*. Similarly, the best response correspondence of the primary user is $\min\left(\bar{Q}_0, Q_0^*\right)$ where $Q_0^* = \frac{1}{2\mu_1} + I_0$ is the best-response of the primary user. In this case, the NE is still unique.

IV SYSTEM PERFORMANCE ANALYSIS

We assume that $\rho_{pk} = \rho_{kp} = \rho_{sp} = \rho_{ps}$ and all channels are static and symmetric during the period of consideration. Following parameter values are used in all numerical simulations: $h_{pk} = 1$, $h_{sk} = 1$, $\rho_{kp} = \rho_{pk} = 0.1$,$\forall k \in K$, $\rho_{jk} = 0.1$, $\forall j,k \in K$, $h_{s0} = 1$, $h_{p0} = 1$, $M = 80$, $\mu_1 = 10$ and $\mu_2 = 100$, and $\sigma^2 = 1$. First, we investigate the performance of the system with the LMMSE receiver. Next, we will compare the overall performance of the systems with the MF and LMMSE detectors at the secondary receiver.

A. *System performance with the LMMSE receiver*

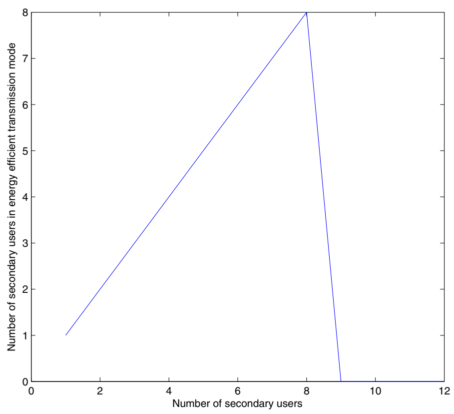

Figure 2: Number of secondary users in energy efficient mode.

In Figs. 2 and 3, we have used $\bar{P}_k = 13$, $\forall k \in K$ and $\bar{\gamma}_0 = 10$ where $\bar{\gamma}_0$ is the required minimum SINR of the primary user. Figure 2 shows the number of secondary users that can achieve the optimum SINR γ^* as a function of the total number of secondary users K. Note that γ^* is the SINR that provides secondary users with the maximum utility. The k-th secondary user suffers the interference not only from other secondary users but also from the primary user. Thus, beyond a certain K, some of the secondary users will not be able to achieve γ^*. In our simulation, for the simplicity of exposition, we have assumed a non-fading scenario so that the fading coefficients of all secondary users are set to be equal to unity. As can be seen from Fig. 2, when $K > 8$, none of the secondary users can transmit in the energy efficient mode that is determined by the secondary user utility function.

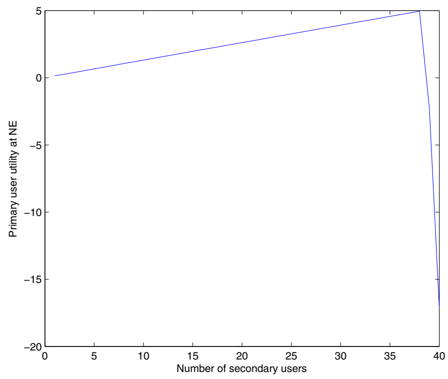

Figure 3: Primary user's utility at the NE.

Figure 3 shows the primary user's utility at the NE as the number of secondary users increases. As can be seen from Fig. 3, when $K \geq 39$, we have $\bar{Q}_0 \leq I_0$ and the primary user cannot achieve its target SINR. Then, the primary user is severely (exponentially) penalized. Thus, the maximum possible number of secondary users that can be afforded by the primary user in this system is $K = 39$. For $K \geq 39$ in Fig. 3, $\bar{Q}_0 \leq I_0$, meaning that the total interference from the secondary users has exceeded the maximum interference $\bar{Q}_0$ that the primary user can tolerate. Beyond $K = 39$, strictly no additional secondary user should be allowed to be added to the system. Otherwise, the primary user's transmission is significantly undermined as reflected by its utility in Fig. 3.

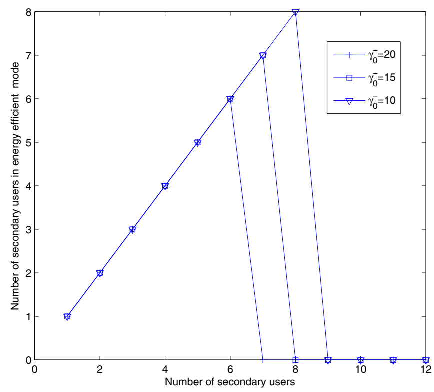

Figure 4: Influence of the primary user's target SINR on the secondary users performance.

In the proposed scheme, the primary user has a target SINR that is determined by its transmission quality requirement which we have denoted by $\bar{\gamma}_0$. If total interference from all secondary users is below the interference cap the primary user sets, the primary user can achieve this target SINR and still gain a positive utility. Otherwise, the primary user cannot achieve its transmission quality and its utility decays very fast (in fact, exponentially). Thus, the target SINR $\bar{\gamma}_0$ determines the primary user's willingness to allow the secondary users to share the spectrum. If the primary user needs a low target SINR, more secondary users can operate in this system and vice versa.

In Fig. 4, we have shown the number of the secondary users that can achieve optimum SINR γ^*, for different values of target SINR $\bar{\gamma}_0$ of the primary user. As the primary user's target SINR increases, primary user imposes more interference on secondary user transmissions. This can be seen from the definition of the primary user's target SINR given in (2). Given all other parameters are fixed, the primary user's transmit power P_0 increases linearly with respect to $\bar{\gamma}_0$. From the design of the power control game, the interference the primary user causes towards the secondary users is a linear function of its target SINR.

B. Performance comparison between different linear receivers

It is well known that the LMMSE receiver has a better multiuser interference suppression capability than the MF receiver. Thus, in the same interference environment, the LMMSE receiver can be expected to support secondary users to achieve γ^* with lower transmit power. This will lead to reduced interference due to secondary users at the primary receiver. In return, the primary user will cause less interference to secondary users since it will be able to achieve its required quality-of-service at a reduced transmit power level. Thus, the LMMSE receiver's superior interference suppression capability can lead to a system in which more secondary users can achieve energy efficient transmissions. This is shown in Fig. 5. Note that, in Figs. 5 and 6, $\bar{P}_k = 20,\ \forall k \in K$ and $\bar{\gamma}_0 = 10$.

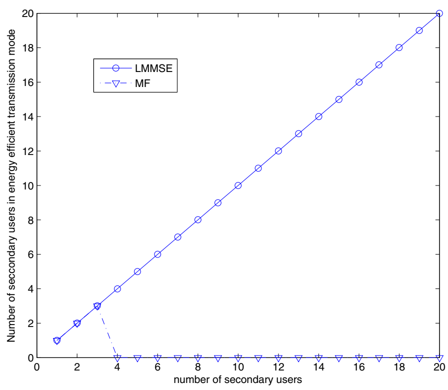

Figure 5: Number of secondary users that can transmit in the energy efficient mode

As can be seen from Fig. 5, with the MF receiver only 3 secondary users can achieve energy efficient transmissions. However, more than 20 secondary users can be supported to achieve the optimum SINR γ^* with the LMMSE-based secondary-system receiver.

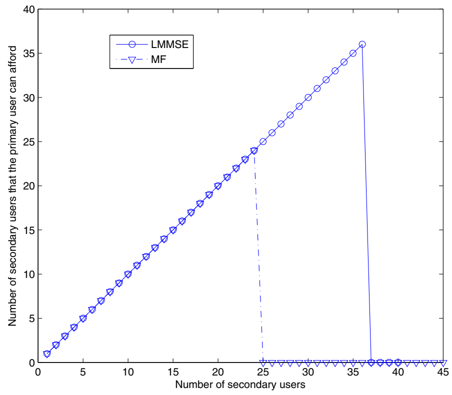

Figure 6: The maximum number of secondary users that can be afforded in the system

In Fig. 6, the maximum number of secondary users that can be afforded by the primary user system so that $\bar{Q}_0 > I_0$ is shown. As can be seen from Fig. 6, when an MF receiver is used, only 24 secondary users can be afforded. When $K > 24$, the interference the secondary users cause exceeds the maximum possible interference cap $\bar{Q}_0$ that the primary user can tolerate. However, with the LMMSE receiver, primary user can afford up to $K \leq 37$ secondary users. The reason is due to the superior interference suppression capability of the LMMSE receiver. When $K > 3$, all secondary users are already transmitting at their maximum power $\bar{P}_k$, for $\forall k \in K$, with the MF receiver. However, when the LMMSE receiver is used, the secondary users can achieve the optimum SINR γ^* with some transmit power $p_k < \bar{P}_k$ up to a much larger value of K. Thus, given the same $\bar{Q}_0$, the system with the LMMSE-based secondary receiver will afford more secondary users than that with the MF receiver. Combined, the results in Figs. 5 and 6 show that the system with the LMMSE receiver can support more secondary users to achieve the energy efficient transmissions. Further, the LMMSE receiver allows more secondary users to be admitted to the system with some level of transmit power, although not necessarily at the energy efficient level, before primary system cannot afford them.

V Conclusion

In this paper, we generalized our previously proposed new game theoretic formulation for power control in a cognitive spectrum sharing system in [1] to the situation where secondary system is equipped with a common LMMSE receiver. We established the existence and the uniqueness of the Nash equilibrium in this modified primary-secondary user power control game. Through numerical simulations, we showed that the system with the LMMSE receiver has superior performance than that with the MF receiver, in terms of both the number of secondary users that can be allowed to transmit with some quality-of-serive as well as the number of secondary users that can transmit in the energy-efficient mode.

REFERENCES

[1] T. Li and S. K. Jayaweera, "A novel primary-secondary user power control game for cognitive radios," in *the 2008 International Symposium on Information Theory and its Applications (ISITA2008)*, Auckland, NZ, Dec. 2008.

[2] J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.

[3] "Report of the spectrum eciency working group," FCC spectrum policy task force, Tech. Rep., Nov. 2002.

[4] J. O'Daniell, "Analysis and design of cognitive radio networks and distributed radio resource management algorithms," Ph.D. dissertation, Virginia Tech., 2006.

[5] T. P. W. Wang, Y. Cui and W. Wang, "Noncooperative power control game with exponential pricing for cognitive radio network," in *Vehicular Technology Conference (VTC2007-Spring)*, April 2007.

[6] F. Meshkati, H. V. Poor, and S. C. Schwartz, "Energy-efficient resource allocation in wireless networks: An overview of game-theoretic approaches," *IEEE Signal Proc. Magazine*, pp. 58–68, May 2007.

[7] D. J. Goodman and N. B. Mandayam, "Power control for wireless data," *IEEE Personal Commun.*, vol. 7, pp. 48–54, April 2000.

[8] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, Feb. 2002.

[9] F. Meshkati, H. V. Poor, S. C. Schwartz, and N. B. Mandayam, "An energy-efficient approach to power control and receiver design in wireless data networks," *IEEE Trans. Commun.*, vol. 53, no. 11, Nov. 2005.

[10] F. Meshkati, A. J. Goldsmith, H. V. Poor, and S. C. Schwartz, "A game-theoretic approach to energy-efficient modualtion in CDMA networks with dealy QoS constraints," *IEEE Trans. Commun.*, vol. 25, no. 6, Aug. 2007.

[11] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, Sep. 1995.

[12] T. Li and S. K. Jayaweera, "Analysis of linear receivers in a target SINR game for wireless cognitive networks," in *Proc. IEEE WICOM'08*, Dalian, China, Oct. 2008.