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A Novel Primary-Secondary User Power Control Game for Cognitive Radios

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Abstract

In cognitive radio networks, secondary users need to access the licensed spectrum while neither disturbing the transmissions of the primary users nor introducing much interference to other secondary users. Hence, proper power controlling is of paramount importance to ensure efficient system operation. The previous work on power control in cognitive networks mainly focuses on the interactions among a set of secondary users. The primary users' influence is rarely considered. In this paper, a new power control game model is formulated, by including the primary users into the player set of the game. The primary users are rewarded for sharing their licensed spectrum with secondary users by setting a reasonable interference cap for the secondary transmissions. However, we ensure a minimum required Quality of Service (QoS) for primary users by severely penalizing their utility if their required transmission quality is not met. Simultaneously, the secondary users achieve energy efficient transmissions while not introducing much interference to both the primary and other secondary users. We prove that the proposed cognitive power control game has a unique Nash equilibrium (NE). The numerical examples show that the proposed game theoretical power control algorithm can provide the secondary users with energy efficient transmissions and the primary users with reasonable monetary rewards, while not compromising their required QoS.

1. INTRODUCTION

Game theory is a collection of tools for analyzing the interactions among rational decision makers. In a wireless network, different users are the players who compete for accessing the spectrum. Many researchers have used the game theoretical methods to analyze the resource allocation in wireless networks. For example, in [1], the authors proposed an energy efficient utility function that was shown to have a unique Nash Equilibrium (NE). In [2], by realizing the NE in the game in [1] may not be optimum, the authors further introduced the concept of Pareto efficiency into the game. They imposed a linear pricing function to gain better overall performance. This energy efficient game was generalized to Linear Minimum Mean Squared Error receiver (LMMSE) in [3], and showed that the modified game also converges to a unique NE due to the quasi-concavity property of the utility function. In [4], the authors generalized this game further by considering the QoS constraints. A summary on game theoretical approaches used in the energy efficient resource allocation in wireless networks can be found in [5].

Cognitive radios is proposed to improve the current static wasteful spectrum utilization by allowing secondary users to access the white spaces in spectrum already allocated to the primary users. Power control is an important issue in this sharing process. In [6] and [7], authors proposed schemes for power control among the secondary users. However, the primary users were not considered as a decision makers, i.e. they don't participate in the spectrum sharing process. Thus, these schemes are similar to the power control schemes in traditional wireless networks. In this paper, on the other hand, we propose a novel power control scheme for cognitive networks where the primary users are also considered as decision makers. They are rewarded for allowing secondary users to share their licensed spectrum. Hence, they have an incentive to leave a reasonable portion of the spectrum for the secondary users when they can still meet their own minimum required QoS. Simultaneously, the secondary users aim to achieve energy efficient transmissions, where they can achieve transmission goals while not causing excessive interference to the primary users.

The remainder of this paper is organized as follows: Sections 2 and 3 introduce the system and game models, respectively. Section 4 extensively defines the utility functions of the primary and secondary users. Section 5 proves that this game has a unique NE guaranteeing that the game is convergent under the best response adaptation. In section 6, we numerically analyze the performance of a dynamic spectrum sharing system under the proposed power control game.

2. SYSTEM MODEL

For the simplicity in exposition, in this paper, we consider a cognitive wireless network with one primary user and K secondary users that form a DS-CDMA system. All secondary users and the primary user operate on the same frequency band. There are one primary user receiver and one secondary user receiver (Generalization to more than one is possible). The cross correlation coefficients between the signalling waveforms of a secondary user and that of a primary user is ρ_{sp}, between a primary user and a secondary user is ρ_{ps}, and the k-th and the j-th secondary users is $\rho_{j,k}$, $\forall k, j \in K$. We assume that all channels are static and symmetric during the period of consideration. The channel gain between the k-th secondary user and the common secondary receiver is h_{sk}, between the k-th secondary user and the primary user receiver (PR) is h_{pk}, between the primary user and the PR is h_{p0}, and between the primary user and the common secondary receiver is h_{s0}.

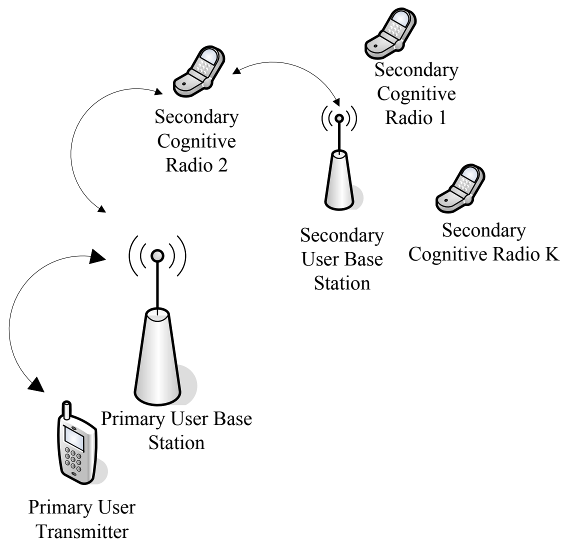

Figure 1: System model

In our proposed formulation, the primary user can adapt its Interference Cap (IC), Q_0, which is defined as the maximum total interference the primary user is willing to tolerate from secondary transmissions. By adjusting the IC, the primary user can control the total transmit power the secondary users impose on the channel. However, at the same time, the primary user should achieve its target SINR to ensure its required transmission quality. All secondary users should also adapt their transmission powers to achieve a certain transmission quality. However, their transmission powers must be carefully controlled in order to ensure low interference to the primary user (within the IC) as well as to other secondary users. We use P_0 and p_k to represent transmission powers of primary user and the k-th secondary user.

3. GAME MODEL

In our proposed cognitive network, the primary user and the secondary users interact with each other by adjusting their own actions. Hence, game theory provides a natural framework to analyze the behavior of this system. First, we are to define the game model:

1. Players: $\mathcal{K} = \{0, 1, 2, ..., K\}$, where 0-th user is the primary user and $k = 1, 2, ..., K$ represents the k-th secondary user.

2. Action space: $\mathcal{P} = \mathcal{Q} \times \mathcal{P}_1 \times \mathcal{P}_2 ... \times \mathcal{P}_K$, where $\mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, ..., K$, represents the k-th secondary user's action set. $\bar{Q}_0$ and $\bar{P}_k$ represent the maximum IC of the primary user and the maximum transmission power of the k-th secondary user. The action vector of all users is: $\mathbf{p} = [Q_0, p_1, ..., p_K]$, $p_k \in \mathcal{P}_k$ and $Q_0 \in \mathcal{Q}$. The action vector excluding the k-th user, for $k = 0, 1, 2, ..., K$ is denoted by $\mathbf{p}_{-k}$.

3. Utility function: We use $u_k(p_k, \mathbf{p}_{-k})$, $\forall k = 1, 2, ..., K$ to represent the k-th secondary user's utility function and $u_0(Q_0, \mathbf{p}_{-0})$ to represent the primary user's utility function.

4. UTILITY FUNCTIONS

The primary user's target SINR is defined as:

$$\bar{\gamma_0} = \frac{h_{p0}^2 P_0}{Q_0 + \sigma^2}, \tag{1}$$

where P_0 and Q_0 represent the primary user's transmission power and IC respectively, and σ^2 is the variance of the additive noise at the primary receiver. Since Q_0 is the maximum possible interference from secondary users the primary user is willing to tolerate, $\bar{\gamma_0}$ represents the least acceptable transmission quality of the primary user. Assuming a Matched Filter (MF) detector at the primary receiver, the primary user's actual

SINR is:

$$\gamma_0^{(P)} = \frac{h_{p0}^2 P_0}{\sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} = \frac{\bar{\gamma}_0 Q_0}{\sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} + \frac{\bar{\gamma}_0 \sigma^2}{\sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} \quad (2)$$

Similarly, the k-th secondary user's received SINR at the common secondary receiver is:

$$\gamma_k^{(s)} = \frac{h_{sk}^2 p_k}{\sum_{j\neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \sigma^2 + h_{s0}^2 \rho_{ps}^2 P_0} = \frac{h_{sk}^2 p_k}{\sum_{j\neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0 Q_0}{h_{p0}^2} + \sigma^2 \left(1 + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0}{h_{p0}^2}\right)} \quad \forall k = 1, 2, ..., K, \quad (3)$$

where in obtaining (3) we have used (1).

4.1. Secondary User Utility

Since the secondary users' transmissions in the cognitive networks are interference to the primary user, they should maximize their transmission energy efficiency, i.e. use the smallest amount of transmission power to achieve the best transmission quality. Thus, a suitable utility function for the k-th secondary user is given by [2],

$$u_k(p_k, \mathbf{p}_{-k}) = \frac{R_k f\left(\gamma_k^{(s)}\right)}{p_k}, \quad (4)$$

where R_k is the transmission rate of the k-th secondary user, $f\left(\gamma_k^{(s)}\right) = \left(1 - e^{\left(-0.5\gamma_k^{(s)}\right)}\right)^M$ is the efficiency function, $\gamma_k^{(s)}$ and p_k are the k-th secondary user's SINR and transmission power, respectively, and M is the number of bits in one packet.

4.2. Primary User Utility

In our formulation, the dynamic spectrum sharing is encouraged by rewarding the primary users for allowing secondary users to operate. However, the transmission quality of the primary user must always be satisfied. Since, $\gamma_0^{(P)} - \bar{\gamma}_0 = \left(Q_0 - \sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j\right) \frac{\bar{\gamma}_0}{\sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2}$, if $Q_0 > \sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j$, then $\gamma_0^{(P)} > \bar{\gamma}_0$, implying that the primary user's quality of service requirement is satisfied. Thus, we define the primary user's utility function as:

$$u_0(Q_0, \mathbf{p}_0) = Q_0 - \mu_1 \left[(Q_0 - I_0)^2 u(Q_0 - I_0) \right] - \mu_2 \left[\left(e^{(I_0 - Q_0)} - 1\right) u(I_0 - Q_0) \right], \quad (5)$$

where $u(.)$ is the step function with $u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$, $I_0 = \sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j$ is the total interference from secondary users to the primary user and μ_1 and μ_2 are positive pricing coefficients.

The pricing functions (the second and the third terms in (5)) are introduced to ensure that the primary user's required QoS is not be undermined. When the primary user's instantaneous SINR is less than the target SINR, i.e. $Q_0 < I_0$, the primary user is significantly penalized because it doesn't achieve its required transmission quality. On the other hand, when its instantaneous SINR is greater than the target SINR, i.e. $Q_0 > I_0$, the primary user is relatively penalized because when the primary user achieves its target SINR, it doesn't need to transmit at too high a power wasting its own power as well as causing more interference to all other users. In other words, when the primary user sets an IC, the shared spectrum should be fully utilized. i.e. the total interference from the secondary users should be as close as possible to that IC.

5. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

5.1. Existence of the NE

From the NE existence theorem 11 in [2], a Nash equilibrium exists in game $G = (K, \mathcal{P}, u_k(.))$, if for all $k = 0, 1, 2, ..., K$: The k-th user's action set, $\mathcal{P}_k$, is a nonempty convex, and compact subset of some Euclidean space $\mathbb{R}^N$, and $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k. Here, $\mathcal{P}_0 = \mathcal{Q}$ and $p_0 = Q_0$ for the primary user.

The power action sets of the primary user and the secondary users are closed subsets of $\mathbb{R}$. Thus, the first condition is satisfied. Furthermore, it's easy to check that the utility functions of the primary user and the secondary users are continuous in $\mathbf{p}$. Finally, since the quasi-concavity of the utility function of the secondary users have been proved in [2], we only need to show the quasi- concavity and the continuity of the utility function of the primary user. Obviously u_0 is continuous in $\mathbf{P}$. Furthermore, when $0 \leq Q_0 \leq I_0$, the primary user's utility function reduces to $u_0 = Q_0 + \mu_2(1 - e^{-(Q_0 - I_0)})$. The second order derivative is $u_0'' = -\mu_2 e^{-(Q_0 - I_0)} < 0$. Thus, it is concave in Q_0. On the other hand, when $I_0 \leq Q_0$, the second order derivative of the primary user's utility function is $u_0'' = -2\mu_1 < 0$, so as can be seen in Fig. 2, the utility function is again concave in Q_0.

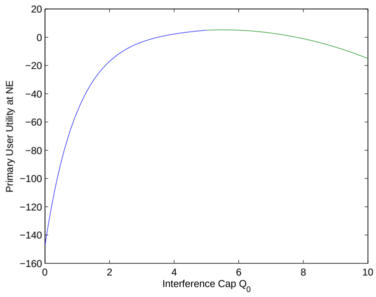

Figure 2: Quasi-Concavity of the primary user's utility function. $I_0 = 5$

Therefore, the utility functions of the primary users and the secondary users can be shown to satisfy all the required conditions, so that there exists at least one NE in this game. In the following, we show that, in fact, this NE is unique.

5.2. Uniqueness of the NE

It has been established in [8] that if the best response correspondences of the primary and the secondary users are standard functions, then the NE in this game will indeed be unique.

The best response correspondence of the secondary users has been shown to be a standard function in [2]. For completeness, below we briefly discuss this.

The best response correspondence of the secondary users can be obtained by setting $u'_k(p_k, \mathbf{p}_{-k}) = 0$, for $k = 1, 2, 3, ..., K$, which leads to $f'(\gamma_k^{(s)})\gamma_k^{(s)} - f(\gamma_k^{(s)}) = 0$. Here,we assume that all the secondary users have the same efficiency function, so that the best response $\gamma_k = \gamma^*$ is the same for all secondary users. Hence, the best response correspondence of the k-th secondary user is the transmit power which provides it with this optimum SINR:

$$r_k^*(\mathbf{p}) = \frac{\gamma^* \left(\sum_{j \neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \sigma^2 (1 + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0}{h_{p0}^2}) + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0 Q_0}{h_{p0}^2} \right)}{h_{sk}^2}. \quad (6)$$

Note that, $r_k^*(\mathbf{p})$ can be shown to be a standard function for $\forall k = 1, 2, ..., K$ by following the similar approach in [9]. Considering the upper bound of the secondary user's action set $\bar{P}_k$, the secondary user's best response correspondence is $\min\{\bar{P}_k, p_k^*\}$, where p_k^* is the k-th secondary user's transmission power which provides it with the optimum SINR γ^*. When some of the secondary users cannot achieve γ^*, they will transmit at $\bar{P}_k$. In this case, the NE is still unique.

In the case of primary users, we first show that the best response correspondence of the primary user utility function never occurs at $Q_0 \leq I_0$. For simplicity in the exposition, below we assume $\bar{Q}_0 \Longrightarrow +\infty$. Note that,

1. when $Q_0 \leq I_0$, $u'_0(Q_0) = 1 + \mu_2 e^{(I_0 - Q_0)} > 0$. Thus, $u_0(I_0) > u_0(Q_0)$, $\forall 0 \leq Q_0 < I_0$.
2. when $Q_0 \geq I_0$, $u'_0(Q_0) = 1 - 2\mu_1(Q_0 - I_0)$. Note that u_0 is continuous in $[I_0, \bar{Q}_0]$. Then, for $I_0 \leq Q_0 < \frac{1}{2\mu_1} + I_0$, u_0 is an increasing function.

Furthermore, when $Q_0 > \frac{1}{2\mu_1} + I_0$, u_0 is a decreasing function. Hence, u_0 achieves its maximum value at $Q_0 = \frac{1}{2\mu_1} + I_0$. Thus, $r_0^*(\mathbf{p}) = \frac{1}{2\mu_1} + I_0$ is the best response correspondence of the primary user utility function. Since $I_0 = \sum_{j=1}^{K} h_{pj}^2 \rho_{sp}^2 p_j$, we have

1. $r_0^*(\mathbf{p}) > 0$, $\forall \mathbf{p} \in P$.
2. Given $\mathbf{p_1} \geq \mathbf{p_2}$, $r_0^*(\mathbf{p_1}) \geq r_0^*(\mathbf{p_2})$.
3. Given $\forall \lambda > 1$, $\lambda r_0^*(\mathbf{p}) = \lambda \frac{1}{2\mu_1} + \lambda I_0$ and $r_0^*(\lambda \mathbf{p}) = \frac{1}{2\mu_1} + \lambda I_0$. Thus, $\lambda r_0^*(\mathbf{p}) > r_0^*(\lambda \mathbf{p})$, for $\lambda > 1$.

Therefore, the best response correspondence of the primary user is a standard function. Note that, in practice, since $\bar{Q}_0$ is finite, when $\frac{1}{2\mu_1} + I_0 \geq \bar{Q}_0$, the primary user sets the IC at $\bar{Q}_0$. However, the NE is still unique in this case. In this situation, the primary user cannot afford this amount of secondary user interference even when they are not working at the energy efficient mode. Hence, the total interference from the secondary users exceeds the maximum amount that the primary user can tolerate.

6. SIMULATION RESULTS AND ANALYSIS

Following parameter values are used in all numerical simulations: $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $h_{pk} = 1$, $,\forall k \in K$, $h_{sk} = 1$, $,\forall k \in K$, $h_{s0} = 1$, $h_{p0} = 1$, $\rho_{sp} = 0.1$, $\rho_{jk} = 0.1$, $\rho_{ps} = 0.1$, $M = 80$, $\bar{\gamma}_0 = 10$, $\mu_1 = 10$ and $\mu_2 = 100$, and $\sigma^2 = 1$. In Figs 3-6 below, we first describe the behavior of the proposed system with $\rho_{jk} = 0.1$ for secondary users.

Figure 3 shows the primary user utility at the NE, as a function of the number of secondary users K. We observe that when $0 < K \leq 3$, all secondary users achieve SINR $= \gamma^*$ which maximizes their utility. When $K > 3$, the network cannot afford these secondary users, i.e. no secondary user can achieve

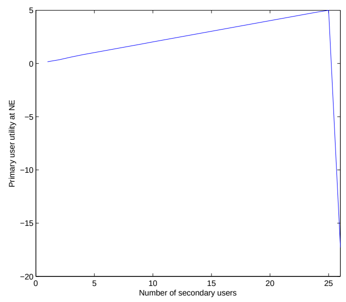

Figure 3: Primary user's utility at the NE

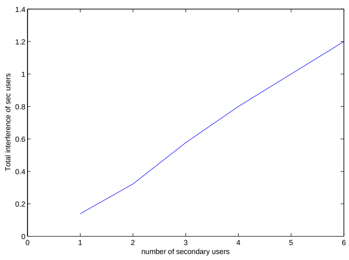

Figure 4: Total Interference from all secondary users

γ^*. Thus, they all transmit at their maximum possible power level of $\bar{P}_k$. It can be shown that the primary user's utility at its best response is $u_0 = \frac{1}{4\mu_1} + I_0$. When $K > 3$, $I_0 = K\bar{P}_k h_{pk}^2 \rho_{sp}^2$. Hence, I_0 increases linearly with K after this point and as a result the primary user's utility also increases as a linear function in K. However, when $K \geq 26$, we have that the $\bar{Q}_0 < I_0$ and the primary user's utility is severely penalized by the exponential pricing function. Figure 4 shows the total interference from the secondary users to the primary user. As discussed above, when $K > 3$, the total interference increases linearly.

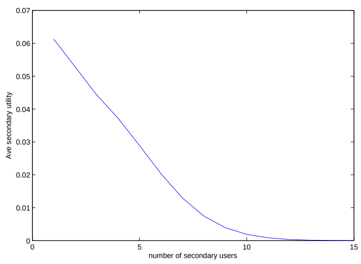

Figure 5: Average secondary user's utility at the NE

Figure 5 reveals the average secondary user utility. Note that, as the number of secondary users increases, each secondary user as well as the primary user sees more interference due to the added secondary users. Thus, to achieve the same optimum SINR, each secondary user has to transmit at a higher power than that with smaller number of secondary users in the system. As can be seen from Fig. 5, this then causes their average utility to decrease.

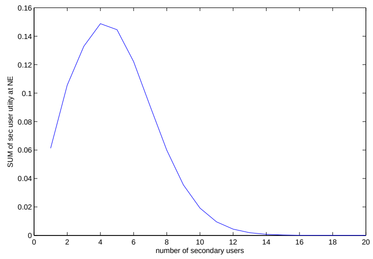

Figure 6: Sum of secondary users' utility at the NE

In Figure 6, we have shown the total utility achieved by all secondary users at the NE. We observe an interesting phenomenon: the sum of all secondary users' utility has a unique maximum at $K = 4$. As the number of secondary users increases, average secondary user utility decreases. When $K < 4$, the decease in the average secondary user utility is dominated by the increase of the number of secondary users. Thus, the summation of the secondary user utility still increases. However, when $K > 4$, the summation of the secondary users utility decreases due to the faster rate of the average secondary user utility decay.

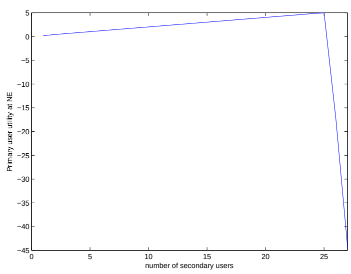

Figure 7: Primary user's utility at the NE

Figures 7-10 show the corresponding results to Figs 3-6 when $\rho_{jk} = 0.8$. From Fig 9, we observe that this system cannot afford even 2 secondary users to achieve their optimum SINR γ^* due to the high cross-correlation among the secondary users. However, the primary user can still afford the same number of secondary users as the case when $\rho_{jk} = 0.1$. This is because that the maximum number of secondary users

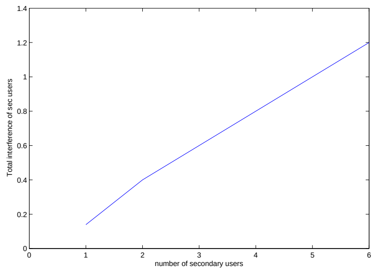

Figure 8: Total Interference from all secondary users

that the primary user can tolerate is determined by $\bar{O}_0$ and $\bar{P}_k$ and is independent of ρ_{jk}.

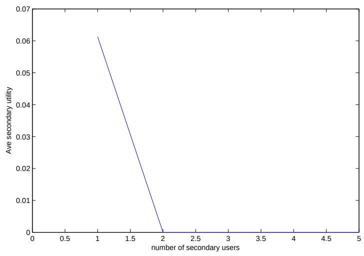

Figure 9: Average secondary user's utility at the NE

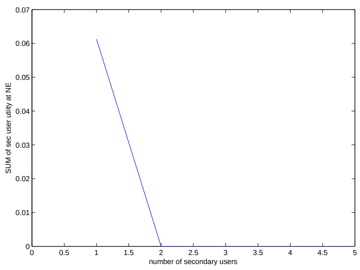

Figure 10: Sum of secondary users' utility at the NE

7. CONCLUSION

In this paper, we proposed a novel primary-secondary user power control game that is suitable for underlay cognitive radio systems. The proposed new formulation allows the primary users to be included in the group of decision makers. Assuming an MF detector of the secondary receiver, we established that the proposed game has a unique NE. The numerical examples show that under this new scheme the primary user is encouraged to share its spectrum with the secondary users while its own transmission quality is guaranteed. Simultaneously, all the secondary users will operate in the energy efficient mode under reasonable parameter settings.

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