Dynamic Spectrum Leasing (DSL) in Dynamic Channels

G. El-howayek, S. K. Jayaweera, K. Hakim,
Department of Electrical and Computer Engineering
University of New Mexico
Albuquerque, NM 87131-0001, USA
Email: {ghowayek, jayaweera, khakim}@ece.unm.edu

G. Vázquez-Vilar, and C. Mosquera
Departamento de Teoría de la Señal y Comunicaciones
Universidad de Vigo
36310 Vigo, Spain
Email: {gvazquez, mosquera}@gts.tsc.uvigo.es

***Abstract*—Dynamic Spectrum Leasing (DSL) was recently proposed in [1]–[4] as a new way to achieve dynamic spectrum sharing (DSS). Unlike previously considered dynamic spectrum access (DSA) proposals, DSL allows for the spectrum owner, called the primary user, to dynamically adjust the amount of interference it is willing to tolerate from secondary users. In response, the secondary users update their transmit powers to maximize a suitably chosen reward function. In previous work, it has been shown that the best response adaptations will converge to a unique Nash equilibrium under the assumptions of a quasi-static channel conditions and for a fixed number of secondary users. In this paper, we investigate the convergence and equilibrium performance of the proposed DSL-game in the presence of slow time-varying fading and time-varying secondary system size. Our results show that while the DSL best response adaptation algorithm is reasonably robust against these dynamics, there is a trade-off between performance and the CSI update rate.**

***Index Terms*—Cognitive radios, dynamic spectrum leasing, DSL, game theory, Rayleigh fading, time-varying channels, time-varying secondary system.**

I. INTRODUCTION

Recent studies [5] have shown that the spectrum is under-utilized in most of the licensed bands. Therefore, the engineering, the economics, and the regulation communities have started searching for better ways to fully utilize the already allocated spectrum bands. One of the solutions that have stimulated a great amount of research activities is the concept of dynamic spectrum sharing (DSS). In [6], the authors provide a game theoretical overview of dynamic spectrum sharing. DSS can be categorized under three models [3]: a) dynamic spectrum allocation, b) open sharing, and c) hierarchical sharing. In a hierarchical sharing based system, there is a primary system that owns licence rights to use a spectrum band and a secondary system that is interested in accessing this spectrum. The proposals under this category can be broadly divided along two concepts: dynamic spectrum access (DSA) and dynamic spectrum leasing (DSL). In DSA, the secondary system is solely responsible for managing the interference level either by spectrum underlay or by spectrum overlay [7]. A great deal of research has been conducted in the DSA networks such as analysis of network users' behaviors, optimality, and fairness among the secondary users. In [8], the authors model the interactions between the primary users and the secondary users as continuous-time Markov chains, by which they capture the effects of the primary user's activities on the secondary users.

In [2], the authors introduced the concept of DSL where unlike in the DSA, the primary system can also be proactive in managing the interference from the secondary system by dynamically adjusting their interference cap (IC), defined as the maximum instantaneous interference level it is willing to tolerate from all secondary users. In [1]–[4], however, the authors assumed a quasi-static environment in which the channel coefficients are constant and the secondary system size is fixed during a block length long enough for the game best response iteration to converge to an equilibrium. In this paper we study the behavior of a DSL system in a dynamic environment, specifically when the channel coefficients change with temporal correlations and the secondary system size changes due to a user arrival process with an arrival rate λ. In quasi-static conditions, the outcome of the round-robin best response iterations of the DSL-game was found in [4] to be the Nash equilibrium. In this paper we analyze the robustness characteristics of the game in a time-varying environment which forces the system to deviate from the actual Nash equilibrium.

In Section II we introduce our signal and system model for DSL in a time-varying environment. Next, Section III describes the game theoretic formulation. In Section IV, we present several simulation results to show the equilibrium in the time-varying scenario and compare it to the quasi-static environment. Finally, Section V concludes the paper by summarizing the results.

II. DSL - BASED RADIO SYSTEM MODEL

We will assume that there is a primary wireless communication system that owns the licence of the spectrum band of interest. The primary system has the rights to lease its spectrum to secondary users. We assume that there is one primary transmitter-receiver link and K_i secondary transmitter-receiver pairs (links) that are active during the i-th symbol interval. The primary user will be labeled by 0 and the K_i secondary links will be labeled 1 through K_i. The time-varying channel coefficients between the k-th transmitter and the j-th receiver is denoted by $h_{jk}(i)$ for $j, k = 0, 1, \cdots, K_i$. The primary user will change its IC, denoted by Q_0, which is the maximum interference that the primary user is willing to tolerate from the secondary users at a given time. Therefore the secondary users choose their transmit powers, denoted by p_k, in such a way that the total interference of the secondary users on the primary, I_0, is less than Q_0.

A. Signal Model

In this paper, we will assume that the primary and the secondary users are transmitting at a symbol rate $1/T$ where T is the symbol period. Let $A_{j,k}^{(i)} = h_{j,k}(i)\sqrt{p_k}$ for $j,k = 0,1,\cdots,K_i$ and σ_j^2 is the variance of j-th receiver noise. As in [2], we may obtain a discrete-time representation of the received signal at the primary receiver during the i-th symbol interval as:

$$\mathbf{r}_{0,i}^{(p)} = A_{0,0}^{(i)} b_{0,i} \mathbf{s}_0^{(p)} + \sum_{k=1}^{K_i} A_{0,k}^{(i)} b_{k,i} \mathbf{s}_k^{(p)} + \sigma_0 \mathbf{n}_0^{(i)}, \qquad (1)$$

where the vectors $\mathbf{r}_{0,i}^{(p)} = \left(r_{0,i,1}^{(p)}, \cdots, r_{0,i,M}^{(p)}\right)$ and $\mathbf{s}_k^{(p)} = \left(s_{k,1}^{(p)}, \cdots, s_{k,M}^{(p)}\right)$ are the vector representation of the received signal $r_{0,i}(t)$ and $s_k(t)$ with respect to the M-dimensional primary basis and $\mathbf{n}_0^{(i)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I_M})$. Similarly, the discrete time representation of the received signals at the j-th secondary receiver for $j = 1,\cdots,K_i$ during the i-th symbol period is

$$\mathbf{r}_{j,i}^{(s)} = \sum_{k=1}^{K_i} A_{j,k}^{(i)} b_{k,i} \mathbf{s}_k^{(s)} + A_{j,0}^{(i)} b_{0,i} \mathbf{s}_0^{(s)} + \sigma_j \mathbf{n}_j^{(i)} \qquad (2)$$

where the vectors $\mathbf{r}_{j,i}^{(s)} = \left(r_{j,i,1}^{(s)}, \cdots, r_{j,i,N}^{(s)}\right)$ and $\mathbf{s}_k^{(s)} = \left(s_{k,1}^{(s)}, \cdots, s_{k,N}^{(s)}\right)$ are the vectors representation of $r_{j,i}(t)$ and $s_k(t)$ with respect to the N-dimensional secondary basis and $\mathbf{n}_j^{(i)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I_N})$.

In the following we assume that the modulation is binary phase shift keying (BPSK), and the detectors are based on the matched filter (MF) receivers. Therefore the primary decisions are given by $\hat{b}_{0,i} = sgn\left(y_{0,i}^{(p)}\right)$ where $y_{0,i}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{r}_{0,i}^{(p)} = A_{0,0}^{(i)} b_{0,i} + \sum_{k=1}^{K_i} \rho_{0k}^{(p)} A_{0,k}^{(i)} b_{k,i} + \sigma_0 \eta_{0,i}^{(j)}$, with $\rho_{0k}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{s}_k^{(p)}$ and $\eta_{0,i}^{(j)} \sim \mathcal{N}(0,1)$. The total secondary interference $I_0^{(i)}$ from all secondary transmissions to the primary-user during the symbol i is $I_0^{(i)} = \sum_{k=1}^{K_i} \left(\rho_{0k}^{(p)}\right)^2 \left(A_{0,k}^{(i)}\right)^2$.

In the following we define the k-th link to be between the k-th secondary receiver and the k-th secondary transmitter. Without loss of generality, we assume that the k-th receiver is only interested in detecting the k-th secondary signal, for $k = 1,\cdots,K_i$ (i.e. k-th link). The decisions on the k-th link is thus given by $\hat{b}_{k,i} = sgn(y_{k,i})$ for $k = 1,\cdots,K_i$ where $y_{k,i} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{r}_{k,i}^{(s)} = A_{k,k}^{(i)} b_{k,i} + \sum_{l=1,l\neq k}^{K_i} \rho_{k,l}^{(s)} A_{k,l}^{(i)} b_{l,i} + \rho_{k,0}^{(s)} A_{k,0}^{(i)} b_{0,i} + \sigma_k \eta_{k,i}^{(s)}$, with $\rho_{k,l}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_l^{(s)}$, for $l = 0,1,\cdots,K_i$, and $\eta_{k,i}^{(s)} \sim \mathcal{N}(0,1)$. For notational convenience, the total interference from all secondary users to the k-th user signal at the k-th receiver, excluding the primary user, will be denoted by $i_k^{(i)} = \sum_{l=1,l\neq k}^{K_i} \left(\rho_{k,l}^{(s)}\right)^2 h_{k,l}^2(i) p_l$.

B. Channel Model

In the following we will assume Rayleigh fading channel coefficients with temporal correlations:

$$h_{(.,.)}(i) = \mathcal{CN}(0, \sigma_{h_{(.,.)}}^2), \qquad (3)$$

where temporal correlation is modeled as a first order Guass-Markov process [9], described via

$$h_{(.,.)}(i) = \sqrt{1-\epsilon^2}\, h_{(.,.)}(i-1) + \epsilon w_{(.,.)}(i), \qquad (4)$$

where the driving noise $w_{(.,.)}(i)$ are iid $\mathcal{CN}(0, \sigma_{h_{(.,.)}}^2)$ and ϵ is the channel variation rate. We assume that the channel state information (CSI) is not instantaneously available to the receivers, and each receiver updates the CSI periodically every L samples. The detectors decisions will thus be based on the estimated CSI defined as:

$$\hat{h}_{(.,.)}(i) = h_{(.,.)}\left(\lfloor i/L \rfloor L\right). \qquad (5)$$

C. Secondary System Size Model

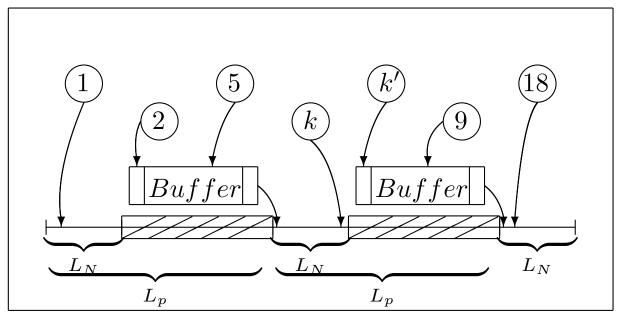
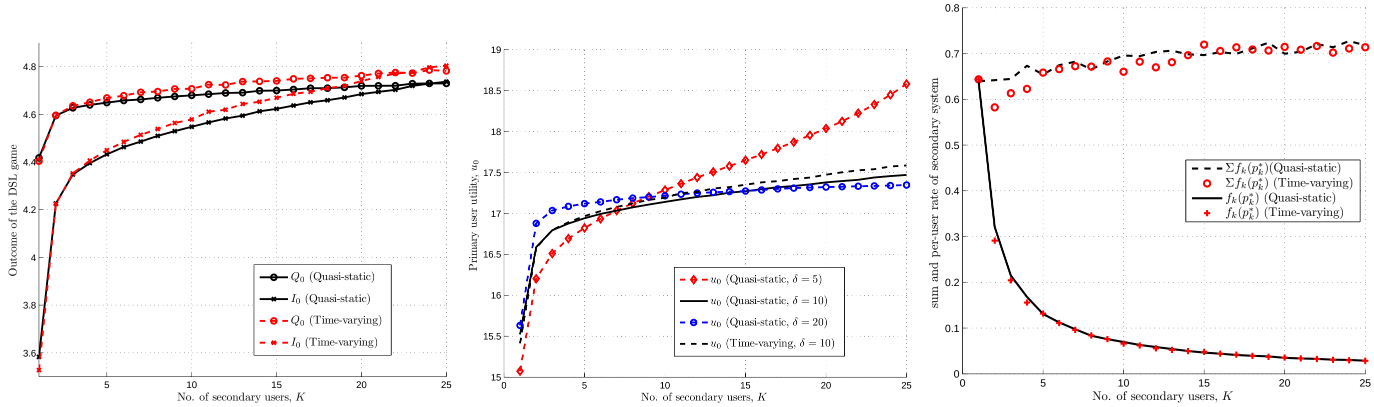


Fig. 1. Time-varying secondary system model.

In this paper, we consider the case in which the secondary system size is time-varying. To that end, we assume that the secondary user arrival process is Poisson with a parameter λ (i.e. the secondary users' average arrival rate per symbol period T) and the service time T_k has an exponential distribution with mean $1/\mu$ where μ is the service rate per symbol period T. The total number of packets for the k-th secondary user $N_{p,k}$ is equal to the number of symbols that can be transmitted during the service time (T_k/T) divided by the total number of symbols per packet L_p (i.e. $N_{p,k} = \lceil T_k/TL_p \rceil$). We assume that L_n number of these symbols are used for best response adaptation.

As shown in Fig. 1, the users that arrive during the best response iterations (i.e. users 1 and 18) can directly join the system. Thus, for $1 \leq i \leq L_N$, we may model the system as $M/M/\infty$ queueing system which is stable for $A \geq 0$ where $A = \lambda/\mu$ is the *offered traffic* [10]. After the best response iteration, we assume that the users start transmitting payload symbols of the packet. Any new arrivals during this period is assumed to be buffered which is the case for users 5 and 9 in Fig. 1. All the buffered users join the system at the end of the current packet.



(a) Game Outcomes at the NE for $\delta = 10$ and $L = 10$. (b) Primary user's utility function u_0 at the NE for $L = 10$. (c) Secondary reward functions at the NE for $\delta = 10$ and $L = 10$.

Fig. 2. Effect of the secondary system size on the time-varying ($\epsilon = 0.1$) case compared with quasi-static case ($\epsilon = 0$) for $L = 10$.

III. Noncooperative DSL-Game Model

A. Game Model

The DSL framework allows for the primary user to interact with the secondary users to adjust the total interference level. The primary changes Q_0 while the secondary users adjust there transmit powers in order to maximize their utility functions. As in [2], it can be formulated as a noncooperative game $(\mathcal{K}, \mathcal{A}_k, u_k(.))$:

- *Players*: $\mathcal{K} = \{0, 1, 2, \cdots, K_i\}$, where we assume that the 0-th user is the primary user and $k = 1, 2, \cdots, K_i$ represents the k-th secondary link.
- *Action space*: $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_{K_i}$, where $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, \cdots, K_i$, represents the k-th secondary user's action set. The action sets are upper bounded by $\bar{Q}_0$ and by $\bar{P}_k$ which represent the maximum possible IC of the primary user and the maximum transmission power of the k-th secondary user, respectively. The action vector of all players is denoted by $\mathbf{a} = [Q_0, p_1, \cdots, p_{K_i}]^T$, where $Q_0 \in \mathcal{Q}$ and $p_k \in \mathcal{P}_k$. We will use the notation $\mathbf{a}_{-k}$ whenever we refer to the action vector excluding the k-th user, for $k = 0, 1, 2, \cdots, K_i$.
- *Utility function*: We denote by $u_0(Q_0, \mathbf{a}_{-0})$ the primary user's utility function and by $u_k(p_k, \mathbf{a}_{-k})$, for $k = 1, 2, \cdots, K_i$, the k-th secondary user's utility function.

As defined in [2], [4], the primary user's target SINR at any given time t is defined in terms of its *assumed* worst-case secondary interference:

$$\bar{\gamma}_0 = \frac{h_{00}^2 p_0}{Q_0 + \sigma_0^2}. \tag{6}$$

The SINR $\bar{\gamma}_0$ is called the worst-case interference since Q_0 is the maximum possible interference that the secondary users are allowed to cause. The instantaneous SINR of the primary user and the k-th secondary link are, respectively, $\gamma_0^{(i)} = \frac{h_{00}^2(i)p_0}{I_0^{(i)}+\sigma_0^2}$ and $\gamma_k^{(i)} = \frac{h_{kk}^2(i)p_k}{i_k^{(i)}+\left(\rho_{k,0}^{(s)}\right)^2 h_{k,0}^2(i)p_0+\sigma_k^2}$.

From [4], the primary and secondary utility functions are, respectively,

$$u_0(Q_0, \mathbf{a}_{-0}) = \left(\bar{Q}_0 - (Q_0 - I_0^{(i)}(\mathbf{a}_{-0}))\right) F(Q_0) \tag{7}$$

$$u_k(p_k, \mathbf{a}_{-k}) = \frac{W_k \log(1+\gamma_k^{(i)})}{1 + e^{\delta\left(I_{0,-k}^{(i)}(\mathbf{a}_{-k}) - Q_0 + \tilde{A}_k^{(i)} p_k\right)}} \tag{8}$$

where $I_{0,-k}^{(i)}(\mathbf{a}_{-k}) = \sum_{j=1, j\neq k}^{K_i} \left(\rho_{0j}^{(p)}\right)^2 \left(A_{0,j}^{(i)}\right)^2$, $\tilde{A}_k^{(i)} = \left(\rho_{0k}^{(p)}\right)^2 h_{0k}^2(i)$, δ is a positive steepness coefficient and $W_k > 0$ is a scaling parameter that can be taken as proportional to the bandwidth. Note, that according to utility function (8), the k-th secondary user reward is the rate $f_k(p_k) = W_k \log(1+\gamma_k^{(i)})$.

B. Nash Equilibrium

In [4], the authors investigated the equilibrium strategies on the proposed game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$. Each user attempts to maximize its utility function (defined in (7) and (8)). A set of necessary conditions on $F(Q_0)$ for u_0 to be quasi-concave and thus the game G to have a unique Nash Equilibrium was given in [3]. In the remainder of this paper, we consider the following primary reward function which satisfies those conditions $F(Q_0) = \log(1+Q_0)$.

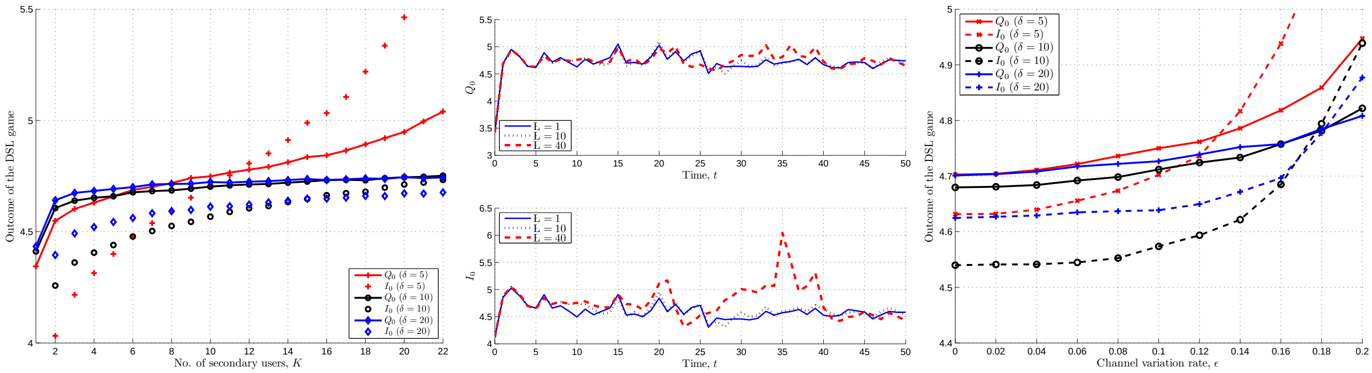
C. Best Response Adaptations

From [4], the best response strategy for the primary user is $r_0(\mathbf{a}_{-0}) = \min\left\{\bar{Q}_0, Q_0^*(I_0^{(i)})\right\}$, where Q_0^* satisfies

$$Q_0^* = (\bar{Q}_0 + I_0^{(i)}) - (1+Q_0^*)\log(1+Q_0^*). \tag{9}$$

On the other hand, the best response strategy of the k-th secondary user is $r_k(\mathbf{a}_{-k}) = \min\left\{\bar{P}_k, p_k^*(Q_0, I_{0,-k}^{(i)}, i_k^{(i)})\right\}$ where p_k^* is the unique solution of the equation:

$$\delta \tilde{A}_k^{(i)} e^{\delta\left(I_{0,-k}^{(i)} - Q_0\right)} \left(1+\gamma_k^{(i)}\right) \log\left(1+\gamma_k^{(i)}\right) = \frac{1}{N_k}\left(e^{\delta\left(I_{0,-k}^{(i)} - Q_0\right)} - e^{\delta \tilde{A}_k^{(i)} p_k}\right).$$



(a) The secondary users interference I_0 in a time-varying case ($\epsilon = 0.1$).

(b) The primary interference cap Q_o while finding the NE.

(c) Game Outcomes for $L = 10$ and $K = 10$ while varying ϵ.

Fig. 3. Effect of the steepness coefficient δ, channel variation rate ϵ and the channel estimation period L.

The best strategy for the k-th secondary user is a function Q_0, $I_{0,-k}^{(i)}$ and $I_k^{(i)}$. The secondary system can measure the total interference at its receiver $I_k^{(i)}$. We will assume that the primary system periodically broadcasts the values of Q_0 and $I_0^{(i)}$. These are the only quantities needed to be exchanged between the primary and the secondary systems. Using an estimated value of the state information $\hat{h}_{0,k}$, and knowing its own transmit power, the k-th secondary user can *estimate* the residual interference $\hat{I}_{0,-k}^{(i)} = I_0^{(i)} - \left(\rho_{0,k}^{(p)}\right)^2 \hat{h}_{0,k}^2(i)p_k$.

IV. Numerical Analysis and Simulations

In this section we investigate the equilibrium behavior of the above DSL-game in the presence of time-varying fading and Poisson distributed user arrivals. First, we compare a quasi-static (ϵ=0) system to that with time-varying channel coefficients assuming a fixed secondary system size $K_i = K$. Next, we allow the secondary system size to be time-varying as described in Section II-C.

For each set of parameters the results are averaged over 2000 channel fading coefficients and secondary system size using Monte Carlo methods. Unless specified otherwise, in what follows simulation parameters were set to $\sigma_j^2 = \sigma_{h_{j,k}}^2 = 1$, $\rho_{j,k} = 1$, $L = 10$, $\delta = 10$ and $W_k = 1$. The maximum IC allowed for the primary user is $\bar{Q}_0 = 10$ and the worst case primary SINR is fixed to $\bar{\gamma}_0 = 1$. The maximum transmit power for the k-th secondary user is set to $\bar{P}_k = 12$.

A. Effect of the Secondary System Size and the Steepness Coefficient

Figure 2(a) shows the outcome of the game versus the fixed secondary system size K. As in the quasi-static case, in the time-varying scenario the safety margin $Q_0 - I_0$ decreases when the number of the secondary users increases. However, in the time-varying system the values of Q_0 and I_0 are slightly higher than those for the quasi-static system. The reason for this increase is due to the incomplete information caused by the outdated CSI. This incomplete information forces the system to deviate from the actual Nash equilibrium. Figure 2(a) shows that the time-varying scenario can accommodate up to 22 secondary users without violating the primary IC whereas in the quasi-static scenario the system can handle up to 24 secondary users.

Figures 2(b) and 2(c) show the primary utility function and the secondary reward function, respectively, at the Nash equilibrium of the DSL system. In Fig. 2(b) we observe a slight increase in the utility function in the time-varying case compared to the quasi-static case. This gain is due to the increased total interference level at the primary receiver as shown in Fig. 2(a). The effect of the steepness coefficient δ in the secondary user utility function is shown in Fig. 2(b) and 3(a). Increasing δ will increase the steepness of the decay of secondary user utility function when $I_0 > Q_0$. Figure 3(a) shows the game outcome in a time-varying environment ($\epsilon = 0.1$) for $\delta = 5, 10$ and, 20. As one would expect, the values of Q_0 and I_0 saturate when δ is large enough. Thus, in what follows the parameter δ is set to be $\delta = 10$.

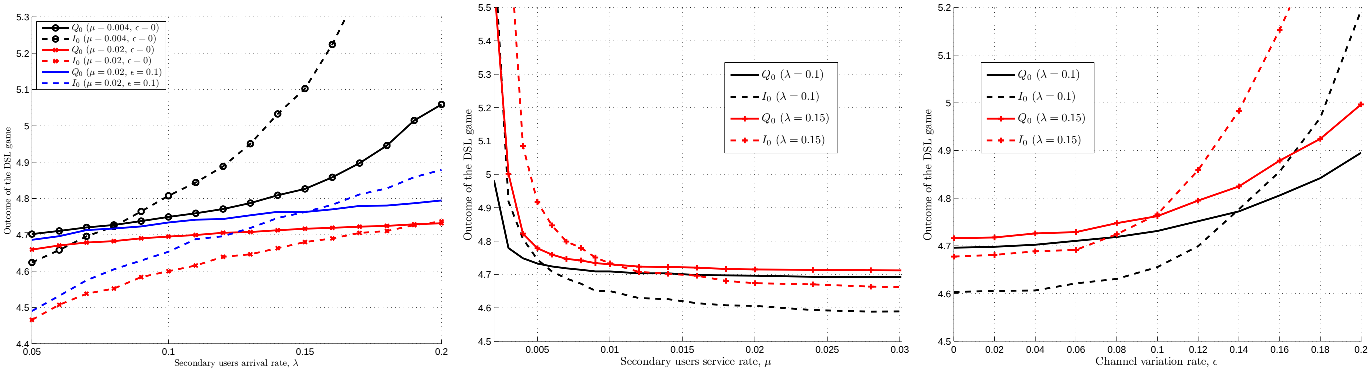
B. Effects of CSI Update Period and Channel Variation Rate

Figure 3(b) shows the evolution of the DSL game outcomes (Q_0 and I_0) during the best response adaptations for different values of L. We have fixed the channel variation rate to $\epsilon = 0.1$ and the number of secondary users to $K = 10$. The convergence deteriorates for large L values as can be observed by comparing $L = 10$ with $L = 40$ in Fig. 3(b).

Figure 3(c) shows Q_0 and I_0 at the system Nash equilibrium as a function of the channel variation rate ϵ. It can be seen in Fig. 3(c) that both the equilibrium IC and the total secondary interference increase when channel variation rate ϵ increases. For a fixed number of secondary users ($K = 10$) the system can tolerate up to 13% channel variation rate when $\delta = 5$. However, for large δ, the system will saturate at $\epsilon = 0.18$.

C. Effect of the Time-Varying Secondary System Size

In this section, we investigate the effect of the time-varying secondary system size K_i. The packet size is assumed to be $L_p = 100$ symbols and the best response iterations are assumed to be terminated after $L_N = 30$ symbols. Figure



(a) The effect of the arrival rate λ.

(b) The effect of the service rate μ on a quasi-static channel ($\epsilon = 0$).

(c) The effect of the channel variation rate ϵ on a time-varying secondary system ($\mu = 0.02$).

Fig. 4. The game outcome for a time-varying secondary system scenario with $L = 10$.

4(a) shows the outcome of the game at Nash equilibrium for different arrival rates λ. The arrival rate λ has the same effect as the fixed secondary system size. When we increase the arrival rate, the mean number of secondary users in the system increases. Therefore the outcomes Q_0 and I_0 of the game increase. For large values of λ, Q_0 limit is violated due to the high number of secondary users available on average in the system. For a service rate $\mu = 0.004$, the maximum value of arrival rate λ that the system can accommodate is 0.08. However if the service rate μ is increased, the performance will improve because on average there will be fewer users in the system. Thus for $\mu = 0.02$, the system can tolerate an arrival rate up to $\lambda = 0.19$. However, Fig. 4(b) shows that for large values of μ (e.g., $\mu \geq 0.01$ when $\lambda = 0.15$) the values of Q_0 and I_0 are almost saturated. This is because, when μ is large, the service time decreases and the number of packets transmitted by the k-th user, $N_{p,k}$, saturates to its minimum value 1.

In Fig. 4(c), we show the outcome of the game at Nash equilibrium as a function of the channel variation rate ϵ. As expected, for smaller secondary arrival rates λ, the system can tolerate higher channel variation rates ϵ. When we decrease λ from 0.15 to 0.1, the system can tolerate a channel variation rate of $\epsilon = 0.14$.

V. Conclusion

In this paper, we investigated the best response convergence and the equilibrium performance of a DSL based spectrum sharing network in the presence of dynamic channel conditions. We showed that the Nash equilibrium is a good approximation for the system equilibrium under the time-varying environment similar to when fading is quasi-static. Under the time-varying conditions and for a reasonable values of the channel variation rate ϵ and the CSI update period L, the equilibrium point is slightly perturbed compared to that under quasi-static conditions due to the outdated channel information. The performance is slightly degraded due to this shift of the system operating point; the maximum number of supported secondary users is decreased. As the channel variation rate ϵ increases, the convergence to the Nash equilibrium deteriorates. To compensate this loss, one can reduces the CSI update period L.

Acknowledgment

G. El-Howayek, S. K. Jayaweera and K. Hakim were supported in part by the Air Force Research Laboratory (AFRL) Space Vehicles Directorate and the National Science foundation (NSF) under the grant CCF-0830545. G. Vázquez-Vilar and C. Mosquera were supported by the Spanish Government under projects SPROACTIVE (ref. TEC2007-68094-C02-01/TCM) and COMONSENS (CONSOLIDER-INGENIO 2010 CSD2008-00010).

References

[1] S. Jayaweera and T. Li, "Dynamic spectrum leasing in cognitive radio networks via primary-secondary user power control games," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3300–3310, June 2009.

[2] S. K. Jayaweera, G. Vazquez-Vilar, and C. Mosquera, "Dynamic spectrum leasing (DSL): A new paradigm for spectrum sharing in cognitive radio networks," *IEEE Trans. Vehicular Technology*, Jan. 2010, accepted for publication.

[3] S. K. Jayaweera and C. Mosquera, "A dynamic spectrum leasing (DSL) framework for spectrum sharing in cognitive radio networks," in *Proc. 43rd Annual Asilomar Conf. on Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2009.

[4] S. Jayaweera, K. Hakim, and C. Mosquera, "A game-theoretic framework for dynamic spectrum leasing (DSL) in cognitive radio networks," in *5th IEEE Workshop on Broadband Wireless Access co-located with Globecom'09*, Honolulu, HI, Nov. 2009.

[5] FCC, "Report of the spectrum efficiency working group," FCC spectrum policy task force, Tech. Rep., Nov. 2002.

[6] Z. Ji and K. Liu, "Cognitive radios for dynamic spectrum access - dynamic spectrum sharing: A game theoretical overview," *IEEE Communications Magazine*, vol. 45, no. 5, pp. 88–94, May 2007.

[7] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Sig. Proc. Magazine*, vol. 24, no. 3, pp. 79–89, May 2007.

[8] B. Wang, Z. Ji, and K. Liu, "Primary-prioritized Markov approach for dynamic spectrum access," in *2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, Dublin, Ireland, Apr. 2007, pp. 507–515.

[9] P. S. Maybeck, *Stochastic models, estimation, and control*, ser. Mathematics in Science and Engineering. U.K.: Academic, 1979, vol. 141.

[10] L. Kleinrock, *Queueing Systems, Volume I: Theory*. New York: Wiley Interscience, 1975, vol. 1.