Analysis of Linear Receivers in a DSL Game for Spectrum Sharing in Cognitive Radio Networks

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***Abstract*—Dynamic Spectrum Leasing (DSL) is a new paradigm for efficient spectrum sharing in cognitive radio networks that was recently proposed in [1]–[3]. Unlike in a hierarchical dynamic spectrum access (DSA) network, DSL allows active participation of both the primary and the secondary users in the spectrum sharing process. In this paper we further generalize the DSL game introduced in [2] by allowing for linear multiuser detectors, in particular the matched filter (MF) and linear minimum mean squared error (LMMSE) receivers, at the secondary base stations. We establish the conditions so that the DSL game has desired equilibrium properties. Performance of the proposed system at the equilibrium is compared through simulations under different linear receivers at the secondary base station.**

I. Introduction

One of the reasons for the growing scarcity of radio spectrum is the inefficiency of traditional fixed spectrum allocation policies [4], [5]. *Dynamic spectrum sharing* (DSS) is considered as an effective way to improve inefficient static spectrum utilization by allowing secondary users to access the so-called white spaces in spectrum already licensed to the primary users. Cognitive radios proposed in [6] are especially suited for realizing such dynamic spectrum sharing due to their ability to observe, learn from and orient to the observed RF environment.

The hierarchical dynamic spectrum access (DSA) methods that have been considered in recent literature presume that a secondary (cognitive) transmitter may access the spectrum band owned by a primary user only on the no- or limited-interference basis to the primary users [7], [8]. Further, the secondary system is almost exclusively responsible for managing the inter-system interference problem due to coexistence. However, recently introduced *dynamic spectrum leasing* (DSL) [1]–[3] is a new paradigm for spectrum sharing in cognitive radio networks in which primary users are allowed to actively manage the secondary interference they are willing to tolerate at any given time. As opposed to DSA techniques considered by many in recent literature, in a DSL-based network the primary users are also active decision makers. Primary users are suitably rewarded for allowing secondary users to share their licensed spectrum, giving them an incentive to leave a reasonable portion of the spectrum for the secondary users whenever they can while meeting their own minimum Quality-of-Service (QoS) requirements. Simultaneously, the secondary users aim to achieve energy efficient transmissions, while not causing excessive interference to the primary users.

Contributions of this paper that distinguishes it from previous literature are as follows: (i) we further generalize the primary system utility function defined in [2], [3], and (ii) the proposed non-cooperative DSL game is generalized to allow for linear multiuser detectors, such as, the matched filter (MF) and the linear minimum mean squared error (LMMSE) receivers at the secondary base stations. We establish the existence of a Nash equilibrium for this modified primary-secondary spectrum-leasing game. The performance of the proposed DSS system is studied with different linear detectors and it was observed that the LMMSE detector outperforms the MF detectors in most of the cases.

The remainder of this paper is organized as follows: Section II describes the DSL-based cognitive radio network made of a primary and a secondary communications system. Section III presents the proposed game-theoretic model for dynamic spectrum leasing in a spectrum sharing cognitive radio network. Sections IV and V discuss the existence of a unique Nash equilibrium for matched-filter (MF) and LMMSE receivers, respectively. Section VI evaluates the performance of a DSL cognitive radio network based on the proposed game model. Finally, Section VII concludes the paper by summarizing our results and outlining possible further work.

II. DSL Based Spectrum Sharing Cognitive Radio Network Model

We assume there is one primary wireless communication system that owns the exclusive rights to the spectrum band of interest and the primary system is willing to allow a secondary system to access this spectrum band whenever it can tolerate and to the maximum possible extent. It is further assumed that there is a single primary transmitter-receiver pair in the primary system. There are K secondary links of interest. For simplicity of exposition, all these links are assumed to belong to the same secondary system. The primary user is denoted as user 0, and the secondary links are labeled as 1 through K. We will refer to k-th transmitter or k-th receiver to mean the transmitter and receiver of the k-th link. The channel gain between the k-th transmitter and the primary receiver and the j-th secondary receiver are denoted by h_{pk} and h_{jk}, respectively, for $k = 0, 1, \cdots, K$ and $j = 1, \cdots, K$. We use p_k to represent transmission power of the k-th user. Such a DSL based cognitive radio network is shown in Fig. 1 with $K = 3$ secondary links. Depending on the type of the secondary network, the receivers of each link may or may not be physically distinct. As can be seen from Fig. 1(a), receivers of Link 1 and Link 2 are different, but in Fig. 1(b), Link 1 and Link 2 share the same receiver.

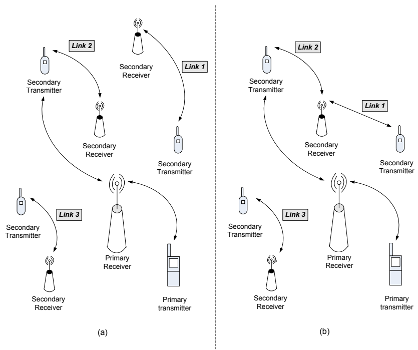

Fig. 1. Cognitive radio system model with $K = 3$ secondary links.

As in [2], we may obtain a discrete-time representation of the received signal at the primary receiver as $\mathbf{r}^{(p)} = A_0 b_0 \mathbf{s}_0^{(p)} + \sum_{k=1}^{K} A_k b_k \mathbf{s}_k^{(p)} + \sigma_p \mathbf{n}^{(p)}$ where $A_k = h_{pk}\sqrt{p_k}$, for $k = 0, 1, \cdots, K$, σ_p^2 is the variance of the zero-mean, additive noise at the primary receiver, $\mathbf{r}^{(p)} = \left(r_1^{(p)}, \cdots, r_M^{(p)}\right)^T$ and $\mathbf{s}_k^{(p)} = \left(s_{k1}^{(p)}, \cdots, s_{kM}^{(p)}\right)$, for $k = 0, 1, \cdots, K$, are the vector representations of $r_p(t)$ and $s_k(t)$ respectively in an M-dimensional basis employed by the primary system and $\mathbf{n}^{(p)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_M)$. Note that, in writing $\mathbf{r}^{(p)}$, we have assumed synchronicity and thus have dropped symbol indices for simplicity of notation. With the conventional matched-filter (MF) detector at the primary receiver, and assuming that primary modulation is BPSK so that $b_0 \in \{+1, -1\}$, the primary decisions are given by $\hat{b}_0 = sgn\left(y_0^{(p)}\right)$, where $y_0^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{r}^{(p)} = A_0 b_0 + \sum_{k=1}^{K} \rho_{0k}^{(p)} A_k b_k + \sigma_p \eta^{(p)}$, with $\rho_{0k}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{s}_k^{(p)}$ and $\eta^{(p)} \sim \mathcal{N}(0, 1)$. The total secondary interference I_0 from all secondary transmissions to the primary-user is

$$I_0 = \sum_{k=1}^{K} \left(\rho_{0k}^{(p)}\right)^2 A_k^2 = \sum_{k=1}^{K} \left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 p_k. \quad (1)$$

Similarly, a discrete-time representation of the received signal at the j-th secondary-system receiver can be written as $\mathbf{r}_j^{(s)} = \sum_{k=1}^{K} B_{j,k} b_k \mathbf{s}_k^{(s)} + B_{j,0} b_0 \mathbf{s}_0^{(s)} + \sigma_s \mathbf{n}_j^{(s)}$ where $B_{j,k} = h_{jk}\sqrt{p_k}$, for $k = 0, 1, \cdots, K$, σ_s^2 is the variance of secondary receiver noise, $\mathbf{r}_j^{(s)} = \left(r_{j1}^{(s)}, \cdots, r_{jN}^{(s)}\right)^T$, $\mathbf{s}_k^{(s)} = \left(s_{k1}^{(s)}, \cdots, s_{kN}^{(s)}\right)^T$, for $k = 0, 1, \cdots, K$, are the N-vector representation of the received signal $r_j^{(s)}(t)$ and $s_k(t)$ with respect to an N-dimensional basis employed by the secondary system, and $\mathbf{n}_k^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$.

III. Game Model for DSL-based Dynamic Spectrum Sharing

In the proposed DSL-based networks, the primary and secondary users interact with each other by adjusting their interference cap and transmit power levels, respectively, in order to maximize their own utilities. We formulate the above system as in following noncooperative DSL game $(\mathcal{K}, \mathcal{A}_k, u_k(.))$:

1) Players: $\mathcal{K} = \{0, 1, 2, \cdots, K\}$, where we assume that the 0-th user is the primary user and $k = 1, 2, \cdots, K$ represents the k-th secondary link.
2) Action space: $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_K$, where $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, \cdots, K$, represents the k-th secondary user's action set. We denote the action vector of all users by $\mathbf{a} = [Q_0, p_1, \cdots, p_K]^T$, where $Q_0 \in \mathcal{Q}$ and $p_k \in \mathcal{P}_k$. It is customary to denote the action vector excluding the k-th user, for $k = 0, 1, 2, \cdots, K$, by $\mathbf{a}_{-k}$.
3) Utility function: We denote by $u_0(Q_0, \mathbf{a}_{-0})$ the primary user's utility function, and by $u_k(p_k, \mathbf{a}_{-k})$, for $k = 1, 2, \cdots, K$, the k-th secondary user's utility function.

At any given time, the primary user's *target* SINR is defined in terms of its assumed worst-case secondary interference: $\bar{\gamma}_0 = \frac{h_{p0}^2 p_0}{Q_0 + \sigma_p^2}$. On the other hand, the primary user's *actual* instantaneous SINR is given by $\gamma_0 = \frac{h_{p0}^2 p_0}{I_0 + \sigma_p^2}$. In DSL-based spectrum sharing, the utilities of primary and secondary users are coupled via mutual interference. To that end, by generalizing [2], [3], we introduce the following utility function for the primary user:

$$u_0(Q_0, \mathbf{a}_{-0}) = \left(\bar{Q}_0 - (Q_0 - I_0(\mathbf{a}_{-0}))\right) F(Q_0). \quad (2)$$

where $F(.)$ is a suitable continuous reward function for the primary system. In this paper we establish conditions on $F(.)$ so that the proposed DSL game has desired equilibrium properties. As a special case of (2), we choose $F(Q_0) = \log(1 + Q_0)$, so that the primary user utility is proportional to the capacity achievable by the secondary system with respect to the primary receiver.

Motivated by network utility considerations, [2], [3] proposed the following utility function for the secondary users:

$$\begin{aligned} u_k(p_k, \mathbf{a}_{-k}) &= (Q_0 - \lambda_s I_0) f(p_k) \\ &= (Q_0 - \lambda_s I_{0,-k} - \lambda_s c_k p_k) f(p_k) \quad (3) \end{aligned}$$

where $f(.)$ is a suitable, non-negative reward function chosen by the secondary system, λ_s is a suitably chosen positive coefficient which controls how strictly secondary users need to adhere to the primary user's interference cap and $I_{0,-k} = I_0 - \left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 p_k$ is the interference from all secondary transmissions at the primary receiver excluding that from the k-th secondary user. In writing (3) we have defined $c_k = \left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 \geq 0$. Again we will use $f(p_k) = W_k \log\left(1+\gamma_k^{(s)}\right)$ throughout this paper as a special case of (3) where $\gamma_k^{(s)}$ is the received SINR of the k-th secondary link for $k = 1, \cdots, K$ and $W_k > 0$ is a scaling parameter that can be taken as proportional to the bandwidth.

IV. DSL Game with the MF Secondary Receiver

We first assume that all secondary transmissions are BPSK and all secondary detectors are based on the MF. Hence the j-th secondary receiver detects the k-th secondary user's symbols as $\hat{b}_k^j = sgn\left(y_k^{s,j}\right)$ where, for $k = 1, \cdots, K$, $y_k^{s,j} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{r}_j^{(s)} = B_{j,k} b_k + \sum_{l=1, l\neq k}^{K} \rho_{kl}^{(s)} B_{j,l} b_l + \rho_{k0}^{(s)} B_{j,0} b_0 + \sigma_s \eta_k^{(s,j)}$ with $\rho_{kl}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_l^{(s)}$, for $l = 0, 1, \cdots, K$, $\rho_{k0}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_0^{(s)}$, and $\eta_k^{(s,j)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{n}_j^{(s)} \sim \mathcal{N}(0,1)$. Hence, the k-th secondary link's SINR is given by

$$\gamma_k^{MF} = \frac{|h_{kk}|^2 p_k}{i_k^{(k)} + \tilde{\sigma}_{s,k}^2} = \frac{p_k}{N_k}, \quad (4)$$

where $i_k^{(k)} = \sum_{l=1, l\neq k}^{K} \left(\rho_{kl}^{(s)}\right)^2 h_l^2 p_l$, $\tilde{\sigma}_{s,k}^2 = \left(\rho_{k0}^{(s)}\right)^2 |h_{k0}|^2 p_0 + \sigma_{s,k}^2$ is the effective noise seen by the k-th user and $N_k = \frac{i_k^{(k)} + \tilde{\sigma}_{s,k}^2}{|h_{kk}|^2}$.

A. Existence of a Nash Equilibrium

With the assumed form of action sets $\mathcal{A}_k$, it is easy to see that the best responses $r_k(\mathbf{a}_{-k})$'s are both compact and convex for all $k = 0, 1, \cdots, K$ [2]. Further, both $u_0(\mathbf{p})$ and $u_k(\mathbf{p})$ are continuous in the action vector $\mathbf{p}$. For the existence of a Nash Equilibrium, the remaining condition that we need to ensure is the quasi-concavity of u_k's for all $k = 0, 1, \cdots, K$ [9]. Let us define a function $\Phi(Q_0)$ as $\Phi(Q_0) = \frac{F(Q_0)}{F'(Q_0)} + Q_0$. It can be seen that u_0 has a local maximum that is indeed a global maximum if $\Phi(Q_0) = \bar{Q}_0 + I_0(\mathbf{a}_{-0})$ has one and only one solution for $Q_0 \in \mathcal{Q}$. Clearly this equation has a solution if $\Phi(Q_0)$ is continuous and $\lim_{Q_0 \to 0} \Phi(Q_0) \leq \bar{Q}_0 + I_0(\mathbf{a}_{-0}) < \lim_{Q_0 \to \infty} \Phi(Q_0)$. This solution would be a global maximum if in addition $\Phi'(Q_0) > 0$ for $Q_0 > 0$. It can be easily verified that $\Phi'(Q_0) > 0$ will be true if $F(Q_0)$ is such that $\frac{F(Q_0)F''(Q_0)}{(F'(Q_0))^2} < 2$. Note also that $\lim_{Q_0\to\infty} \Phi(Q_0) = \infty$ if $\lim_{Q_0\to\infty} \frac{F(Q_0)}{F'(Q_0)} > -\infty$. Hence, a set of necessary conditions on $F(Q_0)$ for u_0 to be quasi-concave is:

1. $F(Q_0)$ is continuous and strictly monotonic for $Q_0 > 0$
2. $F(0) = 0, F'(0) > 0$ and $\lim_{Q_0\to\infty} \frac{F(Q_0)}{F'(Q_0)} > -\infty$
3. $\frac{F(Q_0)F''(Q_0)}{(F'(Q_0))^2} < 2$ for $Q_0 > 0$
4. $0 \leq \bar{Q}_0 + I_0(\mathbf{a}_{-0}) < \infty$.

As can be seen from (3), $u_k(\mathbf{a})$ is continuous in $\mathbf{a}$. Next, consider its first order derivative of $u_k(p_k) = (Q_0 - \lambda_s I_0) W_k \log\left(1+\gamma_k^{MF}\right)$ w.r.t. p_k: $\frac{\partial u_k(p_k)}{\partial p_k} = \frac{W_k g_k^{MF}(p_k)}{\left(1+\gamma_k^{MF}\right)}$ where

$$g_k^{MF}(p_k) = \frac{Q_0 - \lambda_s I_0}{N_k} - \lambda_s c_k \left(1+\gamma_k^{MF}\right) \log\left(1+\gamma_k^{MF}\right). \quad (5)$$

Note that at an interior local extremum point for $p_k \in [0, \infty)$, we should have $g_k^{MF}(p_k) = 0$. As $g_k^{MF}(0) = \frac{1}{N_k}(Q_0 - \lambda_s I_{0,-k})$, for $g_k^{MF}(0) \geq 0$, we should have $\lambda_s \leq \frac{Q_0}{I_{0,-k}}$. Since $g_k^{MF}(\infty) \longrightarrow -\infty$ and $g_k^{MF}(p_k)$ is continuous in p_k, clearly function $g_k(.)$ must have at least one zero crossing. However, because $g_k^{'MF}(p_k) = -\frac{\lambda_s c_k}{N_k}\left(2 + \log\left(1+\gamma_k^{MF}\right)\right) < 0$ for $p_k \geq 0$, there is exactly one zero of $g_k^{MF}(p_k)$ on $[0, \infty]$, implying that $u_k(p_k)$ only has one local extremum point on $p_k \in [0, \infty)$. It follows that the local extremum point is indeed a global maximum of $u_k(.)$ on $[0, \infty)$, implying that $u_k(p_k)$ is quasi-concave in p_k, for each $k = 1, \cdots, K$. From the above discussion it follows that the above game G then has at least one Nash equilibrium.

B. Best response

The best response of the primary user is obtained by setting $u_0'(Q_0) = 0$. For $F(Q_0) = \log(1+Q_0)$, the unique interior solution is given by

$$Q_0^* + (1+Q_0^*)\log(1+Q_0^*) = \left(\bar{Q}_0 + I_0\right). \quad (6)$$

Since $u_0(Q_0)$ is monotonic increasing for $Q_0 < Q_0^*$, the best response is

$$r_0(\mathbf{a}_{-0}) = \min\{\bar{Q}_0, Q_0^*(I_0)\}. \quad (7)$$

On the other hand, the best response of the k-th secondary user to the transmit powers of the other secondary users as well as interference cap set by the primary user is given by the (unique) solution $p_k = p_k^*\left(Q_0, I_{0,-k}, i_k^{(j)}\right)$ of (5). Again, since u_k is quasi-concave in p_k, we have the best response of the k-th secondary user, for $k = 1, \cdots, K$:

$$r_k(\mathbf{a}_{-k}) = \min\{\bar{P}_k, p_k^*\left(Q_0, I_{0,-k}, i_k^{(j)}\right)\}. \quad (8)$$

In this work, we assume that the primary base station broadcasts both Q_0 and I_0 so that each secondary link can compute the residual interference $I_{0,-k}$ since it knows its own transmit power and it may estimate the channel state information c_k.

V. DSL Game with the LMMSE Secondary Receiver

In this generalization, we assume that the secondary system is equipped with LMMSE receivers, while that of primary system is an MF receiver. Then the k-th secondary link's SINR can be written as [10]:

$$\gamma_k^{MMSE} = B_{k,k}^2 \left(\mathbf{s}_k^{(s)}\right)^T \Sigma_{k,k}^{-1} \mathbf{s}_k^{(s)} = h_{kk}^2 p_k \left(\mathbf{s}_k^{(s)}\right)^T \Sigma_{k,k}^{-1} \mathbf{s}_k^{(s)}, \quad (9)$$

where $\Sigma_{k,k} = \sigma^2 \mathbf{I} + B_{k,0}^2 \mathbf{s}_0^{(s)} \mathbf{s}_0^{(s)^T} + \sum_{l=1, l\neq k}^{K} B_{k,l}^2 \mathbf{s}_l^{(s)} \mathbf{s}_l^{(s)^T}$.

A. Existence of a Nash Equilibrium with LMMSE Receiver

The only condition that we need to establish here anew is the quasi-concavity of secondary-user utility as a function of transmit power p_k, when the receiver is based on an LMMSE detector. We consider the first order derivative of

$u_k(p_k, \mathbf{a}_{-k}) = (Q_0 - \lambda_s I_0) W_k \log\left(1+\gamma_k^{MMSE}\right)$ w.r.t. p_k: $\frac{\partial u_k(p_k)}{\partial p_k} = \frac{W_k g_k^{MMSE}(p_k)}{\left(1+\gamma_k^{MMSE}\right)}$ where

$$g_k^{MMSE}(p_k) = (Q_0 - \lambda_s I_0) h_{kk}^2 \left(\mathbf{s}_k^{(s)}\right)^T \Sigma_{k,k}^{-1} \mathbf{s}_k^{(s)} - \lambda_s c_k \left(1+\gamma_k^{MMSE}\right) \log\left(1+\gamma_k^{MMSE}\right). \quad (10)$$

Following the same argument as in Section IV-A, it follows that $u_k(p_k)$ is quasi-concave in p_k, for each $k = 1, \cdots, K$ even in the LMMSE case. Hence the proposed game with LMMSE receiver has at least one NE.

B. Uniqueness of the NE with the LMMSE Receiver

To establish the uniqueness of the NE of the DSL game with secondary LMMSE receiver, we show that the best response correspondence $r_k(\mathbf{a}_{-k})$ is a *standard function* [11] for $k = 0, 1, \cdots, K$. For simplicity of exposition, below we assume $\bar{Q}_0 \longrightarrow \infty$ and $\bar{p}_k \longrightarrow \infty$. We adopt the convention that all the vector inequalities are component-wise.

1) Primary user best response: The primary user best response is given by $r_0(\mathbf{a}_{-0}) = \min\{\bar{Q}_0, Q_0^*(I_0)\}$.

I. *Positivity*: From (6), for $\mathbf{a}_{-0} = \mathbf{0}$, $Q_{0min}^* > 0$. So $r_0(\mathbf{a}_{-0}) > 0$ for $\mathbf{a}_{-0} \geq \mathbf{0}$.

II. *Monotonicity*: Since the left and the right hand side of (6) are increasing functions of Q_0 and $\mathbf{a}_{-0}$, respectively, given $\mathbf{a}_{-0} \geq \mathbf{a'}_{-0}$, $r_0(\mathbf{a}_{-0}) \geq r_0(\mathbf{a'}_{-0})$.

III. *Scalability*: From (6), $Q_0^*(I_0)$ is concave in I_0 since $\frac{d^2 Q_0^*}{dI_0^2} = \frac{-1}{(1+Q_0^*)\left(2+\log(1+Q_0^*)\right)^3} < 0$ for $Q_0^* \geq 0$. It can be easily seen that positivity and concavity of $Q_0^*(I_0)$ together implies scalability. So for $\mu > 1$, we can write $\mu r_0(\mathbf{a}_{-0}) > r_0(\mu \mathbf{a}_{-0})$

Therefore, from the definitions given in [11], the best response correspondence of the primary user is a standard function.

2) Secondary users' best response: The best response correspondence of the k-th secondary user is the transmit power which provides it with the optimum SINR γ^{MMSE^*} given by the solution p_k to the equation $g_k^{MMSE}(p_k) = 0$. Thus the best response correspondence of the k-th secondary user is

$$r_k(\mathbf{a}_{-k}) = \min\left\{\frac{\gamma^{MMSE^*} I_k^{(k)}}{h_{kk}^2}, \bar{p}_k\right\} \quad (11)$$

where $I_k^{(k)} = \left[\left(\mathbf{s}_k^{(s)}\right)^T \Sigma_{k,k}^{-1} \mathbf{s}_k^{(s)}\right]^{-1}$. Since $\frac{\partial \gamma_k^{MMSE}(p_k)}{\partial p_k} = \frac{\gamma_k^{MMSE}}{p_k}$, maximizing the utility function for each user is equivalent to finding optimum SINR γ^{MMSE*}. Note also that γ^{MMSE^*} is independent of k as long as all secondary users have the same reward function.

I. *Positivity*: Since $\gamma^{MMSE^*} > 0$ and $I_k^{(k)} > 0$, the best response correspondence of the k-th secondary user $r_k(\mathbf{a}_{-k}) > 0$ for all $\forall k = 1, 2, \cdots, K$.

II. *Monotonicity*: By following a proof similar to [1], we have that for $\mathbf{a}_{-k} \geq \mathbf{a'}_{-k}$, $I_k^{(k)}(\mathbf{a}_{-k}) > I_k^{(k)}(\mathbf{a'}_{-k})$. Thus,

$$\begin{aligned} r_k(\mathbf{a}_{-k}) &= \frac{\gamma^{MMSE^*} I_k^{(k)}(\mathbf{a}_{-k})}{h_{kk}^2} \\ &\geq \frac{\gamma^{MMSE^*} I_k^{(k)}(\mathbf{a'}_{-k})}{h_{kk}^2} = r_k(\mathbf{a'}_{-k}) \end{aligned}$$

for all $\forall k = 1, 2, \cdots, K$.

III. *Scalability*: For $\mu > 1$, $\mu r_k(\mathbf{a}_{-k}) = \frac{\mu \gamma^{MMSE^*} I_k^{(k)}(\mathbf{a}_{-k})}{h_{kk}^2}$ and $r_k(\mu \mathbf{a}_{-k}) = \frac{\gamma^{MMSE^*} I_k^{(k)}(\mu \mathbf{a}_{-k})}{h_{kk}^2}$. Similar to the proof given in [1], we have that $\mu I_k^{(k)}(\mathbf{a}_{-k}) > I_k^{(k)}(\mu \mathbf{a}_{-k})$. Hence, $\mu r_k(\mathbf{a}_{-k}) > r_k(\mu \mathbf{a}_{-k})$ for all $\forall k = 1, 2, \cdots, K$.

So, the noncooperative DSL game with secondary LMMSE receiver has a unique NE.

VI. Performance Analysis of a Dynamic Spectrum Leasing System

For simplicity of exposition, below we assume that there is only one secondary receiver. We also assume all channel gains in the system to follow Rayleigh distribution with all channel coefficients normalized so that $\mathbb{E}\left\{h^2\right\} = 1$. Other parameters used for simulations are: $W_k = W = 1$, $\bar{Q} = 10$, $\bar{P}_k = 20$, $\bar{\gamma}_0 = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 0.3$ for all k, $\sigma_p^2 = 1$ and $\sigma_s^2 = 0.8$. All our simulation results are obtained by averaging over 1000 fading realizations.

It can be seen from Fig. 2 that with secondary MF receiver, the primary system can support up to $K^* \leq 5$ secondary users before the secondary system violates the primary interference cap at the system NE with $\lambda_s = 0.75$. On the other hand, with secondary LMMSE receiver, the primary system can support up to $K^* \leq 8$ secondary users. This is because of the superior interference suppression capability of the LMMSE receiver. As one would expect, for higher values of λ_s, such as $\lambda_s \geq 1$, the DSL game converges to a NE that does not violate the primary interference cap Q_0.

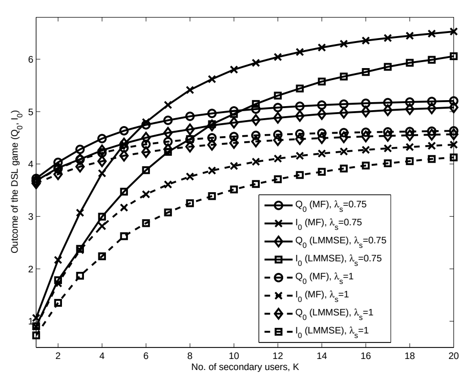

Fig. 2. Outcome (Q_0, I_0) of the DSL game at the system NE as a function of K.

As can be seen from Fig. 3 the primary user utility at the NE with the LMMSE receiver is less than that with the MF receiver. This is because the secondary system can better manage its transmit power with the LMMSE receiver and thus reduce the total interference I_0 at the primary receiver. As seen from Fig. 4, the secondary sum-utility as well as the per-user utility with the LMMSE receiver are much better compared to those with the MF receiver. Note that, the monotonic reduction in per-user utility with K is common to both LMMSE and MF-based receivers. However, with the LMMSE receiver, monotonic reduction in per-user utility is more than offset by the increased number of users in the secondary system.

The minimum transmission quality for the secondary system is defined as the average (over fading) minimum reward achieved by a link at the equilibrium. We denote this minimum required QoS for secondary link k as $f_{min,k} = f_{min}$. Figure 5 shows the outage probability $Pr\left(f_k\left(p_k^*\right) < f_{min}\right)$ of a typical secondary user as a function of K. It is seen that the outage probability increases with K as well as with the minimum QoS requirement. Note here also, the LMMSE-based system ensures a higher QoS due to efficient management of secondary links' transmit powers.

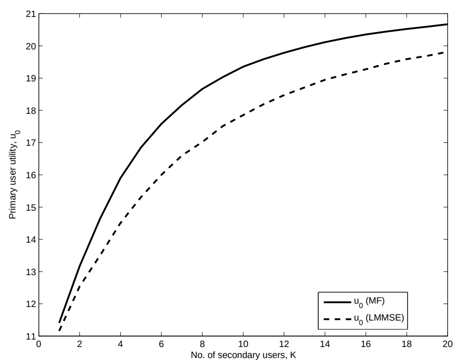

Fig. 3. Primary utility as a function of secondary system size K with $\lambda_s = 0.75$.

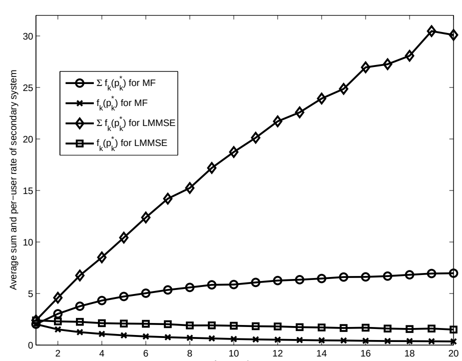

Fig. 4. Average sum-rate and the per-user rate achieved by the secondary system as a function of secondary system size K with $\lambda_s = 0.75$.

VII. CONCLUSIONS

In this paper, we further developed the dynamic spectrum leasing framework of [2] for DSS. We first generalized the primary user utility function defined in [2] for the DSL game. Next we allowed for different linear multiuser detectors, in particular, MF and LMMSE receivers at the secondary receivers. We established the general conditions on the primary user reward function $F(.)$ so as to ensure the existence of a Nash equilibrium. Through a series of simulations, we showed that the system with the LMMSE receiver outperforms that with the MF receiver, in terms of both the allowed secondary system size and the outage probability.

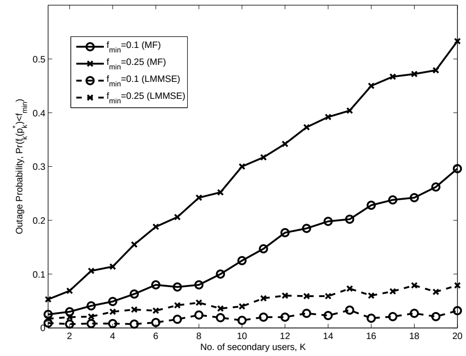

Fig. 5. Outage probability $Pr\left(f_k\left(p_k^*\right) < f_{min}\right)$ of a typical secondary user at the NE of the DSL game for a required QoS requirement f_{min} as a function of secondary system size K with $\lambda_s = 0.75$.

ACKNOWLEDGMENT

K. Hakim and S. K. Jayaweera were supported in part by the Air Force Research Laboratory (AFRL) Space Vehicles Directorate and the National Science foundation (NSF) under the grant CCF-0830545. C. Mosquera was supported by the Spanish Government under projects SPROACTIVE (ref. TEC2007-68094-C02-01/TCM) and COMONSENS (CONSOLIDER-INGENIO 2010 CSD2008-00010).

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