

# Maximizing Throughput in Wireless Multi-Access Channel Networks

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**Abstract**—Recent advances in the physical layer have enabled the simultaneous reception of multiple packets by a node in wireless networks. In this paper, we present a generalized model for the throughput optimization problem in multi-hop wireless networks that support multi-packet reception (MPR) capability. The model incorporates the multi-access channel, which accurately accounts for the achievable capacity of links used by simultaneous packet transmissions. The problem is modeled as a joint routing and scheduling problem. The scheduling subproblem deals with finding the optimal *schedulable sets*, which are defined as subsets of links that can be scheduled or activated simultaneously. We demonstrate that any solution of the scheduling subproblem can be built with  $|E| + 1$  or fewer schedulable sets, where  $|E|$  is the number of links of the network. This result contrasts with a conjecture that states that a solution of the scheduling subproblem, in general, is composed of an exponential number of schedulable sets. Due to the hardness of the problem, we propose a polynomial time scheme based on a combination of linear programming and greedy paradigms. The scheme guarantees the operation of links at maximum aggregate capacity, where the sum of the capacity of the links is maximized and the multi-access channel is fully exploited.

## I. INTRODUCTION

Current communication protocols and architectures for multi-hop wireless networks are mostly based on a single user channel model. In this model, only one transmission can be achieved in a disk of a radio proportional to the distance between the sender and the receiver, centered at the receiver node [1]. As a result, the number of concurrent transmissions in the network is very limited. The seminal paper by Gupta and Kumar [1] demonstrated that the throughput in a connected random wireless network of a set  $V$  of nodes adhering to such communication paradigm scales as  $\Theta\left(\frac{1}{\sqrt{|V|\log|V|}}\right)$ .

To overcome the above limitation, recently researchers studied an alternative paradigm to the single user channel, called multi-packet reception (MPR). In MPR-capable networks, multiple nodes around a receiver can simultaneously transmit to it, and the receiver node can decode multiple packets by exploiting multiuser techniques such as successive interference cancellation (SIC) and CDMA. As a main result, Garcia-Luna-Aceves [2] et al. demonstrated that this paradigm increases the network capacity by a factor  $\Theta(\log|V|)$  with respect to the single user paradigm. Driven by these results, subsequent work considered alternative schemes to approximate to the asymptotic bounds under homogeneous assumptions, such as

nodes transmit to a single base station or access point [3], or nodes are equipped with a single omni-directional antenna [4]. To further reduce complexity, previous work considered the packet as the *only* unit of transmission, neglecting natural restrictions imposed by information theoretic limits of the multi-access channel, or assuming unit link capacities [3], [4], which implies suboptimal channel usage [5]. Celik et al. [3] studied the negative implications of reusing legacy MAC protocols in MPR-capable networks, and how alternative backoff mechanisms can improve throughput and fairness. Karande et al. [6] showed that the network capacity can be further increased if nodes can also transmit to multiple receivers at the same time.

In this paper, we present a throughput optimization model for multi-hop wireless networks with MPR-capability. The model incorporates several features not included in previous work, namely: i) the use of the multi-access channel to accurately accounts for the capacity of links used by simultaneous packet transmissions; ii) general formulation, valid for networks with single or multiple directional or omni-directional transmit antennas per node; iii) a characterization of the scheduling subproblem as a convex optimization; iv) a further convex analysis that demonstrates that any solution of the scheduling subproblem can be built with  $|E| + 1$  or fewer *schedulable sets*, which are defined as subsets of links that can be scheduled or activated simultaneously; and v) a polynomial time scheme to solve the problem based on a combination of linear programming and greedy paradigms.

The paper is organized as follows. Section II presents the antenna and channel models. Section III formulates the throughput optimization problem in MPR networks, and Section IV presents a polynomial time scheme. Section V shows performance studies, and Section VI concludes our work.

## II. ANTENNA AND CHANNEL MODELS

Let  $G = (V, E)$  be a wireless network, where  $V$  is the set of nodes and  $E$  the set of links. Let  $r_{ij}$  be the distance between two nodes  $i$  and  $j$ . There is a link  $(i, j) \in E$  from  $i \in V$  to  $j \in V$  if  $r_{ij} \leq R$ , where  $R$  is the *receiver range* [7].

### A. Antenna Model

The antenna model considered in this paper is the one used in [8]. Sidelobes and backlobes are ignored. We assume that: i) the gain of the antenna is a function of the azimuth angle only;

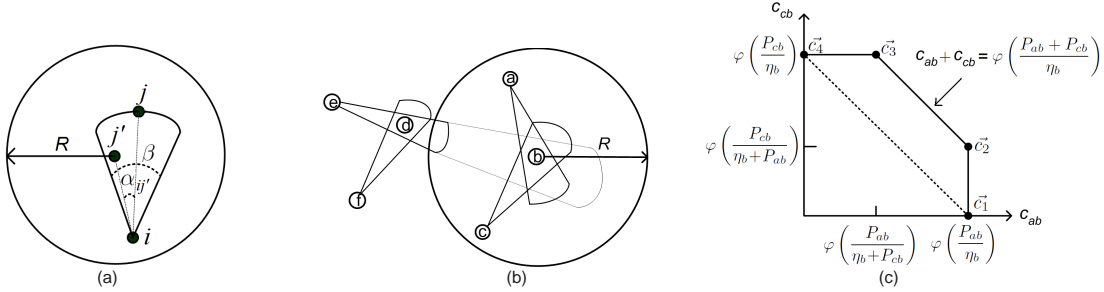


Fig. 1. (a) Radiation pattern model. (b) Simultaneous transmissions to node  $b$ . (c) Capacity region of the multi-access channel for receiver node  $b$  in (b). The region is upper bounded by solid lines. For a single user channel model (TDMA), the capacity region is upper bounded by the dashed line.

it is constant (greater than zero) inside the main lobe, and zero outside it. The beamwidth is denoted as  $\beta$ ; and ii) the axis of the main lobe, the boresight, can be directed to one direction at a time. Fig. 1(a) shows the radiation pattern, where node  $i$  transmits to node  $j$ .  $\alpha_{ij}$  is the angle between the boresight of the transmit antenna at node  $i$  and the direction of an arbitrary node  $j'$ . When a link  $(i, j)$  is scheduled, node  $i$  points out its transmit antenna so that  $\alpha_{ij} = 0$ . To account for the case where a node transmits to multiple receivers at the same time through multiple antennas [6], we denote the number of transmit antennas as  $M$ . Let  $S \subseteq E$  be a *schedulable set* consisting of links which are scheduled at a time. Thus, if

$$\mathcal{R}_i = \{j | (i, j) \in S\} \quad (1)$$

denotes the set of nodes receiving from node  $i$ , we will require that  $|\mathcal{R}_i| \leq M$ .

### B. Multi-Access Channel

Let  $P_0$  be the uniform fixed power level used for any transmission. For an active link  $(i, j)$ , the received power  $P_{ij}$  at node  $j$  decays exponentially with  $r_{ij}$ ,  $P_{ij} = P_0 r_{ij}^{-\gamma}$ , where  $\gamma$  is the path loss exponent. Consider the following scenario: let  $\mathcal{T}_j$  be the set of nodes transmitting to node  $j$ :

$$\mathcal{T}_j = \{i | (i, j) \in S\}. \quad (2)$$

Similarly, let  $\Psi_j$  be the set of simultaneous transmitters interfering with node  $j$ , which point out their transmit antenna such that node  $j$  lies inside the main lobe of them:

$$\Psi_j = \{i' | (i', j') \in S, j \neq j', \text{ and } \frac{-\beta}{2} \leq \alpha_{i'j} \leq \frac{\beta}{2}\}. \quad (3)$$

Then, assuming an Additive White Gaussian Noise (AWGN) channel, the total noise at receiver node  $j$  is:

$$\eta_j = \eta + \sum_{i' \in \Psi_j} P_{i'j}, \quad (4)$$

where  $\eta$  is the variance of the channel noise.  $\eta_j$  is referred as destructive interference [9]. Define the capacity function of a single user of an AWGN channel with bandwidth  $W$  and signal to interference plus noise ratio  $SINR$  as:  $\varphi(SINR) = W \log_2(1 + SINR)$ . Let  $c_{ij}(S)$  be the capacity in bits per second (bps) of a link  $(i, j) \in S$ , when all links in  $S$  are activated. For a set  $\mathcal{T}_j$ , the link capacities can be represented as a link capacity vector  $(c_{i_1j}(S), c_{i_2j}(S), \dots, c_{i_{|\mathcal{T}_j|}j}(S))$ , where nodes  $i_1, i_2, \dots, i_{|\mathcal{T}_j|}$  transmit to  $j$ . For receiver  $j$ , the capacity region is the convex hull of link capacity vectors satisfying:

$$\sum_{i_h \in \mathcal{T}} c_{i_hj}(S) \leq \varphi\left(\frac{\sum_{i_h \in \mathcal{T}} P_{i_hj}}{\eta_j}\right), \quad (5)$$

for all  $T \subseteq \mathcal{T}_j$  [5]. The region is characterized by  $2^{|\mathcal{T}_j|} - 1$  constraints, each corresponding to a nonempty subset of transmitters, and has precisely  $|\mathcal{T}_j|!$  vertices in the positive quadrant, each achievable by SIC.

*Example 1.* Consider the scenario in Fig. 1(b), where  $S = \{(a, b), (c, b), (e, d), (f, d)\}$ , nodes  $a$  and  $c$  transmit to node  $b$  ( $\mathcal{T}_b = \{a, c\}$ ), and node  $e$  interferers with node  $b$  ( $\Psi_b = \{e\}$  and  $\eta_b = \eta + P_{eb}$ ). The capacity region for node  $b$  is shown in Fig. 1(c). The vertices are labeled with vector notations,  $\vec{c}_i = (c_{ab}(S), c_{cb}(S))$ ,  $i \in \{1, \dots, 4\}$ . The aggregate capacity,  $c_{ab}(S) + c_{cb}(S)$ , is maximized when a link capacity vector lies in the segment line between  $\vec{c}_2$  and  $\vec{c}_3$ . The points  $\vec{c}_2$  and  $\vec{c}_3$  can be achieved by using SIC and CDMA. For example,  $\vec{c}_2$  can be obtained in a two-stage SIC decoding process. In the first stage, node  $b$  decodes packet  $p_1$  from node  $c$ , considering the transmission from node  $a$  as part of noise. Therefore,  $c_{cb}$  can be  $\varphi\left(\frac{P_{cb}}{\eta_b + P_{ab}}\right)$ . In the second stage, after packet  $p_1$  has been decoded, it can be subtracted out, thereafter packet  $p_2$  from node  $a$  can be decoded. Thus, the link capacity can be  $\varphi\left(\frac{P_{ab}}{\eta_b}\right)$ . For a general number of transmitters, the points in the capacity region that maximize the sum of the capacity of the links are defined as follows.

*Definition 1:* Let  $\mathcal{T}_j = \{i_1, i_2, \dots, i_{|\mathcal{T}_j|}\}$  be a set of nodes transmitting to a receiver node  $j$ . The corresponding links  $(i_1, j), (i_2, j), \dots, (i_{|\mathcal{T}_j|}, j)$  are said to operate at max-capacity iff  $\sum_{i_h \in \mathcal{T}_j} c_{i_hj} = \varphi\left(\frac{\sum_{i_h \in \mathcal{T}_j} P_{i_hj}}{\eta_j}\right)$ .

In practice, a receiver node can decode only a finite number of packets [2], [3]. Interference models for MPR-capable networks [2] restrict the number of transmissions inside the disk of radio  $R$  centered at a receiver node  $j$  (independently of whether transmissions are intended for node  $j$  or not) to a certain value  $K$ . This constraint is given by Eq. (7):

$$\begin{aligned} \mathcal{T}'_j &= \mathcal{T}_j \cup \{i | i \in \Psi_j \text{ and } r_{ij} \leq R\}, \\ |\mathcal{T}'_j| &\leq K. \end{aligned} \quad (6)$$

To be consistent with previous works [2], [3], [4], we will assume that Eq. (7) is satisfied for a receiver  $j$  to decode packets. We will refer to  $K$  as the *decoding capability*.

### C. Scheduling in MPR-Capable Networks

Denote as  $(M, K, \beta)$ -network a network where every node has a decoding capability of  $K$ , and  $M$  transmit antennas with beamwidth  $\beta$ . Let  $S \subseteq E$  be a set of links simultaneously

scheduled. For any  $(i, j) \in S$ ,  $\mathcal{R}_i$ ,  $\mathcal{T}_j$ , and  $\mathcal{T}'_j$  are defined by Eqs. (1), (2) and (6). Then, we have the following definition.

*Definition 2:* Given an  $(M, K, \beta)$ -network, a set  $S \subseteq E$  is a schedulable set iff  $\forall (i, j) \in S$ :

$$|\mathcal{R}_i| \leq M, \quad (8)$$

$$|\mathcal{T}'_j| \leq K, \quad (9)$$

$$\sum_{i' \in \mathcal{T}} c_{i'j}(S) \leq \varphi \left( \frac{\sum_{i' \in \mathcal{T}} P_{i'j}}{\eta_j} \right), \forall \mathcal{T} \subseteq \mathcal{T}_j. \quad (10)$$

### III. PROBLEM FORMULATION

To formulate the problem succinctly, we first present the routing subproblem, followed by the scheduling subproblem. Then, we formulate the joint routing and scheduling problem.

#### A. Routing

Let  $N$  be the set of flows. Each flow is characterized by a 3-tuple  $(s_n, d_n, f_n)$ , which denotes the source, the destination, and the flow<sup>1</sup> in bps transmitted from  $s_n$  to  $d_n$  respectively. Let  $x_{ij}^n$  be a variable representing the amount of the  $n^{\text{th}}$  flow routed on link  $(i, j)$ . The routing linear program (RT-LP) is defined in Fig. 2. Eq. (11) is the throughput. Eq. (12) represents the flow conservation constraints. Eq. (13) states that the total amount of flow routed through a link  $(i, j)$  cannot exceed  $\varphi \left( \frac{P_{ij}}{\eta} \right)$ , which is an upper bound of its capacity.

$$\begin{aligned} \max F_{RT-LP} &= \sum_{n \in N} f_n \quad (11) \\ \sum_{j:(i,j) \in E} x_{ij}^n - \sum_{j:(j,i) \in E} x_{ji}^n &= \begin{cases} f_n; i = s_n \\ -f_n; i = d_n \\ 0; \text{otherwise} \end{cases} \quad (12) \\ \sum_{n \in N} x_{ij}^n &\leq \varphi \left( \frac{P_{ij}}{\eta} \right); (i, j) \in E \quad (13) \\ f_n, x_{ij}^n &\geq 0; n \in N, (i, j) \in E \quad (14) \end{aligned}$$

Fig. 2. Routing linear program (RT-LP).

#### B. Scheduling

A schedule specifies the schedulable sets and the fraction of time allocated to each set. Let  $\Gamma = \{S_1, S_2, \dots, S_{|\Gamma|}\}$  be the set of *all schedulable sets*. Let  $\lambda_k, 0 \leq \lambda_k \leq 1$ , be a fraction of time allocated to the set  $S_k$ . We may write the time interval  $[0, 1]$  as  $\cup_k [t_k, t_{k+1}]$ , where links in  $S_k$  are activated for the activity period  $t_{k+1} - t_k = \lambda_k, k \in \{1, 2, \dots, |\Gamma|\}$ . We will call the variable  $\lambda_k$  as *activity period variable* corresponding to the schedulable set  $k$ . The schedule restriction can be written as:

$$\sum_{k: S_k \in \Gamma} \lambda_k = 1. \quad (15)$$

Since a link may be activated during multiple activity periods, the amount of flow routed through it must not exceed the sum of its capacity on those periods:

$$\sum_{n \in N} x_{ij}^n \leq \sum_{\forall k \in \{1, 2, \dots, |\Gamma|\} | (i, j) \in S_k} \lambda_k c_{ij}(S_k). \quad (16)$$

<sup>1</sup>Although a flow is characterized by  $(s_n, d_n, f_n)$ , we will also use the term flow to informally refer to  $f_n$ .

$$\max_{\Gamma' \subseteq \Gamma} F_{RTSCH-LP} = \sum_{n \in N} f_n \quad (17)$$

$$\sum_{j:(i,j) \in E} x_{ij}^n - \sum_{j:(j,i) \in E} x_{ji}^n = \begin{cases} f_n; \text{if } i = s_n \\ -f_n; \text{if } i = d_n \\ 0; \text{otherwise} \end{cases} \quad (18)$$

$$\sum_{n \in N} x_{ij}^n \leq \sum_{\forall k | (i, j) \in S_k} \lambda_k c_{ij}(S_k); \quad (i, j) \in E \quad (19)$$

$$\sum_{k: S_k \in \Gamma'} \lambda_k = 1 \quad (20)$$

$$f_n, x_{ij}^n, \lambda_k \geq 0; n \in N, (i, j) \in E, k \in \{1, \dots, |\Gamma'|\} \quad (21)$$

Fig. 3. Routing and scheduling linear program (RTSCH-LP).

#### C. Joint Routing and Scheduling

By incorporating Eqs. (15) and (16) into RT-LP and optimizing over all possible set  $\Gamma' \subseteq \Gamma$  of schedulable sets, the joint routing and scheduling problem is shown in Fig. 3.

The complexity of the routing and scheduling linear program (RTSCH-LP) is mainly determined by the scheduling subproblem. Thus, a key issue is the characterization of it. A schedulable set can be characterized by a *schedulable vector*  $\vec{S}$  of size  $|E|$ . The  $j^{\text{th}}$  element of this vector is set to one if the link  $e_j \in E$  is a member of  $\vec{S}$ , and to zero otherwise. Any schedulable vector  $\vec{S}$  can be regarded as a point in an  $|E|$ -dimensional space, which also becomes a vertex of the convex hull of the set of schedulable vectors. Let  $\vec{u} = (u_1, u_2, \dots, u_{|E|})$  be an  $|E|$ -dimensional *utilization vector*, where  $u_i$  indicates the total fraction of time allocated to link  $e_i \in E$ . By regarding the utilization vector as a point in  $\{0, 1\}^{|E|}$ , then we have the following theorem.

*Theorem 1:* Let  $Co(\Gamma)$  be the convex hull of all schedulable vectors. A solution to the scheduling subproblem given by a set  $\Gamma' = \{S_1, S_2, \dots, S_{|\Gamma'|}\} \subseteq \Gamma$  with corresponding activity periods  $\lambda_1, \lambda_2, \dots, \lambda_{|\Gamma'|}$  is feasible iff the resulting utilization vector  $\vec{u}$  lies within  $Co(\Gamma)$ .

For the proof of *Theorem 1*, please refer to [7]. Hereafter,  $Co(\Gamma)$  and *allocation polytope* will be used interchangeably.

*Example 2.* Consider Fig. 4(a) and assume a half-duplex network with  $K = 1$ . Assume also the existence of two end-

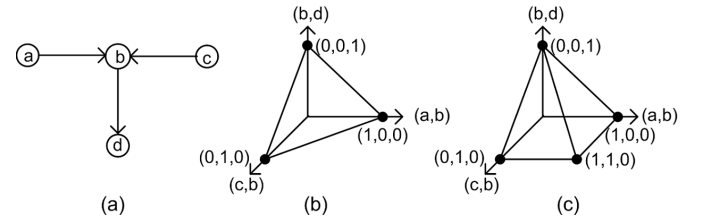


Fig. 4. (a) Network topology with three links:  $(a, b)$ ,  $(c, b)$ , and  $(b, d)$ . (b) Allocation polytope, for  $K = 1$  and half-duplex operation of network in (a). The schedulable vectors are:  $\vec{S}_0 = (0, 0, 0)$ ,  $\vec{S}_1 = (1, 0, 0)$ ,  $\vec{S}_2 = (0, 1, 0)$ , and  $\vec{S}_3 = (0, 0, 1)$ . (c) Allocation polytope, for  $K = 2$ , of network in (a). The schedulable vector  $\vec{S}_4 = (1, 1, 0)$  is included, since node  $b$  can simultaneously decode packets from nodes  $a$  and  $c$ .

to-end flows: flow 1, from node  $a$  to  $d$  routed through node  $b$ ; and flow 2, from node  $c$  to  $d$  routed through node  $b$ . The conservation constraints (Eq. (18)) state that the amount of flow at node  $b$  must be zero, while the amount of flow leaving nodes  $a$  and  $c$  must be maximized. The only schedule that activates the three links includes  $S_1 = \{(a, b)\}$ ,  $S_2 = \{(c, b)\}$ , and  $S_3 = \{(b, d)\}$ . The constraints given by Eq. (19) are:

$$x_{ab}^1 \leq \lambda_1 \varphi \left( \frac{P_{ab}}{\eta_b} \right), x_{cb}^2 \leq \lambda_2 \varphi \left( \frac{P_{cb}}{\eta_b} \right), x_{bd}^1 + x_{bd}^2 \leq \lambda_3 \varphi \left( \frac{P_{bd}}{\eta_d} \right),$$

where  $\eta_b = \eta_d = \eta$ . The scheduling constraint given by Eq. (20) is expressed as:  $\lambda_1 + \lambda_2 + \lambda_3 = 1$ . It requires the scheduling algorithm to allocate time to each schedulable set, such that the resulting utilization vector lies inside the allocation polytope shown in Fig. 4(b). Assume now a multi-access channel with  $K = 2$ . The set  $S_4 = \{(a, b), (c, b)\}$  is now a schedulable set, and the capacities of links  $(a, b)$  and  $(c, b)$  are restricted to lie inside the capacity region shown in Fig. 1(c), where  $\eta_b = \eta$ . Let  $\vec{c}_2$  be the operation point, and  $\Gamma' = \{S_3, S_4\}$ . Then, the constraints given by Eq. (19) are:

$$x_{ab}^1 \leq \lambda_4 \varphi \left( \frac{P_{ab}}{\eta_b} \right), x_{cb}^2 \leq \lambda_4 \varphi \left( \frac{P_{cb}}{\eta_b + P_{ab}} \right), x_{bd}^1 + x_{bd}^2 \leq \lambda_3 \varphi \left( \frac{P_{bd}}{\eta_d} \right),$$

where  $\eta_b = \eta_d = \eta$ , and  $\lambda_3 + \lambda_4 = 1$ . Since  $S_4$  is now a schedulable set, it is included as a vertex of the allocation polytope in Fig. 4(c). From this example, we can see that links  $(a, b)$  and  $(c, b)$  are activated for  $\lambda_4$  seconds, and link  $(b, d)$  for  $\lambda_3$  seconds. In general, we may be interested on activating every link  $e_i \in E$  for certain fraction of time  $u_i$ . *Theorem 2* bounds the number of schedulable sets to achieve this.

*Theorem 2: Any  $\vec{u}$  can be represented as a convex combination of  $|E| + 1$  or fewer schedulable vectors in  $Co(\Gamma)$ .*

*Proof:* *Theorem 2* can be demonstrated by applying Caratheodory's theorem on convex sets [5]. Let  $\Gamma_1 = \{S_1, S_2, \dots, S_{|\Gamma_1|}\} \subseteq \Gamma$  be a schedule with corresponding allocation times  $\lambda_1, \lambda_2, \dots, \lambda_{|\Gamma_1|}$  greater than zero, and utilization vector  $\vec{u} = (u_1, \dots, u_{|E|})$ . We will assume that  $|\Gamma_1| > |E| + 1$ , and show that there is a solution  $\Gamma_2$  that produces the same utilization vector with no more than  $|E| + 1$  schedulable sets. Denote the  $i^{th}$  scalar component of the schedulable vector  $\vec{S}_k$  as  $S_{ki}$ . Then, for any  $e_i \in E$ , the component  $u_i$  of  $\vec{u}$  is:  $\sum_{k: S_k \in \Gamma_1} \lambda_k S_{ki} = u_i; e_i \in E$ . We can formulate a linear program where the optimization variables are activity period variables  $\lambda'_1, \lambda'_2, \dots, \lambda'_{|\Gamma_1|}$ , as shown in Fig. 5. The fundamental theorem of linear programming states that every feasible linear program has a basic feasible solution. In a basic feasible solution, only the basic variables are nonzero. The linear program of Fig. 5 has  $|E| + 1$  basic variables (one per equality constraint). Thus, it has a basic feasible solution with  $|E| + 1$  positive variables, which naturally corresponds to a set  $\Gamma_2 = \{S_k | S_k \in \Gamma_1 \text{ and } \lambda'_k > 0\}$ .  $\diamond$

*Example 3.* Consider Fig. 4(a) and allocation polytope of Fig. 4(c). Let  $\Gamma_1 = \{S_0, S_1, S_2, S_3, S_4\}$  be a schedule with allocation times  $\lambda_i = \frac{1}{5}$  for all  $S_i$ . The schedule produces a vector  $\vec{u} = \sum_{i=0}^4 \lambda_i \vec{S}_i = (\frac{2}{5}, \frac{2}{5}, \frac{1}{5})$ , activating links  $(a, b)$  and  $(c, b)$  for  $\frac{2}{5}$  s, and link  $(b, d)$  for  $\frac{1}{5}$  s. According to *Theorem 2*, we can build a schedule with no more than  $|E| + 1 = 4$  schedulable sets that produces the same utilization vector. A

$$\sum_{k: S_k \in \Gamma_1} \lambda'_k S_{ki} = u_i; e_i \in E \quad (22)$$

$$\sum_{k: S_k \in \Gamma_1} \lambda'_k = 1; \quad (23)$$

$$\lambda'_k \geq 0; k \in \{1, 2, \dots, |\Gamma_1|\} \quad (24)$$

Fig. 5. Linear program to obtain  $\Gamma_2 = \{S_k | S_k \in \Gamma_1 \text{ and } \lambda'_k > 0\}$ .

possible solution is  $\Gamma_2 = \{S_0, S_3, S_4\}$  with corresponding allocation times of  $\lambda_0 = \frac{2}{5}, \lambda_3 = \frac{1}{5}$ , and  $\lambda_4 = \frac{2}{5}$ . The allocation of  $\frac{2}{5}$  s to  $S_0$  implies that the network is idle for  $\frac{2}{5}$  s. Notice also that  $|\Gamma_2| = 3 < |E| + 1$ . In general, however, the number of schedulable sets we should expect is  $|E| + 1$ . For example, for the allocation polytope of Fig. 4(b) and for an utilization vector  $\vec{u} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ , the only set that produces the desired  $\vec{u}$  is  $\Gamma_3 = \{S_0, S_1, S_2, S_3\}$  with allocation times  $\lambda_i = \frac{1}{4}$  for all  $i$ . Thus,  $|\Gamma_3| = |E| + 1$ . *Theorem 2* contrasts with a conjecture that states that a solution of the scheduling subproblem is composed of an exponential number of schedulable sets [10]. The complexity of finding the optimal schedule arises from the large solution space.

#### IV. JRS SCHEME

We present a polynomial time Joint Routing and Scheduling (JRS) scheme for RTSCH-LP. The scheme consists of three steps: i) solve RT-LP; ii) with a Greedy Scheduler (GS) algorithm, create a set  $\Gamma_{GS} \subseteq \Gamma$  of schedulable sets; and iii) solve RTSCH-LP by only considering schedulable sets in  $\Gamma_{GS}$ .

1) *Step 1:* This step solves RT-LP and is intended to identify good paths for each flow, such that  $F_{RT-LP}$  is maximized. The output of step 1 is the set of links assigned a positive flow value by RT-LP, i.e.,  $E_{RT-LP} = \{(i, j) \in E | \sum_{n \in N} x_{ij}^n > 0\}$ .

2) *Step 2:* GS algorithm is shown in Fig. 6. The algorithm schedules, one by one, all links in  $E_{RT-LP}$  until a maximal set is created. Once the set becomes maximal, GS creates a new schedulable set with those links not scheduled yet in previous sets. Line 12 selects the receiver node, denoted by  $j$ , with the largest number of transmitters to that node. Then, lines 15-26 schedule as many links with node  $j$  as receiver as possible, so that the multi-access channel at node  $j$  is fully exploited. At line 16, a link  $(i, j)$  is chosen in decreasing order of distance between nodes  $i$  and  $j$ , to compensate the inferior capacity of the link (first links to be scheduled experience less interference, as explained below). The noise  $\eta_{ij}$  experienced by a link  $(i, j)$  is computed in line 20 and consists of destructive interference (first term) and constructive interference (second term). The latter refers to the power from the links previously scheduled to the same schedulable set, which also have  $j$  as receiver. Line 21 computes the capacity  $c_{ij}(S_k)$  the link  $(i, j)$  will operate at. It can be shown that any set of links scheduled simultaneously operates at max-capacity. For this proof, please refer to [11].

3) *Step 3:* This step solves RTSCH-LP by considering the schedulable sets in  $\Gamma_{GS}$  only. The solution gives the amount of flow  $x_{ij}^n$  routed through each link  $(i, j)$ , and establishes the fraction of time  $\lambda_k$  allocated to each set  $S_k \in \Gamma_{GS}$ .

## Greedy Scheduler (GS)

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1: INPUT:  $E_{RT-LP}$ ,  $G(V, E)$ ;
2: OUTPUT: Set  $\Gamma_{GS} = \{S_1, S_2, \dots, S_{|\Gamma_{GS}|}\}$  of schedulable sets, and link
   capacity  $c_{ij}(S)$ ,  $\forall (i, j) \in S, \forall S \in \Gamma_{GS}$ ;
3:  $\Gamma_{GS} = \{\}$ ;
4:  $k = 0$ ;
5: for all  $j \in V$  do
6:    $nt_j = |\{i | (i, j) \in E_{LP-RT}\}|$ ; // # of remainder transmitters to node  $j$ 
7: end for
8: while  $E_{LP-RT} \neq \{\}$  do
9:    $k = k + 1$ ;
10:   $S_k = \{\}$ ;
11:  while  $\exists (i, j) \in E_{RT-LP}$  and  $\mathcal{E}(S_k \cup \{i, j\})$  do
12:     $j = \arg \max\{nt_j | \exists (i, j) \in E_{RT-LP}$  and  $\mathcal{E}(S_k \cup \{i, j\})\}$ ;
13:     $\Psi_j = \{i' | (i', j') \in S_k, j \neq j', \text{ and } \frac{-\beta}{2} \leq \alpha_{i'j} \leq \frac{\beta}{2}\}$ ;
14:     $\eta_j = \eta + \sum_{i' \in \Psi_j} P_{i'j}$ ;
15:    while  $\exists (i, j) \in E_{RT-LP}$  and  $\mathcal{E}(S_k \cup \{i, j\})$  do
16:       $(i, j) = \arg \max\{r_{ij} | \exists (i, j) \in E_{RT-LP}$  and  $\mathcal{E}(S_k \cup \{i, j\})\}$ ;
17:       $S_k = S_k \cup \{(i, j)\}$ ;
18:       $E_{RT-LP} = E_{RT-LP} - \{(i, j)\}$ ;
19:       $nt_j = nt_j - 1$ ;
20:       $\eta_{ij} = \eta_j + \sum_{t: (t, j) \in S_k} P_{tj}$ ;
21:       $c_{ij}(S_k) = \varphi\left(\frac{P_{ij}}{\eta_{ij}}\right)$ ;
22:      for all  $(i', j') \in S_k | j \neq j'$  and  $\frac{-\beta}{2} \leq \alpha_{i'j'} \leq \frac{\beta}{2}$  do
23:         $\eta_{i'j'} = \eta_{i'j'} + P_{i'j'}$ ; // link  $(i, j)$  adds noise to  $(i', j')$ 
24:         $c_{i'j'}(S_k) = \varphi\left(\frac{P_{i'j'}}{\eta_{i'j'}}\right)$ ;
25:      end for
26:    end while
27:  end while
28:   $\Gamma_{GS} = \Gamma_{GS} \cup S_k$ ;
29: end while

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Fig. 6. Scheduling algorithm.  $\mathcal{E}(S)$  stands for the event  $\mathcal{E}(S) = \{\text{Eqs. (8) and (9) are satisfied for all } (i, j) \in S\}$ .

## V. PERFORMANCE STUDIES

We present a numerical example based on the scheme presented in Section IV, which was implemented as a solver in C language. Although we focus on  $(M, K, \beta)$ -networks, we also evaluated the performance of half-duplex (HD) networks for comparison purposes. Since Eqs. (8) and (9) model networks where nodes can send and receive simultaneously through different interfaces, they should be substituted for the following constraint in HD networks:  $|\mathcal{R}_i| + \left\lceil \frac{|\mathcal{T}_i|}{K} \right\rceil \leq 1$ , for every node  $i$  scheduled in  $S$  as transmitter or receiver. We set the channel bandwidth  $W = 1$  MHz, transmission power  $P_0 = 100$  mW, receiver range  $R = 30$  meters, path loss exponent  $\gamma = 3$ , and channel noise  $\eta = -10$  dB. We generated 4 random flows such that all links of a 30-node random network shown in Fig. 7 were scheduled by JRS. The results include the impact of different

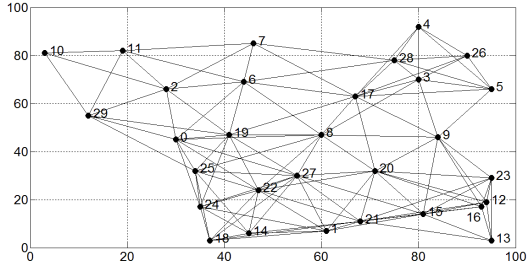


Fig. 7. A 30-node random network. There is a link (solid line) between two nodes if the distance between them is less than or equal to  $R = 30$ .

parameters on the *average node degree of the schedulable sets* found by JRS, which is denoted as  $g(M, K, \beta)$ . It is defined as the number of links per node activated on average:  $g(M, K, \beta) = \frac{1}{|V|} \sum_{i=1}^{|\Gamma_{GS}|} \lambda_i |S_i|$ .

*Impact of the number of transmit antennas.* Figs. 8(a) and 8(b) show the throughput obtained with two different values of  $M$ . For comparison purposes, the results in both figures are normalized to the maximum throughput found by JRS, which is obtained when  $M = 2, K = 5, \beta = \frac{\pi}{4}$ . As expected, the throughput increases with  $K$ . However, for high values of  $K$ , the throughput remains approximately constant and does not improve any further, independently of the beamwidth. The number of links simultaneously scheduled at any time is limited by the number of transmit antennas. This constitutes a *transmission-oriented bottleneck*; as  $K$  increases, transmitting nodes cannot generate enough transmissions to exploit the decoding capability. This fact is illustrated in Figs 8(c) and 8(d), where the average node degrees for  $M = 2$  and  $M = HD$  approach 1.5 and 0.53 as  $K$  increases. The size of schedulable sets cannot get larger, unless  $M$  is increased.

*Impact of the beamwidth.* Referring back to Fig. 8(a) and Fig. 8(b), note that for both  $H = 2$  and  $H = HD$ , and for high values of  $K$ , the results obtained with  $\beta = \frac{\pi}{4}$ ,  $\beta = \frac{\pi}{3}$ , and  $\beta = \frac{\pi}{2}$  converge to the same throughput. The disadvantage of having wider beamwidth antennas may be compensated by increasing the decoding capability. However, we should also highlight the impact of omnidirectional transmissions. Consider the results of Figs. 8(b) and 8(d) for  $\beta = 2\pi$ . For  $K \geq 9$ , even though the average node degree is about the same for all  $\beta$ , the throughput with omnidirectional antennas is notoriously inferior. A reason of this poor performance is the cumulative noise experienced at any receiver node; all transmissions not directed to a given receiver represent additional noise to that receiver. Figs. 8(e) and 8(f) compare the performance of networks with  $\beta = \frac{\pi}{3}$  and  $\beta = 2\pi$ . The results are normalized to the maximum throughput, obtained with  $M = 9, K = 10, \beta = \frac{\pi}{3}$ . Note that, with  $\beta = \frac{\pi}{3}$ , the throughput clearly increases with both  $K$  and  $M$ . On the other hand, with  $\beta = 2\pi$ , increments in  $M$  only do not lead to significant improvement. The better spatial reuse when  $\beta = \frac{\pi}{3}$  leads to larger schedulable sets than those obtained with  $\beta = 2\pi$ . Consequently, larger average node degrees are obtained with the former, as shown in Fig 8(g). For  $K < 6$  and  $M > 1$ ,  $g(M, K, \frac{\pi}{3})$  is more than 3 times  $g(M, K, 2\pi)$ .

*Impact of the decoding capability.* Increasing  $K$  may improve throughput. However, independently of  $\beta$  and for a fixed value of  $K$ , increments on  $M$  may not result on better performances because of a *receiver-oriented bottleneck*; i.e., receiver nodes cannot decode more than  $K$  simultaneous transmissions, even though transmitting nodes may increase the number of transmissions. For a decoding capability of 4 ( $K = 4$ ), Fig. 8(h) shows the throughput as a function of  $\beta$  and  $M$ , normalized to the throughput obtained with  $M = 5, K = 4, \beta = \frac{\pi}{4}$ . For any value of  $\beta$  and small values of  $M$ , say  $M \leq 4$ , the throughput increases almost monotonically. On the other hand, incrementing  $M$  beyond 5

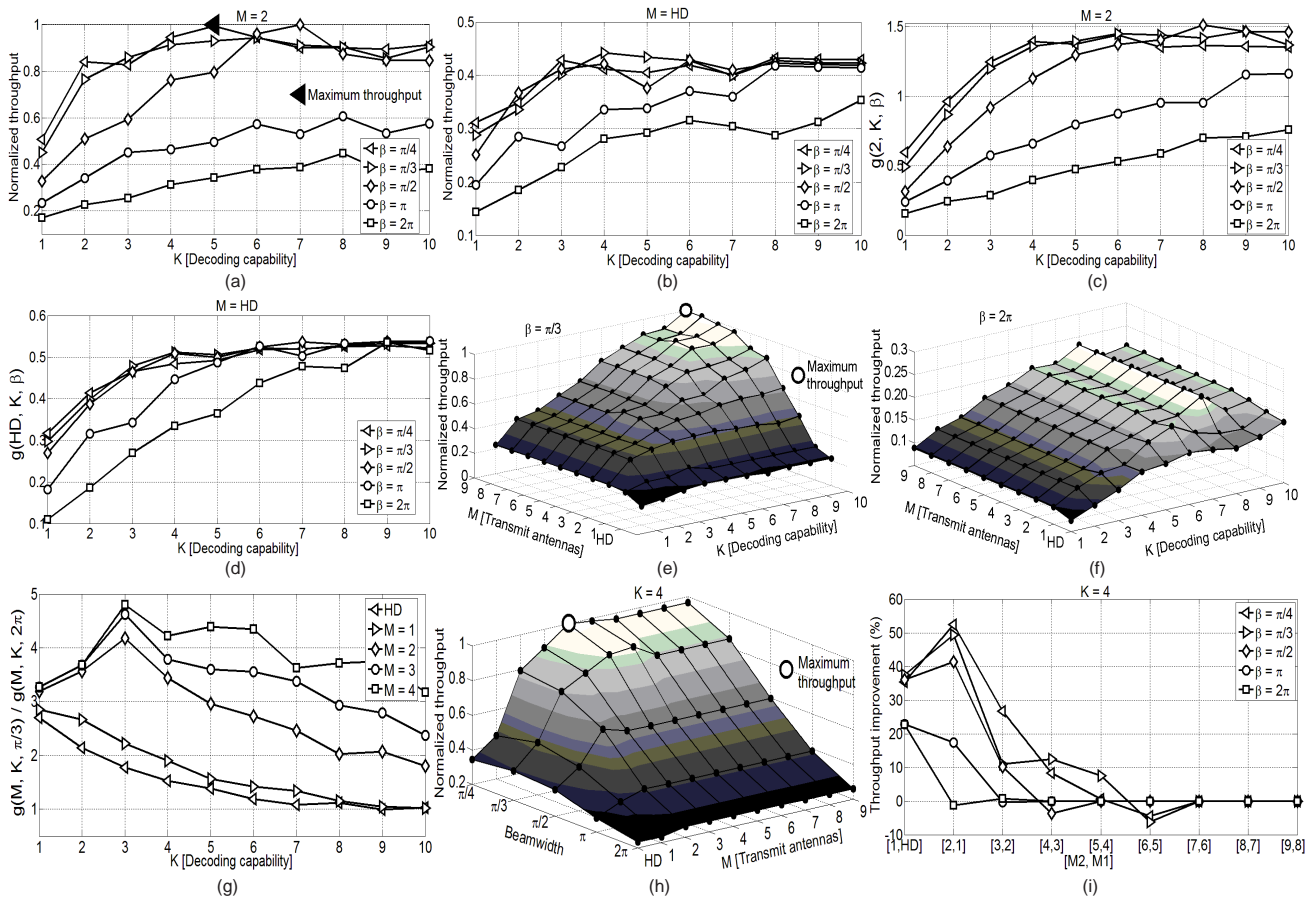


Fig. 8. Numerical results.

does not affect the performance. To quantify the throughput gain produced by a unitary increment on the number of transmit antennas, define throughput improvement ( $TI$ ) as:

$$TI(M2, M1) = \frac{F_{RTSCH-LP}(M2) - F_{RTSCH-LP}(M1)}{F_{RTSCH-LP}(M1)},$$

where  $F_{RTSCH-LP}(M)$  is the throughput obtained with  $M$  transmit antennas. Fig. 8(i) shows that, for any beamwidth, incrementing the number of transmit antennas from HD to 1 implies a throughput improvement of at least 20%. Increments on  $M$  have more impact when they are combined with narrow beamwidths; for example, increasing  $M$  from 1 to 2 has no impact when  $\beta = 2\pi$ . On the other hand, an improvement of at least 40% is obtained when  $\beta \geq \frac{\pi}{2}$ .

## VI. CONCLUSION

We have presented a generalized formulation for the throughput optimization problem in multi-hop MPR-capable wireless networks. To the best of our knowledge, the proposed model is the first joint routing and scheduling formulation that considers the capacity region of the multi-access channel in a network. The model accurately accounts for the capacity of the links used to simultaneously transmit to a common receiver. Additionally, we have proposed a polynomial time scheme that guarantees the operation of links at max-capacity, where the sum of the capacity of the links is maximized and the multi-access channel is fully exploited. Finally, we have

demonstrated that any solution of the scheduling subproblem can be built with  $|E|+1$  or fewer schedulable sets. Future work includes the incorporation of power control to our model.

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