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Information Theoretic Conditions for Tracking in Leader-Follower Systems with Communication Constraints

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Abstract: In this paper, we introduce a general framework for tracking in leader-follower systems under communication constraints, in which the leader and follower systems as well as the corresponding controllers are spatially distributed and connected over communication links. We provide necessary conditions on the channel data rate of each communication link for tracking of the leader-follower systems. By considering the forward and feedback channels as one cascade channel, we also provide a lower bound for the data rate of the cascade channel for the system to track a reference signal such that the tracking error has finite second moment. Examples and simulations are provided to demonstrate some of the results.

Keywords: Networked control systems; Leader-follower system; Information theory

1 Introduction

With the development and application of wireless communications and network science, traditional control systems have been extended in a distributed manner. Control and feedback signals are exchanged among the system's components in the form of information packages through a network, resulting in a *networked control system*.

In networked control system problems, understanding the fundamental relationship between how the control parts and the communication parts of the distributed system interact is significant for controller and communication channel design. Previous work in [1, 2] have shown that stabilization of a linear and time-invariant plant, requires that the channel data rate C to be larger than $\sum_i \max\{0, \log_2(|\lambda_i(A)|)\}$, where the sum is over all unstable eigenvalues of the dynamic matrix of the state-space representation of the plant. The papers [3, 4] have shown that the extra rate $C - \sum_i \max\{0, \log_2(|\lambda_i(A)|)\}$ is critical for performance, as measured by the expected power of the state of the plant. The authors in [4, 5] studied fundamental limitations in disturbance rejection in feedback systems and extended the Bode's integral equation for the case where the preview is made available to the controller via a general, finite capacity, communication channel.

On the other hand, other recent work have been focused on tracking issues in networked feedback systems. The work in [6] shows that a necessary condition for efficient tracking is that the information flow from the reference signal to the output should be greater than the information flow between the disturbance and the output. Meanwhile, the work in [7] define conditions for tracking such that tracking error has finite energy. Following the same approach, in this paper we find conditions for tracking such that the power of the tracking error stays finite. The authors in [7] obtained information theoretical conditions for tracking in linear time-invariant control systems, where the closed loop contains a channel in the feedback loop. The authors provided an upper bound for the mutual information rate between the feedback signal and the reference input signal and showed that this rate must be maximized to improve the tracking performance.

In this paper, we introduce a general framework for tracking in leader-follower systems under communication constraints, where the leader system, follower system and the corresponding controllers are spatially distributed and connected over communication links. The communication channels are used to exchange information and control signals among spatially distributed system components. We consider the particular case in which both the forward link from the reference signal input and the feedback link from the system output contain communication channels with finite data rate.

For this particular problem, we derive necessary conditions on channel data rate of the forward and feedback links for tracking in the leader-follower systems. Then, we show the effect of existence of the feedback link on the required channel data rate of the forward link, when the feedback link is noisy. The channel information loss of the feedback link requires an increase of the channel data rate of the forward link to compensate this effect for tracking between the two systems. By considering the forward and feedback channels as one cascade channel, we also provide a lower bound for the data rate of the cascade channel for the system to track the reference signal such that the tracking error has finite second moment.

The rest of the paper is organized as follows: Section II introduces the notation and the main definitions and properties from information theory. The problem formulation is given in Section III, where we describe the assumptions on communication channels, dynamic systems and the refer-

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ence signal. In Section IV, we show the necessary conditions on individual channels for tracking in leader-follower system and provide a lower bound on the data rate of the cascade channel by considering the forward channel and feedback channel together. In Section V, the above results are extended to the case that leader and follower systems have different system models. In Section VI, we study special cases and demonstrate our results in Section IV. The conclusions and possible extension are provided in Section VII.

2 Definitions and properties

In the following, we present the definitions and properties used in this paper.

Definition 1 (Entropy): For a given discrete random variable $\mathbf{x}$, the entropy is defined by:

$$h(\mathbf{x}) = \sum_{\mathbf{x}} p(\mathbf{x}) \log p(\mathbf{x}),$$

where $p(\mathbf{x})$ is the probability density function of $\mathbf{x}$.

Definition 2 (Mutual Information): The mutual information between discrete random variables $\mathbf{x}$ and $\mathbf{y}$ is defined as

$$I(\mathbf{x};\mathbf{y}) = \sum_{\mathbf{x}\in\mathcal{A}} \sum_{\mathbf{y}\in\mathcal{B}} p(\mathbf{x},\mathbf{y}) \log_2 \frac{p(\mathbf{x},\mathbf{y})}{p(\mathbf{x})p(\mathbf{y})},$$

where $p(\mathbf{x},\mathbf{y})$ is the joint probability density function of $\mathbf{x}$ and $\mathbf{y}$.

Definition 3 (Entropy Rate): For a given stochastic process $\mathbf{a}$, the entropy rate is defined as [11]:

$$h_\infty(\mathbf{a}) = \limsup_{k\to\infty} \frac{h(\mathbf{a}^k)}{k}.$$

Definition 4 (Information Rate): Let $\mathbf{a}$ and $\mathbf{b}$ be stochastic processes. The mutual information rates are defined as [4]:

$$I_\infty(\mathbf{a};\mathbf{b}) = \limsup_{k\to\infty} \frac{I(\mathbf{a}^k;\mathbf{b}^k)}{k}.$$

where $I(\mathbf{a}^k;\mathbf{b}^k)$ is mutual information between $\mathbf{a}^k$ and $\mathbf{b}^k$ and it can be obtained as follows:

$$I(\mathbf{a}^k;\mathbf{b}^k) = h(\mathbf{a}^k) - h(\mathbf{a}^k|\mathbf{b}^k). \tag{1}$$

Definition 5 (Directed Mutual Information and Directed Information Rate): Let $\mathbf{a}$ and $\mathbf{b}$ be stochastic processes. The directed mutual information is defined as follows [4]:

$$I(\mathbf{a}^k \to \mathbf{b}^k) = \sum_{i=1}^{k} I(\mathbf{a}^i;\mathbf{b}(i)|\mathbf{b}^{i-1}),$$

and the directed information rate is given by

$$I_\infty(\mathbf{a}\to\mathbf{b}) = \limsup_{k\to\infty} \frac{I(\mathbf{a}^k\to\mathbf{b}^k)}{k}.$$

Definition 6 (Channel Capacity): For channel CH_i with input $\mathbf{x}_i$ let the corresponding output be denoted by $\hat{\mathbf{x}}_i$, define the error function $E_i(k)$ at time step k as $E_i(k) = \begin{cases} 1 & \mathbf{x}_i \neq \hat{\mathbf{x}}_i \\ 0 & \mathbf{x}_i = \hat{\mathbf{x}}_i \end{cases}$. The channel capacity C_i^{Cap} is defined as the supremum of all achievable rates,

$$C_i^{\mathrm{Cap}} = \sup_{p(\mathbf{x}_i)} I(\mathbf{x}_i;\hat{\mathbf{x}}_i). \tag{2}$$

Properties Assume that $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathbb{R}^n$ are random variables, and f and g are real functions. All the following properties may be found in [11,12].

(P1) $I(\mathbf{a};\mathbf{b}) = I(\mathbf{b};\mathbf{a}) \geq 0$ and $I(\mathbf{a};\mathbf{b}|\mathbf{c}) = I(\mathbf{b};\mathbf{a}|\mathbf{c}) \geq 0$.

(P2) $I((\mathbf{a},\mathbf{b});\mathbf{c}|\mathbf{d}) = I(\mathbf{b};\mathbf{c}|\mathbf{d}) + I(\mathbf{a};\mathbf{c}|(\mathbf{b},\mathbf{d}))$.

(P3) If f and g are measurable functions then $I(f(\mathbf{a});g(\mathbf{b})|\mathbf{c}) \leq I(\mathbf{a};\mathbf{b}|\mathbf{c})$ and equality holds if f and g are invertible.

(P4) Given a function $f : \mathcal{C} \to \mathcal{C}'$, it follows that $I(\mathbf{a};f(\mathbf{c})|\mathbf{c}) = 0$.

(P5) $h(\mathbf{a}|\mathbf{b}) = h(\mathbf{a} - g(\mathbf{b})|\mathbf{b})$.

(P6) $h(\mathbf{a}|\mathbf{b}) \leq h(\mathbf{a})$ with equality if $\mathbf{a}$ and $\mathbf{b}$ are independent.

(P7) Let $\mathbf{a} \in \mathbb{R}^n$ have mean $\boldsymbol{\mu}$ and covariance $\mathrm{Cov}\{\mathbf{a}\}$. Then

$$h(\mathbf{a}) \leq \frac{1}{2}\log_2\big((2\pi e)^n \det(\mathrm{Cov}\{\mathbf{a}\})\big)$$

with equality if $\mathbf{a}$ has a multivariate normal distribution.

(P8) $I((\mathbf{a},\mathbf{b});\mathbf{c}) = I(\mathbf{a};\mathbf{c}|\mathbf{b}) + I(\mathbf{b};\mathbf{c})$.

(P9) Fano's inequality For channel CH_i with input $\mathbf{x}_i$ and corresponding output $\hat{\mathbf{x}}_i$. Let the probability of error as $P_{e,i} = \Pr\{\hat{\mathbf{x}}_i \neq \mathbf{x}_i\}$. Then

$$h(P_{e,i}) + P_{e,i}\log_2|\mathcal{X}_i| \geq h(\mathbf{x}_i|\hat{\mathbf{x}}_i), \tag{3}$$

where $\mathcal{X}_i$ is the alphabet for input $\mathbf{x}_i$.

(P10) $I((\mathbf{a},\mathbf{b});\mathbf{c}) \geq I(\mathbf{b};\mathbf{c})$.

3 Problem formulation

Consider the following networked control system as in Fig. 1. There are two physical systems P_1 and P_2 controlled by corresponding controllers $\mathcal{C}_1$ and $\mathcal{C}_2$ over communication channels CH_i for $i \in \{11, 12, 2, 3, 4\}$ with finite rates. The systems and controllers are spatially distributed and connected over communication links. The communication channels are used to exchange information and control signals among spatially distributed system components such as controllers, actuators and sensors.

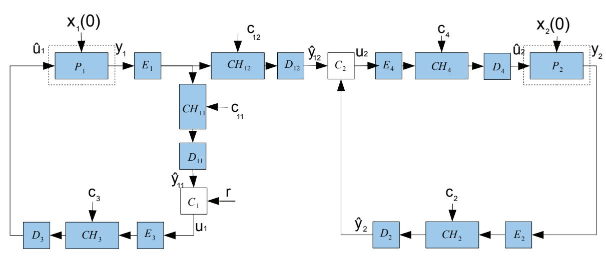

Fig. 1 A general model of a networked control system with two plants.

In our framework, the two linear and time-invariant systems denoted as the leader P_1 and the follower P_2 are assumed to be identical. The framework could be further extended to more general leader-follower network with multiple leaders and followers interconnected over communication links. There are many possible applications of such a general framework including, for example, distributed control of power plants in smart-grid, where control input could be applied to one generator and other generators could act as followers to track the state of that generator. Another possible application is the formation control of homogeneous robots with exterior control applied only to the leader

[8∼10]. However, such general network topologies are beyond the scope of this paper.

Suppose that the reference signal $\mathbf{r}(k)$ is only available for the leader system P_1. The follower system P_2 does not have information of $\mathbf{r}(k)$ and has to track the state of the leader system based only on the output of the plant P_1 over a communication network. The goal is to find the lower bound of channel data rate for channel CH_i to convey enough information to both controllers $\mathcal{C}_1$ and $\mathcal{C}_2$ to generate efficient feedback control signals such that the plants P_1 and P_2 could track $\mathbf{r}(k)$ accurately. Note that, the reference signal $\mathbf{r}(k)$ may not be available for follower system P_2 as a result of the high cost of information delivery to each plant due to long-range spatial separation between systems or a large number of follower systems (not considered in this paper).

In the following, we formulate the discrete-time state-space representation for the leader and follower systems [1]:

$$\begin{aligned}\mathbf{x}_i(k+1) &= F\mathbf{x}_i(k) + G\hat{\mathbf{u}}_i(k),\\ \mathbf{y}_i(k) &= H\mathbf{x}_i(k), \qquad k \geq 0, i = 1, 2,\end{aligned} \tag{4}$$

where the states $\mathbf{x}_i(k)$ takes values in $\mathbb{R}^n$ and the received control input $\hat{\mathbf{u}}_i(k)$ takes values in $\mathbb{R}^r$. The initial state $\mathbf{x}_i(0)$ is a zero mean Gaussian random variable with covariance matrix Σ_{0i}. The state is observed by sensor that generates the measurement $\mathbf{y}_i(k)$ taking values in $\mathbb{R}^q$.

We assume that the pair (F, G) is controllable and the pair (F, H) is observable based on the fact that follower system P_2 tries to track the state of leader system P_1, which requires system state $\mathbf{x}_1$ to be observable and $\mathbf{x}_2$ to be controllable. Since the pair (F, H) is observable, the system state $\mathbf{x}_i$ could be sufficiently determined by output signal $\mathbf{y}_i$. In order to simplify the derivation in next section and achieve theoretical results, we assume that system state could be estimated by $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$ [2], where matrix $L \in \mathbb{R}^{n\times q}$ is a transformation matrix.

Here we define the tracking errors of systems P_1 and P_2 as $\boldsymbol{\xi}_1(k) = \mathbf{r}(k) - \mathbf{x}_1(k)$ and $\boldsymbol{\xi}_2(k) = \mathbf{x}_1(k) - \mathbf{x}_2(k)$, where $\boldsymbol{\xi}_i(k)$ is a stochastic process with mean $\boldsymbol{\mu}_i$ and covariance $\mathrm{Cov}\{\boldsymbol{\xi}_i\}$, for $i = 1, 2$. Assume that the matrix $F = \begin{bmatrix} F_s & 0 \\ 0 & F_u \end{bmatrix}$ where $0 < |\lambda_i(F_s)| < 1$ and $|\lambda_i(F_u)| \geq 1$. Therefore, F^k is invertible $\forall k$.

The encoders and decoders are described as follows:

1) Encoder: At every time step k, encoder ϵ_i calculates and transmits the vector $s_i(k)$ for $i = 1, \cdots, 4$, according to the following functional structure:

$$\begin{aligned}&\mathbf{s}_1(k) = \epsilon_1(\mathbf{y}_1^k),\ \mathbf{s}_2(k) = \epsilon_2(\mathbf{y}_2^k),\\ &\mathbf{s}_3(k) = \epsilon_3(\mathbf{u}_1^k),\ \mathbf{s}_4(k) = \epsilon_4(\mathbf{u}_2^k),\end{aligned}$$

where $\mathbf{s}_i(k)$ takes values in $\mathbb{R}^j$ and $\mathbf{y}_i^k = \{\mathbf{y}_i(1), \cdots, \mathbf{y}_i(k)\}$.

2) Discrete-time memory-less Channel (DMC): Let $\mathfrak{S}_i$ and $\mathfrak{Z}_i$ be given input and output alphabets, along with a white stochastic process, denoted as $\mathbf{c}_i$, with alphabet $\mathfrak{C}_i$. Consider the mapping $\mathcal{F}_i : \mathfrak{S}_i \times \mathfrak{C}_i \to \mathfrak{Z}_i$ for $i \in \{11, 12, 2, 3, 4\}$ such that the following maps: $\mathbf{z}_i(k) = \mathcal{F}_i(\mathbf{s}_i(k), \mathbf{c}_i(k))$, where $\mathbf{c}_i$ is the channel noise.

3) Decoder: We consider the decoder for channel CH_i is of the following form:

$$\begin{aligned}&\hat{\mathbf{y}}_{11}(k) = D_{11}^k(\hat{\mathbf{y}}_{11}^{k-1}, \mathbf{z}_{11}^k),\ \hat{\mathbf{y}}_{12}(k) = D_{12}^k(\hat{\mathbf{y}}_{12}^{k-1}, \mathbf{z}_{12}^k),\\ &\hat{\mathbf{y}}_2(k) = D_2^k(\hat{\mathbf{y}}_2^{k-1}, \mathbf{z}_2^k),\ \hat{\mathbf{u}}_1(k) = D_3^k(\hat{\mathbf{u}}_1^{k-1}, \mathbf{z}_3^k),\\ &\hat{\mathbf{u}}_2(k) = D_4^k(\hat{\mathbf{u}}_2^{k-1}, \mathbf{z}_4^k).\end{aligned}$$

The controllers $\mathcal{C}_1$ and $\mathcal{C}_2$ are defined as follows:

$$\begin{aligned}&\mathcal{C}_1: \quad \mathbf{u}_1(k) = f_1(\mathbf{e}_1^k) \text{ with } \mathbf{e}_1(k) = H\mathbf{r}(k) - \hat{\mathbf{y}}_{11}(k),\\ &\mathcal{C}_2: \quad \mathbf{u}_2(k) = f_2(\mathbf{e}_2^k) \text{ with } \mathbf{e}_2(k) = \hat{\mathbf{y}}_{12}(k) - \hat{\mathbf{y}}_2(k),\end{aligned} \tag{5}$$

where we assume that reference signal $\mathbf{r}(k)$ has finite power such that $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$.

4 Necessary conditions for tracking

In order to be able to derive useful results on channel rate and conditions for optimal control, first we simplify the general model in Fig. 1 as shown in Fig. 2. In Fig. 2, the controllers are assumed to be directly connected to actuators that operate the systems, so that we can assume that the channels CH_3 and CH_4 are lossless with no delays.

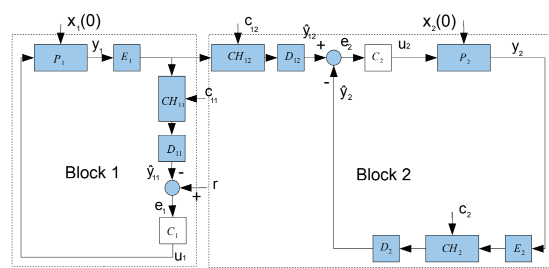

Fig. 2 A simplified model of a networked control system with two plants.

4.1 Necessary conditions on individual channel

4.1.1 Channel CH_{11}

Consider block 1 in Fig. 2. Plant P_1 is not affected by the second plant P_2. The block is a closed-loop system with communication channel in feedback link as in [7]. Before proceeding with our results, we extend Lemma 2 in [7] without assuming that $H = I$ and state it as Lemma 1.

Lemma 1 Consider the closed-loop system in block 1 in Fig. 2, where plant P_1 is an LTI system described by (4). Assume that the pair (F, G) is controllable and (F, H) is observable. Assume $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$ for reference signal $\mathbf{r}(k)$. If $\mathbb{E}[\boldsymbol{\xi}_1(k)^T\boldsymbol{\xi}_1(k)] < \infty$, then

$$\lim_{k\to\infty} \frac{I(\mathbf{x}_1(0); \mathbf{e}_1^k | \mathbf{r}^k)}{k} \geq \lim_{k\to\infty} \frac{I(\mathbf{x}_1(0); \mathbf{e}_1^k)}{k}$$

[1] For ease of mathematical derivation, we only consider identical model systems.

[2] The state-output relation assumption may limit the use of this paper's results in some practical applications. We are currently working on relaxing this assumption on further extensions.

$$\geq \sum_i \max\{0, \log_2(|\lambda_i(F)|)\},$$

where $\mathbf{e}_1^k = H\mathbf{r}^k - \hat{\mathbf{y}}_{11}^k$.

Proof Note that the matrix F can be written in the form $F = \begin{bmatrix} F_s & 0 \\ 0 & F_u \end{bmatrix}$ where F_s corresponds to the stable subspace ($0 < |\lambda_i(F_s)| < 1$) and F_u corresponds to the marginally stable and unstable subspace ($|\lambda_i(F_u)| \geq 1$). If $F = F_s$, from (P1) we just have $I(\mathbf{x}_1(0); \mathbf{e}_1^k) \geq 0$. For any control sequence, the system remains stable. Hence, without loss of generality, we can restrict our attention to matrix $F = F_u$ that contains only marginally stable and unstable eigenvalues.

From the system model of P_1 in (4), the definition of controller $\mathcal{C}_1$ in (5), we may write the system state $\mathbf{x}_1(k)$ as

$$\mathbf{x}_1(k) = F^k\mathbf{x}_1(0) + \sum_{i=0}^{k-1} F^{k-i-1}Gg_1(\mathbf{e}_1^i). \quad (6)$$

With the definition of tracking error $\boldsymbol{\xi}_1(k) = \mathbf{r}(k) - \mathbf{x}_1(k)$, by rearranging the terms in (6), we have

$$-F^{-k}\big(\boldsymbol{\xi}_1(k) - \mathbf{r}(k)\big) = \mathbf{x}_1(0) + \sum_{i=0}^{k-1} F^{-i-1}Gg_1(\mathbf{e}_1^i). \quad (7)$$

For bounded reference signals $\mathbf{r}(k)$ and $\mathbb{E}[\boldsymbol{\xi}_1(k)^T\boldsymbol{\xi}_1(k)] < \infty$, from the triangle inequality we have $\mathbb{E}[\mathbf{x}_1(k)^T\mathbf{x}_1(k)] \leq \mathbb{E}[\boldsymbol{\xi}_1(k)^T\boldsymbol{\xi}_1(k)] + \mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$, implying that the system remains stable. From the definition and properties of the mutual information, we can easily show that

$$\begin{aligned} I(\mathbf{x}_1(0); \mathbf{e}_1^k|\mathbf{r}^k) &\geq I(\mathbf{x}_1(0); \mathbf{e}_1^k), \quad (8)\\ &= h(\mathbf{x}_1(0)) - h(\mathbf{x}_1(0)|\mathbf{e}_1^k), \end{aligned}$$

where we have used the fact that $\mathbf{x}_1(0)$ and $\mathbf{r}^k$ are independent. From (7) and (P5):

$$\begin{aligned} &h(\mathbf{x}_1(0)|\mathbf{e}_1^k) = h\big(-F^{-k}(\boldsymbol{\xi}_1(k) - \mathbf{r}(k))|\mathbf{e}_1^k\big),\\ &\leq h\big(-F^{-k}(\boldsymbol{\xi}_1(k) - \mathbf{r}(k))\big), \quad (9)\\ &\leq \frac{1}{2}\log_2\big((2\pi e)^n \det(\mathrm{Cov}\{-F^{-k}(\boldsymbol{\xi}_1 - \mathbf{r})\})\big), \quad (10)\\ &= \frac{n}{2}\log_2\big(2\pi e\big) + \frac{1}{2}\log_2\big(\det(F^{-k}(F^{-k})^T)\big),\\ &+ \frac{1}{2}\log_2\big(\det(\mathrm{Cov}\{\boldsymbol{\xi}_1 - \mathbf{r}\})\big),\\ &= \frac{n}{2}\log_2\big(2\pi e\big) - k\sum_i \log_2(|\lambda_i(F)|),\\ &+ \frac{1}{2}\log_2\big(\det(\mathrm{Cov}\{\boldsymbol{\xi}_1 - \mathbf{r}\})\big), \end{aligned}$$

where (9) is due to (P6) and (10) is from (P7). Substituting these into (8), we obtain

$$\begin{aligned} &I(\mathbf{x}_1(0); \mathbf{e}_1^k|\mathbf{r}^k) \geq I(\mathbf{x}_1(0); \mathbf{e}_1^k) \geq h(\mathbf{x}_1(0)) - \frac{n}{2}\log_2(2\pi e)\\ &+ k\sum_i \log_2(|\lambda_i(F)|) - \frac{1}{2}\log_2(\det(\mathrm{Cov}\{\boldsymbol{\xi}_1 - \mathbf{r}\})). \end{aligned}$$

Since $\mathbb{E}[\mathbf{x}_1(k)\mathbf{x}_1(k)^T] < \infty$ and $\mathbf{x}_1 = \mathbf{r} - \boldsymbol{\xi}_1$, we have $\log_2\big(\det(\mathrm{Cov}\{\boldsymbol{\xi}_1 - \mathbf{r}\})\big) < \infty$. Finally, if we divide above by k and take the limit $k \to \infty$, then we have $\lim_{k\to\infty} \frac{I(\mathbf{x}_1(0);\mathbf{e}_1^k|\mathbf{r}^k)}{k} \geq \lim_{k\to\infty} \frac{I(\mathbf{x}_1(0);\mathbf{e}_1^k)}{k} \geq \sum_i \log_2(|\lambda_i(F)|)$. If we reintroduce matrix F with some stable eigenvalues, the Lemma follows.

By applying Lemma 1 to plant P_2 and with the definition of tracking error $\boldsymbol{\xi}_2$, we have the following corollary.

Corollary 1 Consider the closed-loop system in block 2 in Fig. 2, where plant P_2 is an LTI system described by (4). Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_2 = L\mathbf{y}_2$. Assume $\mathbb{E}[\hat{\mathbf{y}}_{12}(k)^T\hat{\mathbf{y}}_{12}(k)] < \infty$ for signal $\hat{\mathbf{y}}_{12}(k)$. If $\mathbb{E}[(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))^T] < \infty$, then

$$\begin{aligned} \lim_{k\to\infty} \frac{I(\mathbf{x}_2(0); \mathbf{e}_2^k|\hat{\mathbf{y}}_{12}^k)}{k} &\geq \lim_{k\to\infty} \frac{I(\mathbf{x}_2(0); \mathbf{e}_2^k)}{k}\\ &\geq \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}. \end{aligned}$$

Proof See the proof of Lemma 1.

We now examine necessary conditions on the channel rate of CH_{11} to guarantee $\mathbb{E}[\boldsymbol{\xi}_1(k)^T\boldsymbol{\xi}_1(k)] < \infty$ in plant P_1.

Lemma 2 Consider the closed-loop system given in block 1 in Fig. 2, where the plant P_1 is an LTI system described by (4). The channel CH_{11} is a feedback link with rate C_{11}. Assume finite power for reference signal $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$. If $\mathbb{E}[\boldsymbol{\xi}_1(k)^T\boldsymbol{\xi}_1(k)] < \infty$, then

$$C_{11} \geq I_\infty(\mathbf{r}, \hat{\mathbf{y}}_{11}) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}. \quad (11)$$

Proof By the chain rule (P8) for mutual information , we have

$$I((\mathbf{r}^k, \mathbf{x}_1(0)); \hat{\mathbf{y}}_{11}^k) = I(\mathbf{r}^k, \hat{\mathbf{y}}_{11}^k) + I(\mathbf{x}_1(0); \hat{\mathbf{y}}_{11}^k|\mathbf{r}^k) \quad (12)$$

From (P3) and using the fact that $\mathbf{e}_1^k = H\mathbf{r}^k - \hat{\mathbf{y}}_{11}^k$, we have

$$\begin{aligned} I(\mathbf{x}_1(0); \hat{\mathbf{y}}_{11}^k|\mathbf{r}^k) &= h(\hat{\mathbf{y}}_{11}^k|\mathbf{r}^k) - h(\hat{\mathbf{y}}_{11}^k|\mathbf{x}_1(0), \mathbf{r}^k),\\ &= h(\mathbf{e}_1^k|\mathbf{r}^k) - h(\mathbf{e}_1^k|\mathbf{x}_1(0), \mathbf{r}^k),\\ &= I(\mathbf{x}_1(0); \mathbf{e}_1^k|\mathbf{r}^k). \quad (13) \end{aligned}$$

Substituting (12) into (13), we have

$$I(\mathbf{r}^k, \hat{\mathbf{y}}_{11}^k) = I((\mathbf{r}^k, \mathbf{x}_1(0)); \hat{\mathbf{y}}_{11}^k) - I(\mathbf{x}_1(0); \mathbf{e}_1^k|\mathbf{r}^k). \quad (14)$$

From Lemma 7.9.2 of [11], for a discrete memoryless channel, we have average data rate $kC_{11} \geq I((\mathbf{r}^k, \mathbf{x}_1(0)); \hat{\mathbf{y}}_{11}^k)$. Hence, from (14), we obtain

$$I(\mathbf{r}^k, \hat{\mathbf{y}}_{11}^k) \leq kC_{11} - I(\mathbf{x}_1(0); \mathbf{e}_1^k|\mathbf{r}^k). \quad (15)$$

If we divide (15) by k and take the limit $k \to \infty$, then the result follows from Lemma 1.

From Lemma 2, we know that for the system P_1 to track $\mathbf{r}(k)$ with finite energy error, the channel data rate of CH_{11} should be at least as large as $\sum_i \max\{0, \log_2(|\lambda_i(F)|)\} + I_\infty(\mathbf{r}, \hat{\mathbf{y}}_{11})$. The term $I_\infty(\mathbf{r}, \hat{\mathbf{y}}_{11})$ is the average amount of information about $\mathbf{r}$ contained in channel output $\hat{\mathbf{y}}_{11}$ over time. It can be seen from Fig. 2 that channel CH_{11} conveys the information of $\mathbf{r}$ and the uncertainty of system P_1 to channel output $\hat{\mathbf{y}}_{11}$. Hence, the channel data rate should be larger than the sum of system uncertainty $\sum_i \max\{0, \log_2(|\lambda_i(F)|)\}$ and the mutual information rate between $\mathbf{r}$ and $\hat{\mathbf{y}}_{11}$. In practice, mutual information rate could be estimated by Monte Carlo methods given large amount of data [13, 14].

4.1.2 Channels CH_2 and CH_{12}

If we consider $\mathbf{y}_1(k)$ as the reference signal to plant P_2, then block 2 in Fig. 2 could be considered a closed-loop system with communication channels in both forward and feedback links. In the following lemma, first we examine necessary conditions on the data rate of feedback channel CH_2 to guarantee that the tracking error has finite second moment for plant P_2 to track $\hat{\mathbf{y}}_{12}(k)$.

Lemma 3 Consider the closed-loop system given in block 2 in Fig. 2, where the plant P_2 is an LTI system described by (4). The channel CH_2 is a feedback link with data rate C_2. Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$. Assume $\mathbb{E}[\hat{\mathbf{y}}_{\mathbf{12}}(k)^T\hat{\mathbf{y}}_{12}(k)] < \infty$. If $\mathbb{E}[(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))^T(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))] < \infty$, then

$$C_2 \geq I_\infty(\hat{\mathbf{y}}_{12}, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}. \quad (16)$$

Proof Similar to the proof of Lemma 2, and is omitted.

Lemma 3 shows that if plant P_2 tracks $\hat{\mathbf{y}}_{12}(k)$ with $\mathbb{E}[(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))^T(L\hat{\mathbf{y}}_{12}(k) - \mathbf{x}_2(k))] < \infty$, then (16) should be satisfied. However, our goal is to guarantee $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$ for plant P_2 to track $\mathbf{y}_1(k)$. Therefore, we need to derive necessary conditions on forward channel CH_{12} and determine the interaction of the two channels in such a system.

In the rest of this section, we first start with Lemma 4 which shows a lower bound on the rate of forward channel CH_{12} by assuming that feedback channel CH_2 is lossless and has no delays. Later, we will relax this assumption and arrive at our main result on necessary conditions on both channels CH_{12} and CH_2 in Theorem 1.

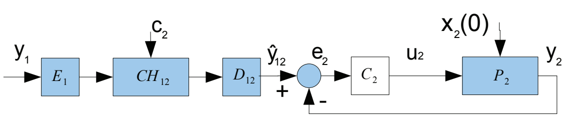

Fig. 3 Closed-loop system (block 2) with communication channel in forward link and channel CH_2 as lossless with no delays.

Lemma 4 Consider the feedback interconnection represented in Fig. 3, where the plant P_2 is an LTI system described by (4). Assume that the channel CH_2 is lossless and has no delays. Assume that encoder ϵ_1 and decoder D_{12}^k are causal and $\mathbb{E}[\mathbf{y}_1(k)^T\mathbf{y}_1(k)] < \infty$. Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$. If $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$, then

$$C_{12} \geq I_\infty(\mathbf{y}_1, \mathbf{y}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\} - h(\mathbf{x}_2(0)), \quad (17)$$

where C_{12} represents the rate of channel CH_{12}. In addition, channel noise $\mathbf{c}_{12} = 0$, then (17) is given by

$$C_{12} \geq I_\infty(\hat{\mathbf{y}}_{12}, \mathbf{y}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\} - h(\mathbf{x}_2(0)), \quad (18)$$

Proof From the chain rule (P8) for mutual information,

$$I((\mathbf{x}_2(0), \mathbf{y}_1^k); \mathbf{y}_2^k) = I(\mathbf{y}_1^k; \mathbf{y}_2^k) + I(\mathbf{x}_2(0); \mathbf{y}_2^k|\mathbf{y}_1^k). \quad (19)$$

From the system model (4) of P_2 and the definition (5) of controller $\mathcal{C}_2$, we may write the output $\mathbf{y}_2(k)$ as

$$\begin{aligned}\mathbf{y}_2(k) &= HF^k\mathbf{x}_2(0) + H\sum_{i=0}^{k-1} F^{k-i-1}Gg_2\big(\hat{\mathbf{y}}_{12}^i - \mathbf{y}_2^i\big),\\ &= \hat{g}_2(\mathbf{x}_2(0), \hat{\mathbf{y}}_{12}(k)).\end{aligned} \quad (20)$$

This shows that $\mathbf{y}_2(k)$ is a function of initial state $\mathbf{x}_2(0)$ and the reference signal $\hat{\mathbf{y}}_{12}$. From (P3) and (20), we obtain

$$\begin{aligned}&I\big((\mathbf{x}_2(0), \mathbf{y}_1^k); \mathbf{y}_2^k\big)\\ \leq &I\big((\mathbf{x}_2(0), \mathbf{y}_1^k); \mathbf{q}^k\big),\\ = &I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k) + I\big(\mathbf{x}_2(0), (\mathbf{x}_2(0), \cdots, \mathbf{x}_2(0))\big),\\ = &I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k) + kh(\mathbf{x}_2(0)),\end{aligned} \quad (21)$$

where $\mathbf{q}^k = \{\mathbf{q}(1), \cdots, \mathbf{q}(k)\}$, $\mathbf{q}(k) = (\mathbf{x}_2(0), \hat{\mathbf{y}}_{12}(k))$ and the second step results from the independence between $\mathbf{x}_2(0)$ and $(\mathbf{y}_1^k, \hat{\mathbf{y}}_{12}^k)$. From (P8), we have

$$\begin{aligned}I((\mathbf{y}_1^k, \hat{\mathbf{y}}_{12}^k); \mathbf{y}_2^k) &= I(\mathbf{y}_1^k; \mathbf{y}_2^k) + I(\hat{\mathbf{y}}_{12}^k; \mathbf{y}_2^k|\mathbf{y}_1^k),\\ &= I(\hat{\mathbf{y}}_{12}^k; \mathbf{y}_2^k) + I(\mathbf{y}_1^k; \mathbf{y}_2^k|\hat{\mathbf{y}}_{12}^k).\end{aligned}$$

Since $I(\hat{\mathbf{y}}_{12}^k; \mathbf{y}_2^k|\mathbf{y}_1^k) = 0$ due to (P4) and $\mathbf{c}_{12} = 0$ and $I(\mathbf{y}_1^k; \mathbf{y}_2^k|\hat{\mathbf{y}}_{12}^k) \geq 0$ due to (P1), we have

$$I(\mathbf{y}_1^k; \mathbf{y}_2^k) \geq I(\hat{\mathbf{y}}_{12}^k; \mathbf{y}_2^k). \quad (22)$$

From the definition of mutual information, we further have

$$\begin{aligned}I(\mathbf{x}_2(0); \mathbf{y}_2^k|\mathbf{y}_1^k) &= h(\mathbf{x}_2(0)|\mathbf{y}_1^k) - h(\mathbf{x}_2(0)|\mathbf{y}_2^k, \mathbf{y}_1^k),\\ &= h(\mathbf{x}_2(0)|\hat{\mathbf{y}}_{12}^k) - h(\mathbf{x}_2(0)|\mathbf{y}_2^k, \hat{\mathbf{y}}_{12}^k) \quad (23)\\ &= I(\mathbf{x}_2(0); \mathbf{y}_2^k|\hat{\mathbf{y}}_{12}^k),\\ &= h(\mathbf{y}_2^k|\hat{\mathbf{y}}_{12}^k) - h(\mathbf{y}_2^k|\mathbf{x}_2(0), \hat{\mathbf{y}}_{12}^k),\\ &= h(\mathbf{e}_2^k|\hat{\mathbf{y}}_{12}^k) - h(\mathbf{e}_2^k|\mathbf{x}_2(0), \hat{\mathbf{y}}_{12}^k), \quad (24)\\ &= I(\mathbf{x}_2(0); \mathbf{e}_2^k|\hat{\mathbf{y}}_{12}^k), \quad (25)\end{aligned}$$

where (23) is due to the independence between $\mathbf{x}_2(0)$ and $(\mathbf{y}_1^k, \hat{\mathbf{y}}_{12}^k)$ and (24) is due to (P5) and the fact that $\mathbf{e}_2^k = \hat{\mathbf{y}}_{12}^k - \mathbf{y}_2^k$. Substitution of (21), (22) and (25) into (19) results in the following:

$I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k) + kh(\mathbf{x}_2(0)) \geq I(\hat{\mathbf{y}}_{12}^k; \mathbf{y}_2^k) + I(\mathbf{x}_2(0); \mathbf{e}_2^k|\hat{\mathbf{y}}_{12}^k)$. We have average channel data rate $kC_{12} \geq I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k)$. By dividing above by k and taking the limit $k \to \infty$, the result follows by using Corollary 1.

Remark 1 From Lemma 4, it could be seen that if $C_{12} < \sum_i \max\{0, \log_2(|\lambda_i(F)|)\} - h(\mathbf{x}_2(0))$, the channel can not convey information at a high enough rate to match the speed of the system dynamics such that the reference signal does not provide any information related to the feedback signal, rendering feedback useless. By comparing Lemma 3 and Lemma 4, we know that there is one more term $-h(\mathbf{x}_2(0))$ in (18), which is due to the fact that $\mathbf{x}_2(0)$ passes through channel CH_2 but does not go through channel CH_{12}. When calculating the channel data rate, the information of $\mathbf{x}_2(0)$ is taken into account in C_2 but not in C_{12}. If we assume $\mathbf{x}_2(0)$ is not a random variable but a deterministic one, there is no uncertainty in $\mathbf{x}_2(0)$ such that $h(\mathbf{x}_2(0)) = 0$ and the bounds on C_2 and C_{12} are the same.

By combining Lemma 3 and Lemma 4, we provide the following theorem which states the necessary conditions on channels CH_{12} and CH_2 together to guarantee $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$ for tracking of $\mathbf{y}_1(k)$ by plant P_2.

Theorem 1 Consider block 2 represented in Fig. 2, where plant P_2 is an LTI system described by (4). Assume that encoders ϵ_1, ϵ_2 and decoders D_{12}^k, D_2^k are causal and $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$. Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$. If $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$, then [3]

$$
\begin{aligned}
&C_2 \geq I_\infty(\hat{\mathbf{y}}_{12}, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}, \text{ and} \\
&C_{12} \geq I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\} \\
&\quad - h(\mathbf{x}_2(0)),
\end{aligned} \tag{26}
$$

where C_2 and C_{12} are the channel rates of CH_2, CH_{12}.

Proof The first equation in (26) results directly from Lemma 3. In the following, we provide the proof for the second equation. From the chain rule (P8) for mutual information, we obtain

$$
I((\mathbf{x}_2(0), \mathbf{y}_1^k); \hat{\mathbf{y}}_2^k) = I(\mathbf{y}_1^k; \hat{\mathbf{y}}_2^k) + I(\mathbf{x}_2(0); \hat{\mathbf{y}}_2^k | \mathbf{y}_1^k). \tag{27}
$$

From (25) and using the fact that $\mathbf{e}_2^k = \hat{\mathbf{y}}_{12}^k - \hat{\mathbf{y}}_2^k$, we have

$$
I(\mathbf{x}_2(0); \hat{\mathbf{y}}_2^k | \mathbf{y}_1^k) = I(\mathbf{x}_2(0); \mathbf{e}_2^k | \hat{\mathbf{y}}_{12}^k). \tag{28}
$$

Using the properties of mutual information, we have

$$
\begin{aligned}
I((\mathbf{x}_2(0), \mathbf{y}_1^k); \hat{\mathbf{y}}_2^k) &\leq I((\mathbf{x}_2(0), \mathbf{y}_1^k); (\mathbf{y}_2^k, \mathbf{c}_2^k)), &(29)\\
&= I((\mathbf{x}_2(0), \mathbf{y}_1^k); \mathbf{y}_2^k), &(30)\\
&\leq I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k) + kh(\mathbf{x}_2(0)), &(31)
\end{aligned}
$$

where (29) results from (P3) and $\hat{\mathbf{y}}_2^k$ is a function of $\mathbf{y}_2^k$ and $\mathbf{c}_2^k$), (29) is due to the independence between $\mathbf{c}_2^k$ and $(\mathbf{x}_2(0), \mathbf{y}_1^k)$ and (31) results from (21). Substitution of (28) and (31) into (27) results in the following

$$
\begin{aligned}
&I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k) + kh(\mathbf{x}_2(0)) \\
&\geq I(\mathbf{y}_1^k; \hat{\mathbf{y}}_2^k) + I(\mathbf{x}_2(0); \mathbf{e}_2^k | \hat{\mathbf{y}}_{12}^k).
\end{aligned}
$$

We have the average channel rate as $kC_{12} \geq I(\mathbf{y}_1^k; \hat{\mathbf{y}}_{12}^k)$. Hence, by dividing above by k and taking the limit $k \to \infty$, the result follows by using Corollary 1.

In Theorem 1, it can be seen that when two channels appear as forward link (CH_{12}) and feedback link (CH_2) as in block 2 in Fig. 2, the rate of the forward channel is affected by the existence of the feedback channel, if the feedback channel is noisy where we have $I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2)$ in (26) instead of $I_\infty(\mathbf{y}_1, \mathbf{y}_2)$ in (17). The physical meaning of Theorem 1 is that the channels CH_{12} and CH_2 should convert information at a high enough rate not just to guarantee system stability by stabilizing the unstable poles of system matrix ($\sum_i \max\{0, \log_2(|\lambda_i(F)|)\}$), but also ensure effective tracking by providing related information between reference signal and feedback signal, which is represented by the mutual information $I_\infty(\hat{\mathbf{y}}_{12}, \hat{\mathbf{y}}_2)$ and $I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2)$.

4.2 Necessary conditions on cascade channel made of CH_{12} and CH_2

Theorem 1 shows necessary conditions on data rate of each individual link for tracking in plant P_2 with finite energy tracking error. It also shows the interaction between forward and feedback channels in this networked feedback system. However, a general overview and abstraction of the necessary conditions on both channels for tracking in such a system is still needed. With regards to information flow, the forward and feedback channels could be connected in a cascade manner. By considering forward and feedback channels together as one cascade channel, we could provide a lower bound on the rate of the cascade channel for tracking in plant P_2 such that $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$.

We may reformulate the structure of block 2 in Fig. 2 as shown in Fig. 4 [4]. In Fig. 4, two channels are connected in cascade with feedback from the output of the second channel to the intermediate node. The first channel $\mathrm{CH}_{12,\mathrm{new}}$ consists of CH_{12} and the lossless link that transmits $\mathbf{x}_2(0)$. The encoder input of the first channel is denoted as $(\mathbf{y}_1, \mathbf{x}_2(0))$ and the decoder output is denoted by $(\hat{\mathbf{y}}_{12}, \mathbf{x}_2(0))$, since $\mathbf{x}_2(0)$ is not affected by the channel noise. For the second channel CH_2, we denote the encoder input as $(\hat{\mathbf{y}}_{12}, \mathbf{x}_2(0))$ and decoder output as $\hat{\mathbf{y}}_2$. Here we consider controller $\mathcal{C}_2$, plant P_2 and encoder ϵ_2 as a macro encoder for the second channel.

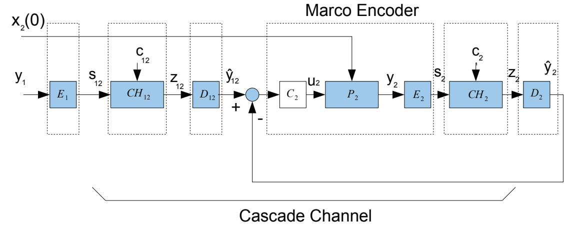

Fig. 4 Closed-loop system (block 2) with communication channels in forward and feedback links.

In this formulation, we reconsider the two channels $\mathrm{CH}_{12,\mathrm{new}}$, CH_2 described in Fig. 4 as one cascade channel $\mathrm{CH}_{\mathrm{cas}}$ with encoder ϵ_1, decoder D_2 and the components in between as the new channel. Here we want to find the minimum channel rate for the cascade channel $\mathrm{CH}_{\mathrm{cas}}$ for plant P_2 to track $\mathbf{y}_1(k)$ with $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$.

[3] The calculation of information rate in practice is difficult due to enormous computation. However, it still could be closely estimated by Monte Carlo techniques given large enough amount [13, 14].

In [4], a similar reformation of closed-loop system with feedback channel is considered. Here we extend the results in [4] to a cascade channel in which both forward and feedback channels are considered. In order to proceed with our results, we first modify Theorem 2.1 in [4] and state it here as Lemma 5.

Lemma 5 Consider the closed-loop system given in Fig. 4, where the plant P_2 is an LTI system described by (4). Assume that the encoders and decoders for the two channels CH_{12} and CH_2 are causal operators. Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$. Let the following assumptions hold:

(A1) The decoder D_2 for the second channel CH_2 satisfies: $\forall k > a, \hat{\mathbf{y}}_2^{a+1,k} = D_2^k(\hat{\mathbf{y}}_2^{1,a}, \mathbf{z}_2^k)$ for some $a \in \mathcal{N}_+$ and a sequence of functions D_2^k, where $\hat{\mathbf{y}}_2^{a+1,k} = \{\hat{\mathbf{y}}_2(a+1), \cdots, \hat{\mathbf{y}}_2(k)\}$ and the output of the decoder D_2 is based on all the received values from channel $\mathbf{z}_2^k$ and the previous output of decoder $\hat{\mathbf{y}}_2^{1,a}$.

(A2) The fading memory condition $\limsup_{k\to\infty} \frac{1}{k} I(\hat{\mathbf{y}}_2^{1,a}; \mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k) = 0$ holds.

Under the above conditions, the following is true:

$$\limsup_{k\to\infty} \frac{1}{k} I(\mathbf{x}_2(0), \mathbf{y}_1^k|\mathbf{z}_2^k) \leq I_\infty(\mathbf{s}_{12} \to \mathbf{z}). \quad (32)$$

Proof We separate the proof into two parts.

1) First, using (P2) and (P10) we can write the following equality, for any given $i \in \{1, \cdots, k\}$:

$$\begin{aligned}&I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}) \quad (33)\\ &\leq I(\mathbf{z}_2(i); \mathbf{s}_{12}^i|\mathbf{z}_2^{i-1}) + I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i).\end{aligned}$$

Now notice that (P2) allows us to rewrite:

$$\begin{aligned}&I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i) \quad (34)\\ &= I((\mathbf{z}_2^i, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})) - I((\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})).\end{aligned}$$

But, from (P3), we know that

$$\begin{aligned}&I((\mathbf{z}_2^i, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1}))\\ &=I((\mathbf{n}(i), \mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})).\end{aligned}$$

where $\mathbf{n}(i)$ represents the additive components in the cascade channel CH_{cas}, including additive noises $\mathbf{c}_{12}(i)$, $\mathbf{c}_2(i)$ and channel output feedback $\hat{\mathbf{y}}_2(i)$. Then, by chain rule, we have

$$\begin{aligned}I((\mathbf{z}_2^i, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})) &= I((\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1}))\\ &+ I(\mathbf{n}(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|(\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i)). \quad (35)\end{aligned}$$

Since $\mathbf{n}(i)$ is independent of $(\mathbf{x}_2(0), \mathbf{y}_1^{i-1})$ given $(\mathbf{s}_{12}^i, \mathbf{z}_2^{i-1})$, we have $I(\mathbf{n}(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|(\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i)) = 0$. Then,

$$\begin{aligned}&I((\mathbf{z}_2^i, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1}))\\ &=I((\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})). \quad (36)\end{aligned}$$

By making use of (34) and (36) we infer that $I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}, \mathbf{s}_{12}^i) = 0$. Together with (P1) and (33), this leads to:

$$I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}) \leq I(\mathbf{z}_2(i); \mathbf{s}_{12}^i|\mathbf{z}_2^{i-1}). \quad (37)$$

From causality (A1), $\mathbf{y}_1^{i,k}$ is independent of $(\mathbf{x}_2(0), \mathbf{y}_1^{i-1}, \mathbf{z}_2^i)$ implying

$$I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^{i-1}) = I(\mathbf{z}_2(i); (\mathbf{x}_2(0), \mathbf{y}_1^{i-1})|\mathbf{z}_2^{i-1}). \quad (38)$$

Substituting (38) in (37) and summing over i from $i = 1$ to $i = k$, then we have

$$I(\mathbf{z}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)) \leq I(\mathbf{s}_{12}^k \to \mathbf{z}_2^k). \quad (39)$$

2) Second, by using (P2) and (P10), we have the following inequality

$$\begin{aligned}I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)) &\leq I(\mathbf{z}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k))\\ &+ I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k). \quad (40)\end{aligned}$$

From (P2) and assumption (A1), we obtain the following:

$$\begin{aligned}&I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k)\\ &= I(\hat{\mathbf{y}}_2^{a+1,k}; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k, \hat{\mathbf{y}}_2^a) + I(\hat{\mathbf{y}}_2^a; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k).\end{aligned}$$

From (P4), we have $I(\hat{\mathbf{y}}_2^{a+1,k}; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k, \hat{\mathbf{y}}_2^a) = 0$. Then,

$$I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k) = I(\hat{\mathbf{y}}_2^a; (\mathbf{x}_2(0), \mathbf{y}_1^k)|\mathbf{z}_2^k). \quad (41)$$

By substitution of (41) in (40) and using the assumption (A2), we obtain:

$$\begin{aligned}&\limsup_{k\to\infty} \frac{1}{k} I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k))\\ &\qquad \leq \limsup_{k\to\infty} \frac{1}{k} I(\mathbf{z}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)),\end{aligned}$$

which, together with (39), completes the proof.

By the definitions of channel rate and directed mutual information, from Lemma 5, we have $C_{\text{cas}} \geq I_\infty(\mathbf{s}_{12} \to \mathbf{z}_2) \geq \limsup_{k\to\infty} \frac{1}{k} I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k))$. Then, we have the following:

$$C_{\text{cas}} \geq \limsup_{k\to\infty} \frac{1}{k} I(\hat{\mathbf{y}}_2^k; (\mathbf{x}_2(0), \mathbf{y}_1^k)). \quad (42)$$

Since $I((\mathbf{y}_1^k, \mathbf{x}_2(0)); \hat{\mathbf{y}}_2^k) = I(\mathbf{y}_1^k, \hat{\mathbf{y}}_2^k) + I(\mathbf{x}_2(0); \hat{\mathbf{y}}_2^k|\mathbf{y}_1^k)$, from (P3) and the fact that $\mathbf{e}_2^k = \hat{\mathbf{y}}_{12}^k - \hat{\mathbf{y}}_2^k = \hat{f}(\mathbf{y}_1^k) - \hat{\mathbf{y}}_2^k$, we have

$$\begin{aligned}I(\mathbf{x}_2(0); \hat{\mathbf{y}}_2^k|\mathbf{y}_1^k) &= h(\hat{\mathbf{y}}_2^k|\mathbf{y}_1^k) - h(\hat{\mathbf{y}}_2^k|\mathbf{x}_2(0), \mathbf{y}_1^k)\\ &= h(\mathbf{e}_2^k|\mathbf{y}_1^k) - h(\mathbf{e}_2^k|\mathbf{x}_2(0), \mathbf{y}_1^k);\\ &= I(\mathbf{x}_2(0); \mathbf{e}_2^k|\mathbf{y}_1^k). \quad (43)\end{aligned}$$

From (43), we have

$$I(\mathbf{y}_1^k, \hat{\mathbf{y}}_2^k) = I((\mathbf{y}_1^k, \mathbf{x}_2(0)); \hat{\mathbf{y}}_2^k) - I(\mathbf{x}_2(0); \mathbf{e}_2^k|\mathbf{y}_1^k). (44)$$

By Corollary 1 and dividing (44) by k and taking the limit $k \to \infty$, we obtain

$$I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) \le \limsup_{k\to\infty} \frac{1}{k} I((\mathbf{y}_1^k, \mathbf{x}_2(0)); \hat{\mathbf{y}}_2^k) - \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}. \quad (45)$$

Substituting (42) in (45), we have

$$I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) \le C_{\text{cas}} - \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}.$$

This result may be summarized in the following theorem:

Theorem 2 Consider the system formulation given in Fig. 4, where the plant P_2 is an LTI system described by (4). Consider the cascade channel, which is the combination of channels CH_{12} and CH_2. Assume that the pair (F, G) is controllable and (F, H) is observable. Let L be a transformation matrix such that $\mathbf{x}_i = L\mathbf{y}_i$ for $i = 1, 2$. Assume that $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$. If $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$, then

$$C_{\text{cas}} \ge I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F)|)\}. \quad (46)$$

Remark 2 Theorem 2 shows that the rate of the cascade channel which includes CH_{12} and CH_2 is lower bounded by the mutual information between $\mathbf{y}_1^k$ and $\hat{\mathbf{y}}_2^k$ and a function of the unstable poles of plant P_2. From the data processing inequality, the rate of cascade channel is less than the rate of each component channel [15]. Since channels $\text{CH}_{12,\text{new}}$ and CH_2 are in cascade connection as in Fig. 4, we have that $C_{12,\text{new}} \ge C_{\text{cas}}$ and $C_2 \ge C_{\text{cas}}$, where $C_{12,\text{new}}$ is the rate of channel $\text{CH}_{12,\text{new}}$. Since the link that transmits $\mathbf{x}_2(0)$ is lossless, the channel rate $C_{12,\text{new}} = C_{12} + h(\mathbf{x}_2(0))$. The result in Theorem 2 is also confirmed by Theorem 1.

Theorem 2 provides a guideline for communication channel design in networked feedback systems by giving a lower bound on the required overall channel rate. In practice, we could adjust the rate of each component channel and the channel orderings to optimize the overall channel rate. Theorem 2 shows that the rate of the cascade channel $I_\infty((\mathbf{y}_1, \mathbf{x}_2(0)); \hat{\mathbf{y}}_2)$ can be estimated in two parts. The first part $I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2)$ could be directly measured by partial input and output relation. The second term $I_\infty(\mathbf{x}_2(0), \hat{\mathbf{y}}_2|\mathbf{y}_1)$ which is caused by internal information loss in the channel could be estimated by the system parameters $\sum_i \max\{0, \log_2(|\lambda_i(F)|)\}$.

5 Extension to different system models

In this section, we consider the extension of above results to tracking in a leader-follower system in which the leader system and follower system have different system models as follows:

$$\begin{aligned} \mathbf{x}_1(k+1) &= F_1\mathbf{x}_1(k) + G_1\hat{\mathbf{u}}_1(k), \\ \mathbf{y}_1(k) &= H_1\mathbf{x}_1(k), \\ &\text{and} \\ \mathbf{x}_2(k+1) &= F_2\mathbf{x}_2(k) + G_2\hat{\mathbf{u}}_2(k), \\ \mathbf{y}_2(k) &= H_2\mathbf{x}_2(k), \qquad k \ge 0, \end{aligned} \quad (47)$$

where the states $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$ take values in $\mathbb{R}^n$ and the received control inputs $\hat{\mathbf{u}}_1(k)$ and $\hat{\mathbf{u}}_2(k)$ take values in $\mathbb{R}^r$. The initial states $\mathbf{x}_1(0)$ and $\mathbf{x}_2(0)$ are zero mean Gaussian random variables with covariance matrices Σ_{01} and Σ_{02}, respectively. The states are observed by sensors that generate the measurements $\mathbf{y}_1(k)$ and $\mathbf{y}_2(k)$ taking values in $\mathbb{R}^q$.

In order to comply to the same formulation as in Section II, the terms $\mathbf{e}_1$ and $\mathbf{e}_2$ are defined as $\mathbf{e}_1 = L_1\mathbf{r} - \hat{\mathbf{y}}_1$ and $\mathbf{e}_2 = L_2^{-1}L_1\hat{\mathbf{y}}_{12} - \hat{\mathbf{y}}_2$, where L_i is an invertible transformation matrix and $\mathbf{x}_1 = L_1\mathbf{y}_1$ and $\mathbf{x}_2 = L_2\mathbf{y}_2$. By following the derivation of above lemmas and theorems, similar results could be obtained for tracking in leader-follower system where the leader system and follower system have different system models. Due to space limitation, we only list the main results here and the proofs follow the same derivation as for Theorems 1 and 2.

Theorem 3 Consider block 2 represented in Fig. 2, where plant P_2 is an LTI system described by (47). Assume that encoders ϵ_1, ϵ_2 and decoders D_{12}^k, D_2^k are causal and that $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$. Assume that the pair (F_i, G_i) is controllable and (F_i, H_i) is observable. Let L_i be an invertible transformation matrix such that $\mathbf{x}_i = L_i\mathbf{y}_i$ for $i = 1, 2$. If $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$, then

$$\begin{aligned} &C_2 \ge I_\infty(\hat{\mathbf{y}}_{12}, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F_1)|)\}, \text{ and} \\ &C_{12} \ge I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F_2)|)\} \\ &\qquad - h(\mathbf{x}_2(0)), \end{aligned} \quad (48)$$

where C_2 and C_{12} are the channel rates of CH_2, CH_{12}.

Theorem 4 Consider the system formulation given in Fig. 4, where the plant P_2 is an LTI system described by (47). Consider the cascade channel, which is the combination of channels CH_{12} and CH_2. Assume that the pair (F_i, G_i) is controllable and (F_i, H_i) is observable. Let L_i be an invertible transformation matrix such that $\mathbf{x}_i = L_i\mathbf{y}_i$ for $i = 1, 2$. Assume that $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] < \infty$. If $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$, then

$$C_{\text{cas}} \ge I_\infty(\mathbf{y}_1, \hat{\mathbf{y}}_2) + \sum_i \max\{0, \log_2(|\lambda_i(F_2)|)\}. \quad (49)$$

6 Examples and simulations

Leader-follower system is defined as a dynamic system in which multiple agents are connected in such a way that followers are controlled or influenced by the behaviors of leaders. In such system, each follower will keep track of the state or output of the leader and generate its own output based on the received information. In this section, we show simulations of a leader-follower system and demonstrate the necessity of above derived conditions for tracking. We consider the close-loop system as in block 2 in Fig. 2. Channels CH_{12} and CH_2 are assumed to be erasure channels with limited data transmission rates R_{12} and R_2 and packet loss erasure probabilities of p_{12} and p_2, respectively. The average data rate of erasure channel is given by $C_i = R_i(1 - p_i)$ [11]. The reference signal satisfies $\mathbb{E}[\mathbf{r}(k)^T\mathbf{r}(k)] \le 10^3$.

We consider a two-part encoder-decoder scheme as follows [7]: encoder ϵ_i converts the input to its binary form, truncates the binary representation to its R_i most significant bits, encapsulates the bits in a packet and sends the packet through the channel. If the packet is received, the decoder D_i extracts the bits in the packet and convert them to its real number representation. Otherwise, the decoder will assume that a zero was sent and outputs zero. The scheme also assumes that the decoder knows exactly the operation of the encoder and that both have access to control signal. Consider system equation of plant P_2 with a simple control law as $\mathbf{x}_2(k+1) = 16\mathbf{x}_2(k) + \mathbf{u}_2(k)$, $\mathbf{y}_2(k) = 15\mathbf{x}_2(k)$ and $\mathbf{u}_2(k) = \hat{\mathbf{y}}_{12}(k) - 1.07\hat{\mathbf{y}}_2(k)$. The control law will drive system state to $\mathbf{x}_2(k) = \mathbf{r}(k)$ if the two channels are lossless and have no delays. The initial state $\mathbf{x}_2(0)$ is Bernoulli distributed with success probability $p_{x_2} = 0.5$.

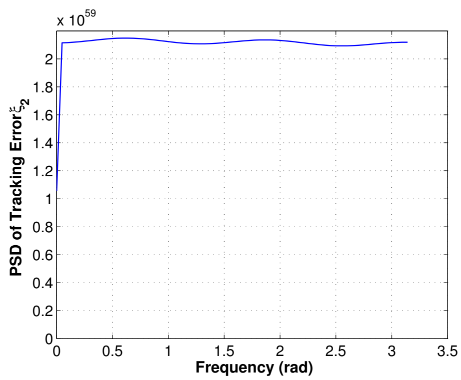

Fig. 5 Example with erasure channels: $C_{12} = 2.5$, $C_2 = 4$.

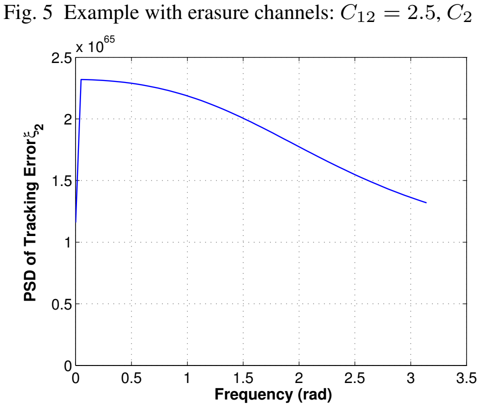

Fig. 6 Example with erasure channels: $C_{12} = 4$, $C_2 = 2.3$.

Our necessary conditions were given in terms of the mutual information rate, which is difficult to compute directly. However, our results impose limits to guarantee that $\mathbb{E}[\boldsymbol{\xi}_2(k)^T\boldsymbol{\xi}_2(k)] < \infty$. Since $\mathbb{E}[\boldsymbol{\xi}_2^T\boldsymbol{\xi}_2] = \frac{1}{2\pi}\int_0^{\pi}\Phi_2(\omega)d\omega$, where $\Phi_2(\omega)$ is the power spectral density of $\boldsymbol{\xi}_2(k)$ [16], we may plot the power spectral density of $\boldsymbol{\xi}_2$ to estimate $\mathbb{E}[\boldsymbol{\xi}_2^T\boldsymbol{\xi}_2]$. From Theorem 1, we know that lower bounds on channel data rate C_2 and C_{12} are 4 bits/timestep, for the above assumed system. Figures 5 and 6 show the power spectral density $\Phi_2(\omega)$ of the tracking error $\boldsymbol{\xi}_2(k)$ of the follower system when only one channel satisfies these necessary conditions. It can be seen that the power spectral density $\Phi_2(\omega)$ is unbounded at every $\omega \in [0,\pi]$. Then, the average power spectrum over an area of $[0,\pi]$ is unbounded. From the above equation, we know that $\mathbb{E}[\boldsymbol{\xi}_2^T\boldsymbol{\xi}_2]$ is no longer finite. However, if the lower bounds are satisfied by both channels as assumed in Fig. 7, the power spectral density is finite. Hence, $\mathbb{E}[\boldsymbol{\xi}_2^T\boldsymbol{\xi}_2]$ stays bounded.

7 Conclusions

In this paper, we considered tracking in leader-follower systems under communication constraints, where the system components are distributed and connected over communication links with finite data rates. We provided lower bounds on the channel rate of each communication link as necessary conditions for tracking in such a leader-follower system. We also showed examples to demonstrate our results. The results in this work provide fundamental limitations in terms of information quantities on communication links which can have important roles on control design in leader-follower systems. Limitations in both overall channel and individual channel are provided and it should be taken into account for designing new control system with communication constraints. Our future work is to extend the leader-follower system to more general framework in which multiple leaders and followers are interconnected as a network with more general graph topologies.

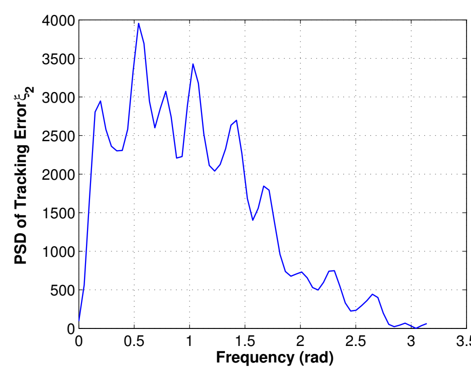

Fig. 7 Example with erasure channels: $C_{12} = 4$, $C_2 = 9$

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