# Power Efficient Analog Forwarding for Correlated Data Fusion in Wireless Sensor Networks

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***Abstract*— In this paper we consider the problem of optimal power allocation for fusion of a deterministic signal in an inhomogeneous wireless sensor network (WSN) with correlated observations. We assume that each distributed node performs analog-relay amplifier local processing on its observation and transmits locally processed data to the fusion center over a wireless channel. We also assume that the channel between the fusion center and sensors undergoes fading and the fading coefficients are assumed to be known to the transmitter. We derive exact fusion error probability and an easy to optimize upper bound for the fusion error probability that is valid for sufficiently small correlations. The transmit power is allocated to sensor nodes to keep the fusion error probability bound under a required threshold while minimizing the total power spent by the network. It is shown that the optimal scheme inactivates the sensor nodes with poor observation quality and low fading coefficients. For the remaining active sensors the transmit power is determined by the individual channel gains, local observation quality, required fusion error probability bound and the correlation coefficient. From numerical results we see that this optimal scheme has a significant gain over the uniform power allocation scheme when either local observations are good or the number of sensors is large and the correlation coefficient is sufficiently small. It is also shown that the optimal power allocation scheme can be implemented distributively with a minimal feedback from the fusion center.***

Key words—Data fusion, sensor networks, decentralized detection, correlated observations, hypothesis testing.

## I. Introduction

Decentralized detection is more attractive in many Wireless Sensor Networks (WSN) applications over the centralized approach since it drastically reduces communication resource requirements. In decentralized detection, each node sends a summary of its observations to the fusion center [1], [2] in contrast to that in centralized detection. The fusion center makes use of partially processed data from local nodes to make the final decision. Since only a summary of observations is transmitted, at the expense of some performance reduction, decentralized detection is reliable and survivable compared to centralized detection.

The fusion performance of a decentralized detection system in a low power WSN is limited by resource constraints, namely power and bandwidth. In a typical WSN, communication and computing capabilities of sensor nodes can be limited due to design considerations such as small battery and available bandwidth. For example, it may be impractical to replace or recharge the batteries due to cost and operating environment considerations. Therefore, the power management is a core issue in designing a WSN. Power should be allocated to the sensor nodes to obtain optimal fusion performance while conserving total power of the network.

The problem of distributed detection and fusion performance under resource constraints has been considered by many authors in literature. In [3], it was shown that when the network is subjected to a joint power constraint, having identical sensor nodes (i.e. each node using the same transmission scheme) is asymptotically optimal in binary decentralized detection. When the whole system is subjected to a total average power constraint [4] shows that it is better to combine as many not-so-good local decisions as possible rather than relying on a few very good local decisions in the case of deterministic signal detection. The optimal power scheduling for distributed detection in WSN has recently been considered in [5], where they developed the optimal power allocation scheme with respect to the so-called J-divergence performance index. The optimal power scheduling scheme for decentralized estimation to achieve a target Mean Squared Error (MSE) at the fusion center (with independent observations) has also been considered in [6], [7]. The minimum energy decentralized estimation with correlated data was addressed in [8].

In a typical WSN, sensor observations are likely to be correlated, especially when the sensors are densely deployed. In this paper we consider the optimal power allocation for data fusion in a wireless sensor network when local observations are correlated. We consider a WSN consisting of a fusion center and a large number of spatially separated sensors. Each distributed sensor node collects observations, computes a local message and transmits it to the fusion center. We specifically assume that each node performs analog-relay amplifier processing on its own observation. Each node sends the locally processed data to the fusion center over a dedicated noisy wireless channel. We derive the exact, as well as an upper bound, for the fusion error probability which is easy to optimize when the local observation correlations are sufficiently small. The optimal power is allocated to keep the fusion error probability bound under a required threshold. As we will show that the optimal power allocation offers significant energy savings when compared to the uniform power allocation scheme. The proposed scheme has a simple distributed implementation with minimal feedback from the fusion center.

The remainder of this paper is organized as follows: Section II formulates the fusion problem. In Section III the optimal fusion performance is analyzed. Optimal power allocation scheme is given in Section IV. Section V presents the performance results and finally section VI gives concluding remarks.

## II. Fusion Problem Formulation

We consider a binary hypothesis testing problem in an $n$-node distributed wireless sensor network. The $k$-th sensor observation under each hypothesis is given by,

$$\begin{aligned} H_0 : z_k &= v_k;\ k = 1, 2, ...., n \\ H_1 : z_k &= x_k + v_k;\ k = 1, 2, ..., n\ , \end{aligned} \tag{1}$$

where $v_k$ is the observation noise and $x_k$ is the signal to be detected. In vector notation (1) becomes, $\mathbf{z} = \mathbf{x} + \mathbf{v}$, where $\mathbf{v}$ is a zero mean Gaussian $n$- vector of noise samples with covariance matrix $\mathbf{\Sigma_v}$. We consider the detection of a deterministic signal so that $x_k = m$ for all $k$. Let us define the local signal-to-noise ratio as $\gamma_0 = \frac{m^2}{\sigma_v^2}$ where $\sigma_v^2$ is the observation noise variance. The prior probabilities of the two hypotheses, $H_1$ and $H_0$ are denoted by $P(H_1) = \pi_1$ and $P(H_0) = \pi_0$, respectively.

Each node processes its own observation to produce a local decision $u_k(z_k)$ and sends it to the fusion sensor. Here we assume that amplify-and-forward local processing is used at each node, according to which each node retransmits an amplified version of its own observation to the fusion center. This class of sensors has been shown to perform well when the observations at the sensor nodes are corrupted by additive noise [7], [9]. The local decisions sent to the fusion center are, $u_k = g_k z_k;\ k = 1, 2, ...n$ where $g_k$ is the relay amplifier gain at node $k$. The received signal at the fusion center is $r_k = h_k g_k z_k + w_k; k = 1, 2, ....n$, where $h_k$ is the channel fading coefficient and the receiver noise $w_k \sim N(0, \sigma_w^2)$ is assumed to be independent and identically distributed. Under each hypothesis, the received signal $r_k$ is given by

$$\begin{aligned} H_0\ : r_k &= n_k;\ \ k = 1, 2, ...., n \\ H_1\ : r_k &= h_k g_k x_k + n_k;\ \ k = 1, 2, ...., n \end{aligned}$$

where $n_k = h_k g_k v_k + w_k$. Defining $\mathbf{r} = [r_1, \cdots, r_n]^T$, in vector notation, $\mathbf{r} = \mathbf{A}\mathbf{x} + \mathbf{n}$ under $H_1$ and $\mathbf{r} = \mathbf{n}$ under $H_0$, where $\mathbf{A} = \text{diag}(h_1 g_1, h_2 g_2, ...., h_n g_n)$. The detection problem at the fusion center can be formulated as,

$$\begin{aligned} H_0 &: \mathbf{r} \sim p_0(\mathbf{r}) = \mathcal{N}(0, \mathbf{\Sigma_n}) \\ H_1 &: \mathbf{r} \sim p_1(\mathbf{r}) = \mathcal{N}(\mathbf{A}\mathbf{m}, \mathbf{\Sigma_n}) \end{aligned} \tag{2}$$

where $\mathbf{\Sigma_n} = \mathbf{A^T \Sigma_v A} + \sigma_\mathbf{w}^\mathbf{2}\mathbf{I}$, and $\mathbf{m} = m\mathbf{e}$ where $\mathbf{e}$ is the $n$-length vector with all ones. The log-likelihood ratio (LLR) for the detection problem (2) can be written as, $T(\mathbf{r}) = m\mathbf{e}^T\mathbf{A}\mathbf{\Sigma_n^{-1}}\mathbf{r} - \frac{1}{2}m^2\mathbf{e}^T\mathbf{A}\mathbf{\Sigma_n^{-1}}\mathbf{A}\mathbf{e}$. It is well known that optimal fusion tests should be threshold tests on the above LLR. Thus the optimal Bayesian decision rule at the fusion center is given by,

$$\delta(\mathbf{r}) = \begin{cases} 1 & \text{if } T(\mathbf{r}) \geq \log \tau \\ 0 & \text{if } T(\mathbf{r}) < \log \tau, \end{cases} \tag{3}$$

where $\tau$ is the threshold given by $\tau = \frac{\pi_1}{\pi_0}$ (assuming minimum probability of error Bayesian fusion).

## III. Analysis of Optimal Fusion Performance

The LLR $T(\mathbf{r})$ has the following distribution under the two hypotheses:

$$\begin{aligned} H_0 : &\quad T(\mathbf{r}) \sim \mathcal{N}(-\frac{1}{2}m^2\mathbf{e^T A \Sigma_n^{-1} A e},\ m^2\mathbf{e^T A \Sigma_n^{-1} A e}) \\ H_1 : &\quad T(\mathbf{r}) \sim \mathcal{N}(\frac{1}{2}m^2\mathbf{e^T A \Sigma_n^{-1} A e},\ m^2\mathbf{e^T A \Sigma_n^{-1} A e}). \end{aligned} \tag{4}$$

Hence the probability of error at the fusion center for a Bayesian detector is given by

$$P(E) = Q\left(\frac{1}{2}\sqrt{m^2\mathbf{e^T A \Sigma_n^{-1} A e}}\right)\ , \tag{5}$$

where $Q$-function is defined by $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{\zeta^2}{2}}d\zeta$ and the prior probabilities are assumed to be equal in (5).

It is not clear how to evaluate $\mathbf{\Sigma_n^{-1}}$ in (5) analytically for a general $\mathbf{\Sigma_v}$ unless it has a diagonal structure. In the following we consider a specific sensor network model and obtain an upper bound for $P(E)$ in (5) that is valid for small correlations. To that end we assume a 1-D sensor network in which adjacent nodes are separated by distance $d$ and correlation between observations at nodes $i$ and $j$ is proportional to $\rho_0^{d|i-j|}$ where $|\rho_0| \leq 1$. Letting $\rho_0^d = \rho$, $\mathbf{\Sigma_v}$ can be written as

$$\mathbf{\Sigma_v} = \sigma_v^2 \begin{bmatrix} 1 & \rho & . & . & . & \rho^{n-2} & \rho^{n-1} \\ \rho & 1 & . & . & . & \rho^{n-3} & \rho^{n-2} \\ . & . & . & . & . & . & . \\ \rho^{n-2} & \rho^{n-3} & . & . & . & 1 & \rho \\ \rho^{n-1} & \rho^{n-2} & . & . & . & \rho & 1 \end{bmatrix}. \tag{6}$$

When $\rho$ is sufficiently small, we may approximate (6) by its tri-diagonal version by dropping second and higher order terms of $\rho$. Now, using Bergstrom's inequality [10], for any two positive definite matrices $\mathbf{P}$ and $\mathbf{Q}$ it can be shown that

$$(\mathbf{e^T P^{-1} e}) \geq \frac{(\mathbf{e^T (P+Q)^{-1} e})(\mathbf{e^T Q^{-1} e})}{(\mathbf{e^T Q^{-1} e} - \mathbf{e^T (P+Q)^{-1} e})}. \tag{7}$$

Since $m^2\mathbf{e^T A \Sigma_n^{-1} A e} = m^2\mathbf{e^T}(\mathbf{\Sigma_v} + \sigma_\mathbf{w}^\mathbf{2}\mathbf{A^{-2}})^\mathbf{-1}\mathbf{e}$ we take $\mathbf{P} = (\mathbf{\Sigma_v} + \sigma_\mathbf{w}^\mathbf{2}\mathbf{A^{-2}})$ and define the matrix $\mathbf{Q}$ such that,

$$\mathbf{Q} = \sigma_v^2 \begin{bmatrix} 1 & -\rho & . & . & -\rho^{n-2} & -\rho^{n-1} \\ -\rho & 1 & . & . & -\rho^{n-3} & -\rho^{n-2} \\ . & . & . & . & . & . \\ -\rho^{n-2} & -\rho^{n-3} & . & . & 1 & -\rho \\ -\rho^{n-1} & -\rho^{n-2} & . & . & -\rho & 1 \end{bmatrix}.$$

For small enough $\rho$ it can be shown that $\mathbf{e^T Q e} > 0$. In fact, when $\mathbf{\Sigma_v}$ has the tri-diagonal structure (implying only the adjacent node observations are correlated), it can be shown that for any $|\rho| < \frac{n}{2(n-1)}$, we will have $\mathbf{e^T Q e} > 0$. In general, if $\mathbf{\Sigma_v}$ was in (6), this will be true for small enough $\rho$. The noise covariance matrix (6) can be used in many applications in 1-D sensor networks such as traffic monitoring or in industrial monitoring, where the sensors are equally spaced. The tri-diagonal version of (6) is a more realistic model when the correlation coefficient $\rho$ is small, since then the second and higher order terms of $\rho$ in (6) are negligible. From (7) it can be shown that

$$\mathbf{e^T}(\mathbf{\Sigma_v} + \sigma_\mathbf{w}^2 \mathbf{A^{-2}})^{-1}\mathbf{e} \geq \left( \frac{1}{\sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2}} - \frac{1}{D} \right)^{-1}, \quad (8)$$

where $D = \mathbf{e^T Q^{-1} e}$. From (5) and (8), we then have the following upper bound for the fusion error probability when the observations are correlated:

$$P(E) \leq Q\left( \frac{m}{2} \left( \frac{1}{\sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2}} - \frac{1}{D} \right)^{-\frac{1}{2}} \right). \quad (9)$$

For $\rho = 0$, $D = n/\sigma_v^2$. Then,

$$\lim_{g_k^2 \to \infty \; k=1,\cdots,n} \left( \frac{1}{\sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2}} - \frac{1}{D} \right)^{-1} = \frac{n}{\sigma_v^2}.$$

That is the fusion error probability bound (9) has a performance floor of $Q\left(\frac{\sqrt{n\gamma_0}}{2}\right)$ for infinite amplifier gains.

On the other hand, when observation noise is i.i.d., the exact fusion error probability is given by $P(E) = Q\left(\frac{m}{2}\sqrt{\sum_{k=1}^n \frac{h_k^2 g_k^2}{h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2}}\right)$ (note that when the observation noise is i.i.d. $\mathbf{\Sigma_n^{-1}}$ in (5) can be evaluated analytically) and it has a performance floor, $P(E) = Q\left(\frac{\sqrt{n\gamma_0}}{2}\right)$ as $g_k^2 \to \infty; k = 1, 2, \cdots, n$. It can be seen that both the exact fusion error probability and the proposed error probability bound exhibit the same performance for i.i.d. observations when the local amplifier gain is infinite. An experimental analysis of the tightness of the bound is illustrated in Fig. 1, which we will discuss in Section V.

## IV. Optimal Power Allocation

In the following, we derive a power allocation scheme that minimizes the total power spent by the whole sensor network subjected to the constraint that the fusion error probability be less than a given threshold. When the observation noise is independent the exact fusion error probability (5) can be easily evaluated analytically and the optimal power allocation scheme was derived in [11]. However, when observations are correlated, the exact error expression (5) is not easy to optimize analytically. Thus, here we use the bound (9) obtained above for our power optimization. The optimal power allocation problem can be formulated as

$$\min_{g_k \geq 0 \text{ for } k=1,\cdots,n} \sum_{k=1}^n g_k^2 \, ,$$

such that

$$Q\left( \frac{m}{2} \left( \frac{1}{\sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2}} - \frac{1}{D} \right)^{-\frac{1}{2}} \right) \leq \epsilon,$$

where $\epsilon$ is the required threshold for fusion error probability. The above problem can alternatively be written as,

$$\min_{g_k \geq 0 \text{ for } k=1,\cdots,n} \sum_{k=1}^n g_k^2 \, ,$$

such that $q - \sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2} \leq 0$ and $g_k \geq 0$; $k = 1, 2, \cdots, n$ where $q = \left(\frac{1}{\beta^2} + \frac{1}{D}\right)^{-1}$ and $\beta = \frac{2Q^{-1}(\epsilon)}{m}$ (Note that, $q > 0$ since $D > 0$). Then the Lagrangian for the above optimization problem is given by

$$G(L, \lambda_0, \mu_k) = \sum_{k=1}^n g_k^2 + \lambda_0 \left[ q - \sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2} \right] + \sum_{k=1}^n (-g_k)\mu_k, \quad (10)$$

where $\lambda_0 \geq 0$ and $\mu_k \geq 0$ for $k = 1, 2, \cdots, n$. The optimal solution should satisfy the following KKT conditions:

$$2g_k - \lambda_0 \frac{2g_k h_k^2 \sigma_w^2}{(2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2)^2} - \mu_k = 0; \; k = 1, 2, \cdots \quad (11)$$

$$\lambda_0 \left[ q - \sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2} \right] = 0 \quad (12)$$

$$\sum_{k=1}^n (-g_k)\mu_k = 0$$

$$q - \sum_{k=1}^n \frac{h_k^2 g_k^2}{2h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2} \leq 0$$

$$g_k \geq 0; \; k = 1, 2, \cdots, n \quad (13)$$

In order to find a solution that satisfies above KKT conditions, let us assume that $\lambda_0 \neq 0$ and $\mu_k = 0$ for $k = 1, 2, \cdots, n$. Then, for $g_k \neq 0$, from (11) we have,

$$g_k^2 = \frac{\sigma_w^2}{2h_k^2 \sigma_v^2} \left[ \frac{\sqrt{\lambda_0} h_k}{\sigma_w} - 1 \right]. \quad (14)$$

Let us define the set $\Phi$ such that $\Phi = \{k; g_k \neq 0\}$. From (12) and (14), we get $\sqrt{\lambda_0} = \frac{\sigma_w \sum_{k \in \Phi} \frac{1}{h_k}}{|\Phi| - 2\sigma_v^2 q}$ where $|\Phi|$ denotes the cardinality of $\Phi$. Let us define a function $f(.)$ as $f(k) = \frac{k - 2\sigma_v^2 q}{h_k \sum_{j=1}^k \frac{1}{h_j}}$ Suppose that, without loss of generality, $h_1 \geq h_2 \geq \cdots \geq h_n$. Then it can be shown that (see the Appendix), we can find a unique $K_1$ such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$ for $1 \leq K_1 \leq n$. The value of $K_1$ can be found by searching for the maximum integer $k$ such that $f(k) < 1$ where $k = 1, \cdots, n$. With this notation, we have

$$\sqrt{\lambda_0} = \frac{\sigma_w}{h_{K_1} f(K_1)}. \quad (15)$$

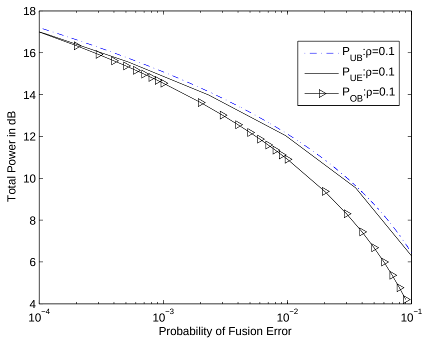


Fig. 1. Total power Vs. the fusion error probability: $\gamma_0 = 5dB$, $n = 100$

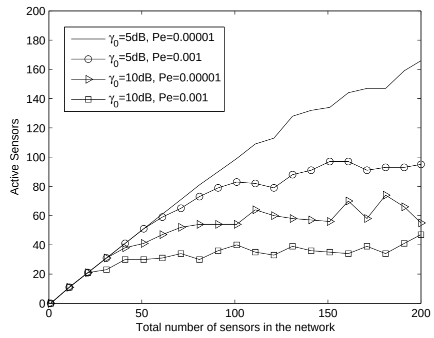


Fig. 2. Active sensors Vs. total number of sensors for $\rho$=0.1

From (14) and (15) we get, $g_k^2 = \frac{\sigma_w^2}{2h_k^2\sigma_v^2}\left[\frac{h_k}{h_{K_1}f(K_1)} - 1\right]$ which satisfies the KKT condition (13) if $n > 2\sigma_v^2 q$. Suppose $n \leq 2\sigma_v^2 q$, and assume $\lambda_0 = 0$ and $\mu_k \neq 0$ for $k = 1, 2, \cdots, n$. From the KKT conditions it can be seen that there is no non-trivial solution for $g_k$ whenever $\mu_k \neq 0$ for $k = 1, 2, \cdots, n$. Thus the solution to the optimal power allocation problem can be given as,

$$g_k^2 = \begin{cases} \frac{\sigma_w^2}{2h_k^2\sigma_v^2}\left[\frac{h_k \sum_{j=1}^{K_1}\frac{1}{h_j}}{(K_1 - 2\sigma_v^2 q)} - 1\right] & ;\text{if } k < K_1 \text{ and} \\ & n - 2\sigma_v^2 q > 0 \\ 0 & ;\text{if } k > K_1 \\ & \text{and } n - 2\sigma_v^2 q > 0 \\ \text{infeasible} & ;\text{if } n - 2\sigma_v^2 q \leq 0 \end{cases} . \quad (16)$$

Note from (16) that to achieve the required fusion error probability at the fusion center the total number of the active sensors should be greater than $2\sigma_v^2 q$ in the optimal solution.

The implementation of optimal power allocation scheme shown in (16) has in general centralized structure; i.e. the fusion center determines each node's power values and sends to each node. But this can be implemented distributively using side information from the fusion center. If each node is aware of its channel fading coefficients (via feedback assuming a block fading channel) then once the fusion center determines and broadcasts $\sqrt{\lambda_0}$ as side information to the nodes, each node can determine its power as in (14).

## V. Performance Results

In our numerical results we assume that fading coefficients $h_k$ of the channel between sensors and the fusion center to be Rayleigh distributed with unit mean. Our performance measure is the total power expenditure defined as $P_{OB} = \sum_{k=1}^{n} g_k^2$ where $g_k$'s are given by (16). In contrast, in a uniform power allocation scheme each node transmits locally processed data to the fusion center with equal power irrespective of the quality of local observation and the channel. The corresponding total power spent with uniform power allocation is $P_{UB} = ng^2$, where $g$ is given by the expression, $q = g^2 \sum_{k=1}^{n} \frac{h_k^2}{2h_k^2 g^2 \sigma_v^2 + \sigma_w^2}$. The total power under uniform power allocation for the exact $P(E)$ is given by $P_{UE} = ng^{'2}$ and $g^{'2}$ is given by,

$$g^{'2}\mathbf{e^T H}(g^{'2}\mathbf{H\Sigma_v H} + \sigma_\mathbf{w}^\mathbf{2})^{-1}\mathbf{He} = \beta^2$$

where $\mathbf{H} = \text{diag}(h_1, h_2, \cdots, h_n)$.

From Fig. 1, it can be seen that the optimal power allocation scheme based on the fusion error probability bound has significant performance over the uniform power allocation based either on the exact or bound to the fusion error probability. It also shows how tight the bound is at least with uniform power allocation. It can be seen from Fig. 1 that the total optimal power required to keep the fusion error probability bound under a given threshold is less than the total uniform power required to keep the exact fusion error probability under the same threshold. Also the bound is well performed when the required threshold is not significantly low.

Observe from Fig. 2 that only a small number of sensors are active when the local observation quality is good and when the required fusion error probability bound is not significantly low. It is depicted in Fig. 3 that the gain of the optimal scheme over the uniform scheme increases when the required fusion error probability is relatively high since then only a small number of sensor are active in the optimal scheme conserving total power of the system. Figure 3 also shows that the total power spent by the system decreases when the number of sensors in the system increases since it is not necessary to operate all the sensors in large power levels because large number of replicas of the same signal contributes to the fusion decision.

From Fig. 4 we can see that when $\gamma_0$ is high the gain of the optimal power allocation scheme over the uniform scheme is high. This is because when $\gamma_0$ is high the optimal scheme only needs a small number of active sensors. Figure 4 also depicts that the network needs more power when the correlation coefficient of the observation noise is high since the new information added by each additional sensor then decreases leading to degraded fusion performance.

## VI. Conclusion

In this paper we derived an upper bound for the fusion error probability in a WSN when local observation noise correlation is sufficiently small. The optimal power allocation scheme over the distributed nodes was derived to keep the upper bound of

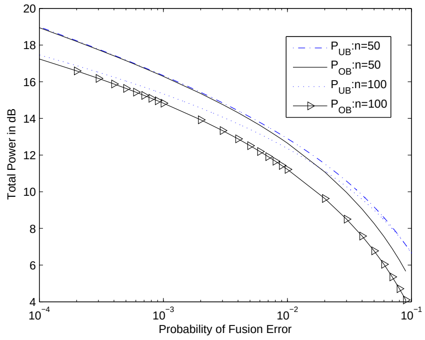


Fig. 3. Optimal Total Power and Fusion Error Probability; $\rho = 0.1$, $\gamma_0 = 5dB$

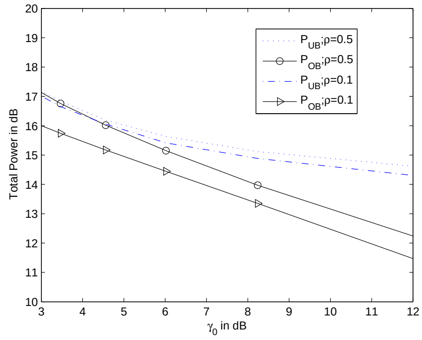


Fig. 4. Total Power Vs. $\gamma_0$ for $n = 100$ and $P(E) = 10^{-3}$

the fusion error probability under a required threshold. The derived upper bound for the fusion error probability is valid for 1-D WSN models with equally spaced sensors which can be justified in certain applications. We showed that the optimal power allocation scheme saves total power in the system when compared with the uniform power allocation by inactivating nodes with poor observation quality and bad channels. The gain of optimal power allocation scheme over the uniform power allocation scheme becomes more prominent when $\gamma_0$ is high. We also showed that the proposed optimal power allocation scheme has a distributed implementation with a small feedback from the fusion center.

## APPENDIX

In this Appendix, we show the existence of a unique $K_1$, where $1 \leq K_1 \leq n$ is such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$ where $f(k) = \frac{(k-2\sigma_v^2 q)}{h_k \sum_{j=1}^{k} \frac{1}{h_j}}$ and we have assumed $h_1 > h_2 > \cdots, h_n$. When $k = 1$, $f(1) = \frac{(1-2\sigma_v^2 q)}{h_1 \cdot \frac{1}{h_1}} < 1$; since $q$ is positive. So, $f(k) > 1$ is not possible for all $k = 1, 2, \cdots, n$. Therefore there are two possibilities:

- $f(k) < 1$ for all $1 \leq k \leq n$: In this case we set $K_1 = n$.
- There exists a unique $K_1$ such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$, where $1 \leq K_1 \leq n$:

The uniqueness of $K_1$ implies that for any $k \geq K_1 + 1$, we should have that $f(k) \geq 1$. This can be proved by showing that if $f(k) \geq 1$, then $f(k+1) \geq 1$. When $f(k) \geq 1$, it implies that $f(k+1) = \frac{(k+1-2\sigma_v^2 q)}{h_k \sum_{j=1}^{k+1} \frac{1}{h_j}} = \frac{(k-2\sigma_v^2 q)+1}{(h_k \sum_{j=1}^{k} \frac{1}{h_j}+1)-(h_k - h_{k+1}) \sum_{j=1}^{k} \frac{1}{h_j}}$. The second term of the denominator of the last equality is positive since we have assumed that $h_k > h_{k+1}$ and $k - 2\sigma_v^2 q > 0$ for $f(k) \geq 1$. Hence $f(k+1) > \frac{(k-2\sigma_v^2 q)+1}{h_k \sum_{j=1}^{k} \frac{1}{h_j}+1} > 1$ as required.

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