OPTIMAL NODE PLACEMENT IN DECISION FUSION WIRELESS SENSOR NETWORKS FOR DISTRIBUTED DETECTION OF A RANDOMLY-LOCATED TARGET

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ABSTRACT

We consider the problem of optimal (fixed) wireless sensor network (WSN) design for distributed detection of a randomly-located target. A distributed one-dimensional (1-D) WSN model with equal spacing d between any two adjacent nodes is assumed. We first model the target as being randomly located following an exponential distribution with a known parameter, and the channel between the nodes and the fusion center to be AWGN with path loss attenuation. A simplified decision fusion rule for the high observation signal-to-noise ratio (SNR) regime and its Bayesian error probability are derived, which then is used to optimize the parameters of the WSN. The optimal sensor placements are obtained in the limit of a large sensor system and the analytical properties of the obtained solution are discussed with corresponding numerical examples. It is shown that in many cases deviation from optimal inter-node spacing can cost significant performance penalty. Finally, the results are generalized to a fading wireless channel and for any target-location distribution specified only via its second-order statistics. It is shown that the optimal sensor placements essentially stays the same regardless of whether the channel is AWGN or fading, and are insensitive to operating SNR's either at the fusion center or at the local sensor nodes.

I. INTRODUCTION

In many distributed wireless sensor networks (WSN's) preserving the node power is crucial, especially when they are deployed in difficult/hostile environments that prevent replacing or recharging the batteries. In such situations, initial optimization of network design can help save precious battery life thereby extending the network lifetime. In this paper, we formulate and solve this problem for a one-dimensional (1-D) WSN, made of n equi-spaced nodes, employed to detect a randomly-located target.

Remote/distributed target detection and decision fusion have been researched extensively over the years in various contexts (see [1–5], and references therein). However, in this paper, our focus is specifically on the optimal design of wireless sensor networks for such an application. In recent years the concept WSN's has gained considerable attention in various detection, tracking and/or monitoring applications due to their versatility and flexibility. In many cases, however, distributed wireless sensor networks operate under strict constraints on node power and available communication bandwidth [6, 7]. These inherent resource constraints make judicial system design extremely important in order to extend the lifetime of the network.

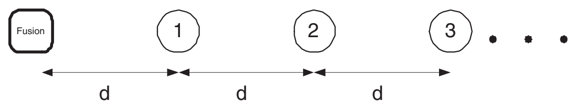

Figure 1: 1-D Equi-spaced Sensor Network.

In this paper, we formulate the problem of optimal design of a one-dimensional wireless sensor network to detect a randomly-located target. The WSN is assumed to be made of n equi-spaced nodes, where d denotes the distance between any two adjacent nodes as shown in Fig. 1. The distributed nodes make independent binary decisions on whether the target is present or not based on their own observations, and wirelessly relay these local decisions to the fusion center. The fusion center observes a noisy version of the local decisions and makes a final decision on whether the target is present or not. The wireless channel suffers from both path loss due to attenuation as well as random fading. The amount of path loss is a function of the distance between a particular node and the fusion center and, of course, the path loss exponent of the wireless channel. In addition, the quality of local decisions from distributed nodes is also a function of the relative location of the target at any given time with respect to a specific nodes. Hence, the fusion error probability performance critically depends on the node locations of the wireless sensor network which are essentially determined by the inter-node spacing parameter d. The goal in this paper is to propose an optimal WSN design by deriving the best d that leads to the minimum achievable Bayesian fusion probability of error.

It is interesting to note that the inter-node spacing d also directly relates to the infrastructure cost of the wireless sensor network as well as the coverage area. If one were to use a too small value of d in designing the network, this will require more sensor nodes to cover a given area leading to more infrastructure cost. on the other hand, for a fixed number of nodes n it will limit the coverage area. On the other hand, if d were to be too large the sensor network could be sparse leading to poor sensing performance, as well as needless waste of transmit power over long communication distances. Of course, the optimal d would provide the correct trade-off between these extremes leading to the best possible performance.

The remainder of this paper is organized as follows: Section II. details the assumed models for the sensor network, target and the wireless communication system. In Section III. we derive the optimal decision fusion strategy in the regime of high local observation SNR and analyze its error probability. Section IV. derives the optimal sensor placements that leads to the minimum possible Bayesian fusion performance for different target-location models and wireless channel models. Numerical examples that shows superior performance of optimal sensor networks designed according to the derived results are also presented in Section IV.. Finally, Section V. concludes the paper.

II. System model

In this paper, we consider the detection of a randomly-located target using an n-node, one-dimensional wireless sensor network. The target is also assumed to be located randomly in a 1-D space. Denoting the target absent and present hypotheses by H_0 and H_1, respectively, the k-th sensor observations under the hypothesis H_j, for $j = 0, 1$, are given by

$$H_j : y_k \quad = \quad x_{j,k} + v_k, \tag{1}$$

where observation noise v_k is assumed to be a sequence of iid (independent and identically distributed), zero-mean random variables. Due to node power and bandwidth constraints, the k-th node makes a local binary decision $\delta_k(y_k) \in \{0, 1\}$ on the true hypothesis that is to be sent to a fusion center. Note that, each node makes its decision based on its own observation, so there is no need for any inter-sensor communication. Let us denote by $P_{f_k}(\mathbf{r}_t)$ and $P_{d_k}(\mathbf{r}_t)$ the false-alarm and detection probabilities, respectively, associated with the k-th node's binary decisions $\delta_k(y_k)$'s, conditioned on the target location being at location $\mathbf{r}_t$.

The local decisions are transmitted to the fusion center over a noisy, wireless channel via antipodal signalling. Hence, the transmit symbol from node k is given by $u_k = 2\delta(y_k) - 1$ where $u_k \in \{+1, -1\}$. Assuming orthogonal sensor-to-fusion center communication, the equivalent complex-baseband representation of the received signal at the fusion center due to the k-th node can be written as

$$z_k \quad = \quad g_k h_k u_k + w_k, \tag{2}$$

where g_k is the transmit amplitude at the k-th node, h_k is the (in general) complex fading coefficient between k-th node and the fusion center and w_k is the additive receiver noise that is assumed to be zero-mean, complex-Gaussian with variance σ^2. Throughout this paper, we assume equal power allocation at all nodes, so that $g_k = g$ for all k. For convenience, let $\mathbf{z} = [z_1, \cdots, z_n]^T$ denote the received signal vector at the fusion center from all sensor nodes.

In a spatially distributed wireless sensor network, the distance between any given node and the fusion center varies from node to node. If the distance from the fusion center to the k-th node is d_k, the received power from that node is attenuated in proportion to $d_k^{-\alpha}$, where $\alpha \geq 2$ is the path loss exponent of the wireless channel and usually can be $2 \leq \alpha \leq 6$ [8]. In addition, in a wireless channel there is short-term fading that is typically modeled as being random (for convenience, in this paper we do not explicitly model shadowing effects). Thus, in general, the propagation channels seen by nodes in a distributed, fixed wireless sensor network are inhomogeneous. Accordingly, we assume that the channel fading coefficients are independent but non-identically distributed such that $h_k = \alpha_k \sqrt{\gamma_k}$ where α_k is a random variable with $\mathbb{E}\left\{|\alpha_k|^2\right\} = 1$ accounting for random channel variations that is independent from the node-location dependent γ_k. In particular, in a Rayleigh fading channel, without a line-of-sight (LOS) component, α_k is zero-mean, complex-Gaussian with independent real and imaginary parts. Note that $\mathbb{E}\{|h_k|^2\} = \mathbb{E}\left\{|\alpha_k|^2\right\}\gamma_k = \gamma_k$. Thus γ_k represents the received channel power level at the fusion center due to transmissions from node k as a function of distance between them.

In a fixed wireless sensor network the dependence of γ_k's over the sensor nodes is different depending on the spatial distribution of the nodes. In this paper, we consider a one-dimensional fixed sensor network as shown in Fig. 1 where sensor nodes are equi-spaced along a straight line so that $d_k = kd$. In this case γ_k's represent the average received power level at the fusion center due to node k:

$$\gamma_k \quad = \quad \frac{\gamma_d}{k^\alpha}. \tag{3}$$

where $\alpha \geq 2$ is the path loss exponent of the wireless channel and γ_d is the average received channel power from the first node when it transmits at a unit power. The average received channel power per node (averaged over all nodes) is

$$\bar{\gamma}_d \quad = \quad \frac{1}{n}\sum_{k=1}^{n}\gamma_k \;=\; \frac{\gamma_d}{n}\sum_{k=1}^{n}\frac{1}{k^\alpha} \;\approx\; \frac{\gamma_d}{n}\zeta(\alpha), \tag{4}$$

where $\zeta(\alpha) = \sum_{k=1}^{\infty}\frac{1}{k^\alpha}$, for $\alpha > 1$, is the Riemann-zeta function. Thus (3) can be written in terms of the parameter $\bar{\gamma}_d$ as

$$\gamma_k \quad = \quad \frac{n\bar{\gamma}_d/\zeta(\alpha)}{k^\alpha}. \tag{5}$$

III. Optimal Decision Fusion Receiver and Its Performance

The optimal detection procedures at the fusion center for binary hypothesis testing problem are the likelihood ratio tests (LRT's) based on the received signal vector $\mathbf{z}$ [9]. Assuming coherent detection, the required likelihood ratio (LR) at the fusion center is

$$L(\mathbf{z}|\mathbf{h}) \quad = \quad \prod_{k=1}^{n}\frac{p(z_k|h_k, H_1)}{p(z_k|h_k, H_0)}, \tag{6}$$

where $\mathbf{h} = (h_1, \cdots, h_n)^T$. The conditional density $p(z_k|h_k, H_1)$ can be written as:

$$\begin{aligned} p(z_k|h_k, H_1) &= \sum_{u_k}\mathbb{E}_{\mathbf{r}_t}\left\{p(u_k|\mathbf{r}_t, H_1)p(z_k|h_k, u_k)\right\} \\ &= \frac{1}{\sqrt{2\pi}\sigma}\Bigg[\mathbb{E}_{\mathbf{r}_t}\left\{P_{d_k}(\mathbf{r}_t)\right\}e^{-\frac{|z_k - g_k h_k|^2}{2\sigma^2}} \\ &\quad + \left(1 - \mathbb{E}_{\mathbf{r}_t}\left\{P_{d_k}(\mathbf{r}_t)\right\}\right)e^{-\frac{|z_k + g_k h_k|^2}{2\sigma^2}}\Bigg], \end{aligned} \tag{7}$$

where $\mathbb{E}_{\mathbf{r}_t}\{.\}$ denotes the expectation with respect to the random target location and we have let

$$p(u_k|\mathbf{r}_t, H_1) = \begin{cases} P_{d_k}(\mathbf{r}_t) & \text{if} \quad u_k = 1 \\ 1 - P_{d_k}(\mathbf{r}_t) & \text{if} \quad u_k = -1 \end{cases}.$$

Similarly, we have

$$p(z_k|H_0) = \frac{1}{\sqrt{2\pi}\sigma}\Bigg[\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\} e^{-\frac{|z_k - g_k h_k|^2}{2\sigma^2}} + (1 - \mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}) e^{-\frac{|z_k + g_k h_k|^2}{2\sigma^2}}\Bigg], \quad (8)$$

where again we have let

$$p(u_k|\mathbf{r}_t, H_0) = \begin{cases} P_{f_k}(\mathbf{r}_t) & \text{if} \quad u_k = 1 \\ 1 - P_{f_k}(\mathbf{r}_t) & \text{if} \quad u_k = -1 \end{cases}.$$

If the k-th node location is $\mathbf{d}_k$, then assuming a Rayleigh fading channel between the target location and the k-th node, we may reasonably approximate the false-alarm probability $P_{f_k}(\mathbf{r}_t)$ as

$$P_{f_k}(\mathbf{r}_t) \approx \frac{1}{2\left(1 + \frac{\gamma_0}{|\mathbf{d}_k - \mathbf{r}_t|^2}\right)}, \quad (9)$$

where γ_0 denotes the local observation SNR at each distributed node (assumed to be the same for all nodes). Under these conditions it is also reasonable to assume that

$$P_{d_k}(\mathbf{r}_t) \approx 1 - P_{f_k}(\mathbf{r}_t), \quad (10)$$

an approximation that we will make use of throughout the rest of the paper.

In order to obtain a useful characterization of the optimal fusion rule that facilitates performance analysis and network design optimization, in the following we investigate the LR obtained by substituting (7) and (8) in (6) in the high observation SNR regime of $\gamma_0 \gg 1$. When local observation SNR γ_0 at distributed nodes is large so that $P_{f_k}(\mathbf{r}_t) \ll 1$ and $1 - P_{d_k}(\mathbf{r}_t) \ll 1$, for all $k = 1, \cdots, n$, we may simplify the LR to obtain

$$L(\mathbf{z}|\mathbf{h}) \approx \prod_{k=1}^{n} \frac{\mathbb{E}_{\mathbf{r}_t}\{P_{d_k}(\mathbf{r}_t)\}}{1 - \mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}} e^{\frac{2 g_k Re\{h_k^* z_k\}}{\sigma^2}}. \quad (11)$$

Using (10), the corresponding log likelihood ratio (LLR) is

$$\log L(\mathbf{z}|\mathbf{h}) = \frac{2}{\sigma^2}\sum_{k=1}^{n} g_k Re\{h_k^* z_k\}. \quad (12)$$

From (12) we note that optimal fusion tests compare the linear decision fusion statistic $\log L(\mathbf{z}|\mathbf{h})$ to the threshold $\log(\tau)$, where τ is a threshold determined by the particular optimality criteria (for example, Bayesian vs. Neyman-Pearson):

$$\delta(\mathbf{z}) = \begin{cases} 1 & \\ & \text{if} \quad T(\mathbf{z}) \quad \begin{matrix} \geq \\ \\ < \end{matrix} \quad t \\ 0 & \end{cases}. \quad (13)$$

where we have defined

$$T(\mathbf{z}) = \sum_{k=1}^{n} g_k Re\{h_k^* z_k\}, \quad (14)$$

and

$$t = \frac{\sigma^2}{2}\log\tau. \quad (15)$$

For example, in the case of Bayesian fusion with the minimum probability of fusion error optimality criterion, $\tau = 1$, and the threshold t simplifies to $t = 0$.

To analyze the performance of the above coherent detector, we assume that in the case of a large sensor network (i.e. large n), the decision statistic T is a normal random variable under both hypotheses. When local observation quality is good, it can then be shown that

under H_0:

$$T(\mathbf{z}) \sim \mathcal{N}\left(\sum_{k=1}^{n} g_k^2 |h_k|^2 \left(2\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t - 1)\}\right), \frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2 |h_k|^2\right),$$

and

under H_1:

$$T(\mathbf{z}) \sim \mathcal{N}\left(\sum_{k=1}^{n} g_k^2 |h_k|^2 \left(1 - 2\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}\right), \frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2 |h_k|^2\right), \quad (16)$$

where we have used the approximation (10) in obtaining (16). The false-alarm and detection probabilities at the fusion center can then be derived to be, respectively

$$P_F = \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{t + \sum_{k=1}^{n} g_k^2 |h_k|^2 \left(1 - 2\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2 |h_k|^2}}\right)\right\}, \quad (17)$$

and

$$P_D = \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{t - \sum_{k=1}^{n} g_k^2 |h_k|^2 \left(1 - 2\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2 |h_k|^2}}\right)\right\}. \quad (18)$$

Using the fact that when local decision quality is good $t = 0$, the Bayesian fusion error probability can be written as

$$P_e = \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{\sum_{k=1}^{n} g_k^2 |h_k|^2 \left(1 - 2\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2 |h_k|^2}}\right)\right\}. \quad (19)$$

IV. Optimal Sensor Network Design

A. *Exponential Target in a 1-D Sensor Network with No Short-term Fading*

When there is no short-term fading the channel coefficients are simply given by $|h_k|^2 = \gamma_k$ where γ_k is the received power at the fusion center from the k-th node when it uses a unit transmit power. The target location is assumed to follow an exponential distribution with a known parameter D_t. Assuming $\gamma_0 \gg 1$, and using (9), we can approximate the average local false-alarm probability as

$$\mathbb{E}_{\mathbf{r}_t}\left\{P_{f_k}(\mathbf{r}_t)\right\} \approx \frac{D_t^2}{2\gamma_0}\left[1+\left(1-\frac{d_k}{D_t}\right)^2\right]. \quad (20)$$

We can obtain the false-alarm, detection and Bayesian error probabilities by substituting (5) and (20) in (17)-(19). In particular, the fusion center error probability can be written as

$$P_e \approx Q\left(\frac{\sqrt{2n\bar{\gamma}_d\gamma_c}}{\zeta(\alpha)}\sum_{k=1}^{n}\frac{\left(1-2\mathbb{E}_{\mathbf{r}_t}\left\{P_{f_k}(\mathbf{r}_t)\right\}\right)}{k^\alpha}\right) \quad (21)$$

$$= Q\left(\sqrt{2n\bar{\gamma}_d\gamma_c}\left[1-\frac{D_t^2}{\gamma_0}\left(2-2\frac{d}{D_t}\frac{\zeta(\alpha-1)}{\zeta(\alpha)}+\frac{d^2}{D_t^2}\frac{\zeta(\alpha-2)}{\zeta(\alpha)}\right)\right]\right) \quad (22)$$

where, in (22) we have used (20) with $d_k = kd$ and defined the fusion center SNR as $\gamma_c = \frac{g^2}{\sigma^2}$. In (22) we require that $\alpha > 3$ for Riemann-zeta functions to be well-defined. Note that (22) indicates an asymptotic performance floor as γ_0 tends to infinity, since

$$\lim_{\gamma_0 \longrightarrow \infty} P_e = Q\left(\sqrt{2n\bar{\gamma}_d\gamma_c}\right). \quad (23)$$

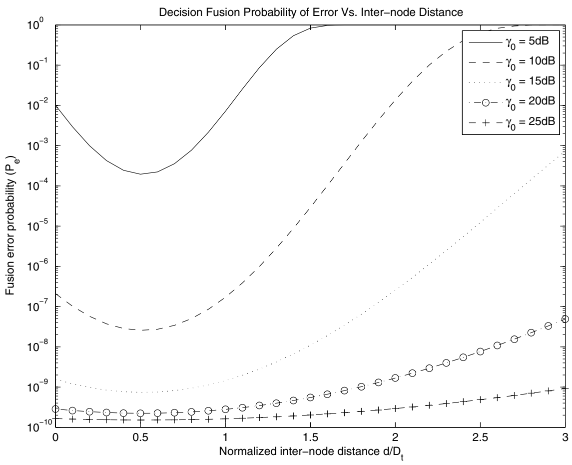

Figure 2: Dependance of Decision Fusion Error Probability on Normalized Inter-node Spacing $\frac{d}{D_t}$ for $D_t = 1$, $\alpha = 3.5$, $\bar{\gamma}_d = 1$, $\gamma_c = 0$ dB and $n = 20$.

Of course, this is usually the case in many decision fusion problems with distributed decisions: the fusion performance will ultimately be limited by the quality of the communication channel between the distributed nodes and the fusion center (as characterized by the channel SNR γ_c) no matter how good the local decisions are [10, 11]. This behavior is shown in Fig. 2 for the assumed parameters of $\alpha = 3.5$, $D_t = 1$, $\bar{\gamma}_d = 1$, $\gamma_c = 0$dB and $n = 20$ (for convenience, in Fig. 2 we have used the normalized inter-node distance defined as $\frac{d}{D_t}$). Observe from Fig. 2 that the fusion error probability finally converges to the same asymptotic limit (23) regardless of the actual inter-node distance d. However, the inter-node distance determines how large γ_0 needs to be for P_e to achieve its asymptotic value (23) for a given n and channel SNR γ_c. Intuitively, this reflects the trade-off between the inter-node distance d and the required local SNR γ_0.

Figure 2 also shows the non-uniform dependence of fusion error probability on inter-node distance d. Clearly, there is an optimal $d = d_0$ that leads to the minimum error probability for any given target parameter D_t, channel SNR γ_c and the observation SNR γ_0. As seen from Fig. 2, even a small deviation from the optimal d may lead to significant degradation of the final fusion error probability. Hence, the node placements need to be optimized for the best possible performance. It can easily be seen that both Neyman-Pearson (in which the detection probability P_D is maximized subject to a required false-alarm probability P_F threshold) and Bayesian optimality amounts to maximizing the argument of the Q-function in (21). Equivalently, it is enough to minimize $\sum_{k=1}^{n}\frac{\mathbb{E}_{\mathbf{r}_t}\left\{P_{f_k}(\mathbf{r}_t)\right\}}{k^\alpha}$ as a function of the inter-node distance d. Due to the quadratic structure of (20), it follows easily that the optimal inter-node distance d_0 is given by

$$d_0 = \frac{\zeta(\alpha-1)}{\zeta(\alpha-2)}D_t. \quad (24)$$

Observe that the optimal inter-node distance d_0 is independent of the actual operating SNR's (either channel or local) and only dependent on the target parameter D_t and the path loss exponent α. This makes the design of wireless sensor networks based on (24) simpler and more general. Figure 3 shows the optimal d_0 as a function of the path loss exponent. Note that for large α, $d_0 \longrightarrow D_t$. However, real wireless channels usually have $2 < \alpha \leq 6$ and very large α's are simply unrealistic. Hence, the optimal d_0 for a given channel environment and a target needs to be carefully determined via (24).

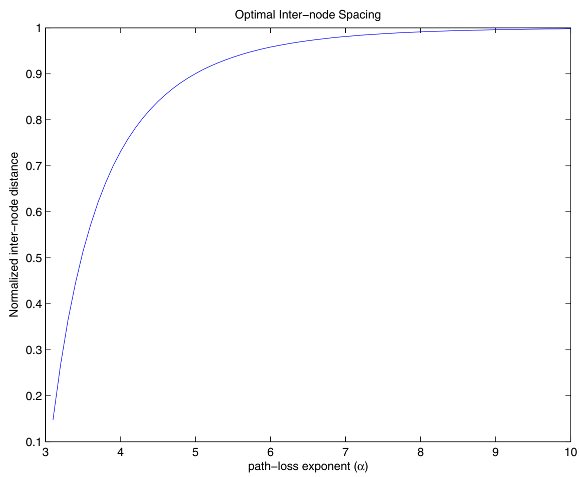

Figure 3: Optimal Normalized Inter-node Spacing $\frac{d}{D_t}$ as a Function of Path Loss Exponent.

The resulting minimum achievable fusion error probability with $d = d_0$ can easily shown to be

$$P_e \approx Q\left(\sqrt{2n\bar{\gamma}_d\gamma_c}\left[1-\frac{D_t^2}{\gamma_0}\left(2-\frac{\zeta^2(\alpha-1)}{\zeta(\alpha)\zeta(\alpha-2)}\right)\right]\right).$$

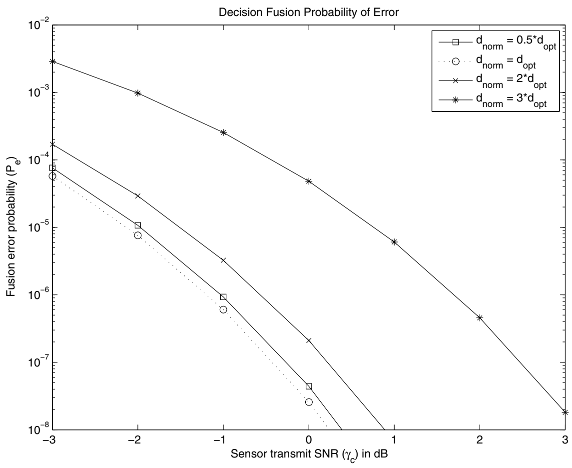

Figure 4: Decision Fusion Error Probability Versus Channel SNR γ_c for $D_t = 1$, $\alpha = 3.5$, $\bar{\gamma}_d = 1$, $\gamma_0 = 10$ dB and $n = 20$.

Figure 4 shows the performance penalty in terms of the required channel SNR that results from not using the optimal inter-node distance in designing a sensor network. The assumed parameters in Fig. 4 are $\alpha = 3.5$, $D_t = 1$, $\bar{\gamma}_d = 1$, $\gamma_0 = 10$dB and $n = 20$. For example, from Fig. 4 we observe that at $P_e = 10^{-6}$, there is about 3dB penalty in channel SNR γ_c if we were to use $d = 3d_0$ rather than $d = d_0$. This shows that in some cases, there can be severe waste of node power if sensors are placed without regard to the optimal choice of locations. Interestingly, Fig. 4 also shows that the SNR penalty is severe when $d > d_0$ compared to when $d < d_0$ (by the same factor). However, smaller d values lead to large number of required nodes to cover the same area (node density increases), which of course is costly. On the other hand, larger d values allow covering the same area with fewer sensing nodes. The above d_0 of course gives the optimal inter-node spacing that provides the correct trade-off between the error performance and required node density.

B. *Target Specified via Second-order Statistics in a 1-D Sensor Network with No Short-term Fading*

It can be shown that the optimal inter-node distance derived in the previous section in fact holds for a general class of target location models. Indeed, if we were to assume that the target location is randomly distributed with mean location $\mathbb{E}\{r_t\} = m_t$ and variance $Var(r_t) = \sigma_t^2$, then under the assumption of high observation SNR regime ($\gamma_0 \gg 1$), we can show that

$$P_e \approx Q\left(\sqrt{2n\bar{\gamma}_d\gamma_c}\left[1 - \frac{\left(m_t^2 + \sigma_t^2 - 2dm_t\frac{\zeta(\alpha-1)}{\zeta(\alpha)} + d^2\frac{\zeta(\alpha-2)}{\zeta(\alpha)}\right)}{\gamma_0}\right]\right). \quad (25)$$

By comparing the above fusion error probability expression, it is easily seen that (22) is in fact a special case of (25) since the assumed exponential target location distribution has $m_t = D_t$ and $\sigma_t^2 = D_t^2$. It is also easy to show that the optimal inter-node distance d_0 that minimizes (25) is given by

$$d_0 = \frac{\zeta(\alpha-1)}{\zeta(\alpha-2)}m_t. \quad (26)$$

Note that, again the optimal inter-node distance is only a function of the mean target location m_t and the path loss exponent α. In particular, it does not depend on the operating SNR values at either the fusion center or the distributed nodes. The resulting minimum fusion error probability is given by

$$P_e \approx Q\left(\sqrt{2n\bar{\gamma}_d\gamma_c}\left[1 - \frac{\sigma_t^2}{\gamma_0}\left(1 + \frac{m_t^2}{\sigma_t^2}\left(1 - \frac{\zeta^2(\alpha-1)}{\zeta(\alpha)\zeta(\alpha-2)}\right)\right)\right]\right).$$

It can be verified that the performance characteristics observed in the previous section in the case of an exponentially distributed target hold verbatim in general for the second-order-specified target in the asymptotically large observation SNR regime. In particular, there is a performance floor determined by the channel SNR for large observation SNR γ_0, as observed previously for an exponentially-located target.

C. *Target Specified via Second-order Statistics in a 1-D Wireless Sensor Network with Fading*

We can further show that the above optimal inter-node spacing expression derived above holds true even when there is channel fading between the fusion center and the distributed nodes (in addition to path attenuation). When there is random fading the channel coefficients can be written as $|h_k|^2 = \alpha_k\gamma_k$ where α_k is a random variable that specifies the fading distribution (assumed to be the same for all k) and γ_k is as defined above. We assume that α_k is normalized so that $\mathbb{E}\{|\alpha_k|^2\} = 1$. As in the previous section we assume that the target location is specified via its second-order statistics: $\mathbb{E}\{r_t\} = m_t$ and $Var(r_t) = \sigma_t^2$ (Note that, the assertions we make below are then also applicable to the case of exponential target location discussed earlier in Section A. without fading). Assuming high observation SNR regime at the distributed nodes, the fusion error probability can derived to be

$$P_e \approx \mathbb{E}_{\mathbf{h}}\left\{Q\left(\sqrt{\frac{2n\bar{\gamma}_d\gamma_c}{\zeta(\alpha)}}\frac{\sum_{k=1}^{n}\frac{|\alpha_k|^2}{k^\alpha}\left(1 - \frac{\left(m_t^2+\sigma_t^2-2dm_tk+d^2k^2\right)}{\gamma_0}\right)}{\sqrt{\sum_{k=1}^{n}\frac{|\alpha_k|^2}{k^\alpha}}}\right)\right\}.$$

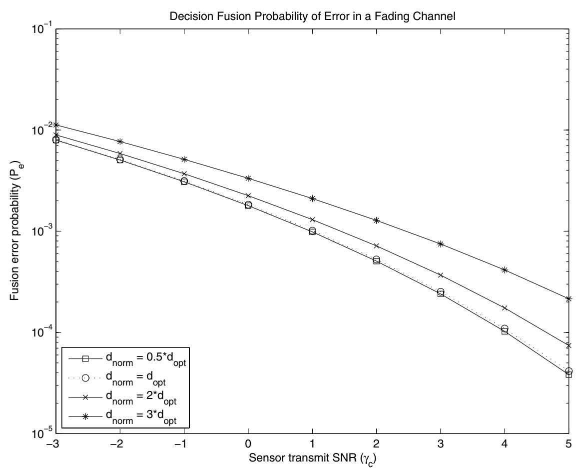

Figure 5: Decision Fusion Error Probability Versus Channel SNR γ_c for $D_t = 1$, $\alpha = 3.5$, $\bar{\gamma}_d = 1$, $\gamma_0 = 20$ dB and $n = 20$ in the Presence of Rayleigh Fading.

While optimizing the above error probability directly is difficult, when the number of sensor nodes n is very large it can be argued that the average performance (over the fading realizations) is approximately maximized by choosing the optimal d that maximizes the average SNR. Assuming further that this is approximately the same as maximizing the expected value of the numerator of the argument of the Q-function in (27), it can be shown that when n is sufficiently large the optimal inter-node spacing is approximately given by $d_0 \approx m_t \frac{\zeta(\alpha-1)}{\zeta(\alpha-2)}$, which is the same as (24). In Figure 5 we have shown the fusion error probability in the presence of small-scale random fading (modeled as being Rayleigh) on top of path loss attenuation. Note that every point on Fig. 5 was obtained by averaging over 100 independent fading samples. As can be seen from Fig. 5, the above optimal inter-node d_0 indeed provides a very good approximation. Furthermore, as in an AWGN channel, the performance degradation is more severe for sub-optimal d values larger than the optimal d_0 compared to those that are smaller.

V. Conclusions and Future Work

We considered the problem of optimal design of fixed wireless sensor networks for distributed target detection with decision fusion. The optimal fusion receiver and its error probability performance were derived assuming high local observation SNR regime at distributed nodes. The optimal inter-node spacing d was derived for a 1-D, equi-spaced WSN with no short-term fading in detecting an exponentially located target. It was shown that the derived optimal inter-node spacing expression also holds true approximately when fading is present and also generalizes to a target location distribution specified via only the second order statistics. Further, the optimal node locations are only a function of channel path loss exponent and the mean target location. In particular, they do not depend on operating SNR either at the fusion center or at local nodes. These properties of the optimal inter-node spacing simplify the design of optimal WSN's since the optimality is essentially preserved under various network conditions (fading, no fading, second-order target and operating SNR's). It is of further interest to investigate optimal sensor placement problem for other commonly assumed sensor network models as well as target models.

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