Power Efficient Distributed Estimation in a Bandlimited Wireless Sensor Network

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***Abstract*— The problem of distributed estimation of a non-random parameter in a Wireless Sensor Network (WSN) with, in general, correlated observations over a bandlimited channel is addressed. Each distributed node performs amplify-and-forward (AF) processing on its noise corrupted local observations and sends the locally processed data to the fusion center over a wireless channel sharing a common bandwidth. The bandwidth constraint is taken into account by assigning a direct sequence code division multiple access (DS-CDMA) signaling waveform for each node. The communication with orthogonal, equi-correlated and perfect correlated signaling waveforms is considered. We find the Best Linear Unbiased Estimator (BLUE) based on the observations at the fusion center. Assuming perfect synchronization in sensor transmissions, we first analyze the asymptotic Mean Squared Error (MSE) performance with correlated observations assuming equal node power and identical channel gains. Next the optimal power allocation scheme is derived to minimize the total power consumption of the network meeting a required MSE. We show that the optimal power scheduling scheme with perfect correlated signaling waveforms has a significant performance over that of using orthogonal signaling. The impact of synchronization errors on MSE performance is also analyzed and it is shown that with small synchronization errors the use of perfect correlated signaling waveforms performs well when compared with orthogonal channels.**

I. Introduction

In most distributed estimation approaches, it is assumed that the sensors transmit their observations to the fusion center over a set of orthogonal channels. However, the use of bandlimited (or non-orthogonal) channels has been attracted considerable attention in the context of wireless sensor networks since the available bandwidth of the system is fixed regardless of the number of nodes.

The use of bandlimited channels in WSNs has been considered by many recent works. In [1] the estimation over Type-Based Multiple Access (TBMA) was considered where each sensor transmits its observations using certain signaling in a shared channel. They have shown that TBMA is asymptotically optimal in the limit of large number of sensors if the channel gains are identical. Power efficient distributed estimation of a random parameter over bandlimited channel was considered in [2]. The asymptotic performance analysis based on non-orthogonal channels for distributed detection was addressed in [3], [4].

In addition to limited bandwidth, an important issue to be considered in WSNs is node power, since sensors are usually equipped with small size batteries that can be expensive and/or difficult to replace. A considerable work has been done on power constrained WSNs for distributed estimation in the literature, to name a few [2], [5], [6]. In [5] the minimum energy decentralized estimation with correlated data was addressed. They have exploited the knowledge of noise covariance matrix to select quantization levels at nodes and minimum power was derived accordingly to meet a target MSE. In [6], the optimal power scheduling scheme meeting a required target MSE at the fusion center (with independent observations) was considered assuming quantized decisions at local nodes. It was shown that the optimal power scheduling scheme decreases the quantization resolutions of the nodes correspond to bad channels or poor observation qualities. In [2], the same problem was addressed with AF processing at local nodes.

In this work, the estimation of a non-random parameter over a bandlimited channel with, in general, correlated observations is considered with analog forwarding at local nodes. Each node is assigned a signaling waveform (or code) which corresponds to DS-CDMA. We consider the cases where signaling waveforms are orthogonal, equi-correlated and perfectly correlated. Assuming perfect synchronization in sensor transmissions, first we analyze the asymptotic MSE performance for correlated observations with equal power at nodes and identical channel gains. Next, we derive the optimal power allocation schemes for the communication with orthogonal and perfectly correlated codes to achieve a required MSE performance at the fusion center. It is shown that the optimal power scheduling scheme for perfectly correlated channels has a better performance over that of the orthogonal channels. We also discuss the effect of the synchronization errors on the estimation performance.

The remainder of this paper is organized as follows. Section II presents the sensor network model and formulates the estimation problem. In Section III, the asymptotic MSE performance is analyzed for correlated observations assuming equal power at sensor nodes and identical channel gains. Assuming channels undergo fading, the optimal power allocation schemes for orthogonal and non-orthogonal communication are presented in Section IV. In Section V, the effect of the synchronization errors in sensor transmissions on estimation performance is discussed. The conclusions of this work are given in Section VI.

II. Sensor Network Model

Consider a WSN with n spatially separated sensors. Each sensor has a measurement z_k of a non-random parameter θ:

$$z_k = \theta + v_k; \;\; k = 1, 2, \cdots, n$$

where v_k's are assumed to be zero mean correlated additive noise with covariance matrix $\mathbf{\Sigma_v}$. We assume that θ has a finite range so that its average energy is finite. Let us define the local signal-to-noise ratio $\gamma_0 = \frac{P_s}{\sigma_v^2}$ where P_s is the average power of the parameter to be estimated and σ_v^2 is the noise variance of each v_k. Each node performs AF processing on its observation with a gain of g_k. The k-th node is assigned a signaling waveform $\mathbf{s}_k$ normalized such that $\mathbf{s}_k^T\mathbf{s}_k = 1$, for $k = 1, \cdots, n$. The number of degrees of freedom in the signaling waveform is assumed to be N so that $\mathbf{s}_k$ is a length N vector for $k = 1, \cdots, n$. Then the transmitted signal u_k at each sensor node is given by $u_k = g_k z_k \mathbf{s}_k$. A sufficient statistic for the estimation of θ at the fusion center is given by the output of a bank of n filters matched to the signalling waveforms $\mathbf{s}_k$'s. Assuming perfect synchronization in sensor transmissions, the matched filter output is given by [7],

$$\mathbf{y} = \mathbf{RAz} + \mathbf{w} \tag{1}$$

where $\mathbf{R}$ is the code cross correlation matrix, $\mathbf{A} = \mathrm{diag}(h_1 g_1, \cdots, h_n g_n)$ where h_k's are the channel fading coefficients and $\mathbf{w}$ is the filtered Gaussian noise vector distributed as $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2\mathbf{R})$ where σ_w^2 is the receiver noise power at the fusion center. In this paper we assume that $\mathbf{R}$ has the following form which is a common assumption in practice: $\mathbf{R} = \begin{bmatrix} 1 & \rho & . & . & . & \rho & \rho \\ \rho & 1 & . & . & . & \rho & \rho \\ . & . & . & . & . & . & . \\ \rho & \rho & . & . & . & \rho & 1 \end{bmatrix}$ where $|\rho| \leq 1$. The Best Linear Unbiased Estimator (BLUE) at the fusion center based on matched filter output $\mathbf{y}$ can shown to be (which is the same as MVUE when the noise is Gaussian),

$$\hat{\theta}(\mathbf{y}) = \frac{\mathbf{e^T AR\Sigma_n}^{-1}\mathbf{y}}{\mathbf{e^T AR\Sigma_n}^{-1}\mathbf{RAe}},$$

where $\mathbf{\Sigma_n} = \mathbf{RA\Sigma_v AR} + \sigma_w^2\mathbf{R}$ and $\mathbf{e}$ is the n-length vector with all ones. The resulting MSE is given by,

$$\mathrm{MSE}(\hat{\theta}) = \left(\mathbf{e^T AR\Sigma_n}^{-1}\mathbf{RAe}\right)^{-1}. \tag{2}$$

III. Asymptotic MSE Performance

For asymptotic analysis, we assume that each node has the same amplification factor g and identical channel gains, $h_k = 1$ for $k = 1, \cdots, n$. Then MSE in (2), using matrix inversion lemma, can shown to be

$$\mathrm{MSE}(\hat{\theta}) = \frac{1}{\frac{g^2}{\sigma_w^2}\left(\mathbf{e^T Re} - \frac{g^2}{\sigma_w^2}\mathbf{e^T}\left[(\mathbf{R\Sigma_v R})^{-1} + \frac{g^2}{\sigma_w^2}\mathbf{R}^{-1}\right]^{-1}\mathbf{e}\right)}.$$

Let us denote, $\mathbf{Z}_n = \left((\mathbf{R\Sigma_v R})^{-1} + \frac{g^2}{\sigma_w^2}\mathbf{R}^{-1}\right)$. Further, let us assume the noise covariance matrix $\mathbf{\Sigma_v}$ has the Gauss-Markov model, so that

$$\mathbf{\Sigma_v} = \sigma_v^2 \begin{bmatrix} 1 & \rho_d & . & . & . & \rho_d^{n-2} & \rho_d^{n-1} \\ \rho_d & 1 & . & . & . & \rho_d^{n-3} & \rho_d^{n-2} \\ . & . & . & . & . & . & . \\ \rho_d^{n-1} & \rho_d^{n-2} & . & . & . & \rho_d & 1 \end{bmatrix} \tag{3}$$

where $|\rho_d| \leq 1$. It is easy to see that with noise covariance matrix (3), $\mathbf{Z}_n$ becomes a circulant matrix for sufficiently large n . Since the inverse of a circulant matrix is also circulant, $\mathbf{Z}_n^{-1}$ is a circulant matrix. It can be shown that for large n [8],

$$\mathbf{e^T Z}_n^{-1}\mathbf{e} = n\lambda_{\mathbf{Z}^{-1},M} \tag{4}$$

where $\lambda_{\mathbf{Z}^{-1},M}$ is the largest eigenvalue of $\mathbf{Z}_n^{-1}$. By using eigenvalue decomposition (EVD) and exploiting the fact that all circulant matrices have same eigenvectors, we have $\mathbf{Z}_n^{-1} = U\left[\Lambda_\mathbf{R}^{-1}\Lambda_\mathbf{v}^{-1}\Lambda_\mathbf{R}^{-1} + \frac{g^2}{\sigma_w^2}\Lambda_\mathbf{R}^{-1}\right]^{-1}U^*$ where $\Lambda_\mathbf{R}$ and $\Lambda_\mathbf{v}$ are diagonal matrices of eigenvalues of $\mathbf{R}$ and $\mathbf{\Sigma_v}$ respectively. U is a unitary matrix where columns of U contain eigenvectors of an $n \times n$ circulant matrix. The m-th eigenvalue of $\mathbf{Z}_n^{-1}$ is given by $\lambda_{\mathbf{Z}_n^{-1},m} = \frac{\sigma_w^2\lambda_{\mathbf{R},m}^2\lambda_{\mathbf{v},m}}{\sigma_w^2 + g^2\lambda_{\mathbf{R},m}\lambda_{\mathbf{v},m}}$. Now, (4) becomes,

$$\mathbf{e^T Z}_n^{-1}\mathbf{e} = n\frac{\sigma_w^2\lambda_{\mathbf{R},M}^2\lambda_{\mathbf{v},M}}{\sigma_w^2 + g^2\lambda_{\mathbf{R},M}\lambda_{\mathbf{v},M}}$$

where $\lambda_{\mathbf{R},M}$ and $\lambda_{\mathbf{v},M}$ are maximum eigenvalues of $\mathbf{R}$ and $\mathbf{\Sigma_v}$ respectively. It can be shown that [4], [8], for large n, $\lambda_{\mathbf{R},M}$ equals to $(1+\rho(n-1))$ for $0 \leq \rho \leq 1$ and $1-\rho$ for $-1 \leq \rho < 0$ respectively and $\lambda_{\mathbf{v},M} = \frac{\sigma_v^2(1+|\rho_d|)}{(1-|\rho_d|)}$ for $|\rho_d| < 1$. Then MSE asymptotically is given by,

$$\mathrm{MSE}(\hat{\theta}) = \frac{(1-\rho_d)\sigma_w^2 + g^2\sigma_v^2(1+\rho_d)(1-\rho+(n\rho)^+)}{ng^2(1-\rho_d)(1-\rho+(n\rho)^+)}$$

where $(x)^+$ equals 0 for $x < 0$, and otherwise equals to x.

1) ρ=0: Orthogonal Communication: For orthogonal channels, the cross correlation between codes is $\rho = 0$. Then the asymptotic MSE is given by,

$$\mathrm{MSE}(\hat{\theta}) = \frac{(1-\rho_d)\sigma_w^2 + g^2\sigma_v^2(1+\rho_d)}{ng^2(1-\rho_d)}. \tag{5}$$

2) ρ=1: Perfect correlation between codes : In this case, each node uses same signalling code. Then the MSE asymptotically is given by,

$$\mathrm{MSE}(\hat{\theta}) = \frac{(1-\rho_d)\sigma_w^2 + ng^2\sigma_v^2(1+\rho_d)}{n^2g^2(1-\rho_d)}. \tag{6}$$

It is clear from (5) and (6), that the use of non-orthogonal channels improves the MSE performance. Figure 1 shows the derived asymptotic MSE performance and the exact MSE as a function of n for a given ρ_d. It can be seen that the derived asymptotic expression for MSE is a good approximation for the exact MSE even with relatively small n. The figure also shows that the MSE performance is improved by increasing code cross correlation. This is because, with AF local processing and non-orthogonal channels, the distributed sensor system tends to act as a cooperative beam-former. For $\rho = 1$, the system has a perfectly directed beam towards the fusion center that

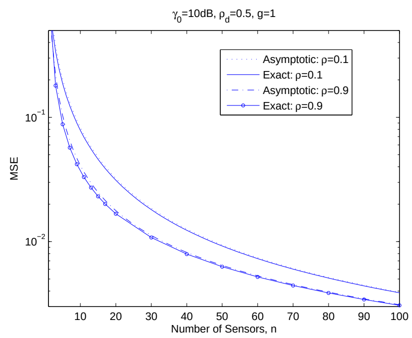


Fig. 1. MSE as a function of number of sensors n

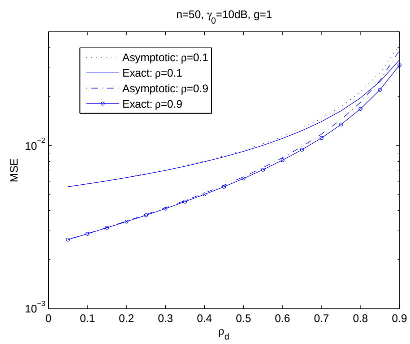


Fig. 2. The dependence of MSE on local observation correlation parameter ρ_d

exploits the full coherent gain. In contrast, when $\rho = 0$, a set of orthogonal channels are used for sending information regarding the same estimator and does not have the cooperative beam-forming gain. In Fig. 2 the dependence of MSE on local observation correlation ρ_d is shown. It is observed that when observation correlation is larger, the MSE performance is degraded. This is because of the fact that the new information added by the additional sensors decreases as the correlation increases. However, it is shown that by increasing the code cross correlation ρ, a better performance can be achieved even when the observations are highly correlated.

IV. Optimal Power Allocation in Fading Channels

In the following we assume the channels between sensors and the fusion center undergo fading. The objective is to allocate the node power in an optimal way such that the minimum power is spent by the network to achieve a desired MSE performance at the fusion center. The optimization problem can be formulated as

$$\min_{g_k \geq 0, k=1,\cdots,n} \sum_{k=1}^{n} g_k^2 \quad \text{such that } \mathrm{MSE}(\hat{\theta}) \leq D_0 \tag{7}$$

where D_0 is the required MSE threshold at the fusion center.

A. $\rho = 0$ and i.i.d observations

When the observations are i.i.d., $\mathbf{\Sigma_v} = \sigma_v^2 \mathbf{I}$. Since $\rho = 0$, $\mathbf{R} = \mathbf{I}$. Then MSE in (2) becomes $\mathrm{MSE}(\hat{\theta}) = \left(\sum_{k=1}^{n} \frac{h_k^2 g_k^2}{\sigma_v^2 h_k^2 g_k^2 + \sigma_w^2}\right)^{-1}$. Letting $D = \frac{1}{D_0}$, the optimization problem (7) becomes

$$\min_{g_k \geq 0, k=1,\cdots,n} \sum_{k=1}^{n} g_k^2 \quad \text{such that}$$
$$D - \sum_{k=1}^{n} \frac{h_k^2 g_k^2}{h_k^2 g_k^2 \sigma_v^2 + \sigma_w^2} \leq 0 \tag{8}$$

The optimal solution g_k^{*2} for (8) can shown to be,

$$g_k^{*2} = \begin{cases} \frac{\sigma_w^2}{h_k^2 \sigma_v^2} \left[\frac{h_k \sum_{j=1}^{K_1} \frac{1}{h_j}}{(K_1 - D\sigma_v^2)} - 1 \right] & ; \text{ if } f(k) - 1 < 0 \text{ and } \\ & \quad n > D\sigma_v^2 \\ 0 & ; \text{ if } f(k) - 1 > 0 \text{ and } \\ & \quad n > D\sigma_v^2 \\ \text{infeasible} & ; \text{if } n < D\sigma_v^2 \end{cases} \tag{9}$$

where assuming, without loss of generality, $h_1 \geq h_2 \geq \cdots \geq h_n$, $f(k) = \frac{(k - D\sigma_v^2)}{h_k \sum_{j=1}^{k} \frac{1}{h_j}}, 1 \leq k \leq n$ and K_1 is found such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$ for $1 \leq K_1 \leq n$. Note that letting $\sqrt{\delta_0} = \sigma_w \frac{\sum_{k=1}^{K_1} \frac{1}{h_k}}{K_1 - D\sigma_v^2}$, for $f(k) - 1 < 0$ and $n > D\sigma_v^2$, the optimal g_k^{*2} can be written as, $g_k^{*2} = \frac{\sigma_w^2}{h_k^2 \sigma_v^2} \left(\frac{h_k \sqrt{\delta_0}}{\sigma_w} - 1 \right)$. Hence, assuming channel state information (CSI) is available at sensor nodes, once the fusion center broadcasts $\sqrt{\delta_0}$, each node can determine its power using $\sqrt{\delta_0}$ as a side information.

B. $\rho = 1$ and i.i.d. observations

For $\rho = 1$ with i.i.d. observations, the MSE in (2) is shown to be

$$\mathrm{MSE}(\hat{\theta}) = \frac{\sigma_v^2 \sum_{k=1}^{n} h_k^2 g_k^2 + \sigma_w^2}{\left(\sum_{k=1}^{n} h_k g_k\right)^2}. \tag{10}$$

Since $\mathrm{MSE}(\hat{\theta})$ in (10) is not convex over g_k's a variable transformation as in [2] is done to obtain a convex programming problem for (7). Let $q_k = h_k g_k$ for $k = 1, 2, \cdots, n$ and $s = \sum_{k=1}^{n} q_k$. Then $g_k = \frac{q_k}{h_k}$ and the optimization problem becomes,

$$\min_{q_1,\cdots,q_n;s} \sum_{k=1}^{n} \frac{q_k^2}{h_k^2} \text{ such that}$$
$$\sum_{k=1}^{n} q_k^2 + \frac{\sigma_w^2}{\sigma_v^2} \leq ds^2 \text{ and } s = q_1 + \cdots + q_n,$$

where $d = \frac{D_0}{\sigma_v^2}$. By solving the above optimization problem, the optimal g_k^{*2} can shown to be,

$$g_k^{*2} = \frac{\mu^2}{4} \frac{h_k^2}{(1 + \lambda_0 h_k^2)^2}, \; k = 1, 2, \cdots, n \tag{11}$$

where λ_0 can be found numerically by solving the equation $\sum_{k=1}^{n} \frac{\lambda_0 h_k^2}{(1+\lambda_0 h_k^2)} = \frac{1}{d}$ and μ is given by, $\mu = 2\frac{\sigma_w}{\sigma_v} \left(\frac{1}{\lambda_0^2 d} - \sum_{k=1}^{n} \frac{h_k^4}{(1+\lambda_0 h_k^2)^2} \right)^{-\frac{1}{2}}$. The optimal total power spent is $P_{total} = \sum_{k=1}^{n} g_k^{*2} = \frac{\sigma_w^2}{\sigma_v^2} \lambda_0$. From (11), it can be seen that the optimal power has a distributed structure with λ_0 and μ as side information from the fusion center assuming CSI is available at the transmitter.

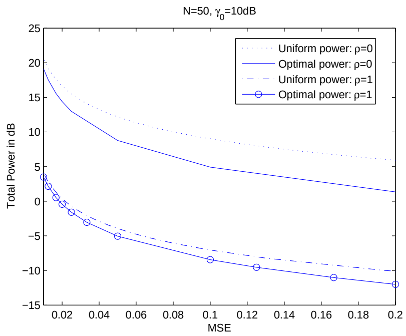


Fig. 3. Comparison of optimal power and uniform power for $\rho = 0$ and $\rho = 1$

The performance of the optimal power allocation schemes derived in sections IV-A and IV-B are shown in Fig. 3. As observed in Section III, it is seen that the MSE performance is improved as ρ increases. Also it is observed that the derived optimal power allocation scheme has a better performance compared to the uniform power allocation scheme especially when the number of sensors in the system is large and/or the required MSE is not significantly small.

It is noted from Sections IV-A and IV-B that for orthogonal communication ($\rho = 0$), it is optimal to activate the sensors with good channel quality and high local SNR while turning off the sensors with poor channel and local SNR quality. However, in the case where the channels are perfectly correlated ($\rho = 1$), it is optimal to combine all the observations irrespective of the channel and the local SNR quality. This is because, for $\rho = 1$, the system has a perfectly directed beam towards the fusion center that exploits a n factor of coherent gain when there are n sensors in the network. Therefore, for $\rho = 1$, in the optimal power allocation scheme, all the sensors are active to exploit the full coherent gain at the fusion center in contrast with $\rho = 0$ case where there is no cooperative beamforming gain.

C. $\rho = 1$ *and correlated observations*

In this case, MSE (2) can shown to be,

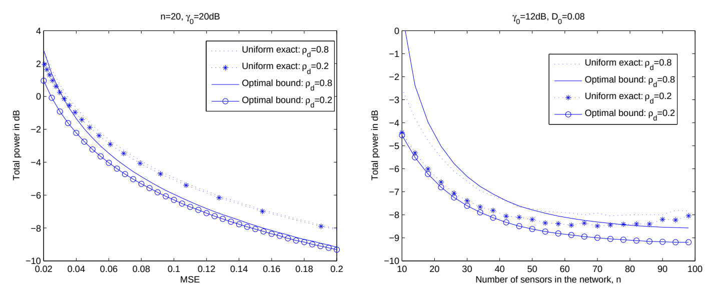
$$\mathrm{MSE}(\hat{\theta}) = \frac{\mathbf{e^T A \Sigma_v A e} + \sigma_w^2}{(\mathbf{e^T A e})^2}. \tag{12}$$

Since when the observations are correlated, it is difficult to obtain an analytical closed form solution for the optimal power allocation problem (7), using the fact that the Rayleigh quotient of a Hermitian matrix is upper bounded by its maximum eigenvalue, we find the following upper bound for the MSE (12), $\mathrm{MSE}_B(\hat{\theta}) = \frac{\lambda_M \sum_{k=1}^n h_k^2 g_k^2 + \sigma_w^2}{(\sum_{k=1}^n h_k g_k)^2}$ where λ_M is the maximum eigenvalue of $\mathbf{\Sigma_v}$. Now the optimal power allocation scheme is found to keep the MSE bound under a desired threshold D_0. Following a similar procedure as in Section IV-B, the optimal power can shown to be

$$g_k^2 = \frac{\mu'^2}{4} \frac{h_k^2}{(1 + \lambda_0' \lambda_M h_k^2)^2}, \ k = 1, \cdots, n \tag{13}$$

where λ_0' is found by solving the expression $\lambda_0' \sum_{k=1}^n \frac{h_k^2}{(1+\lambda_0' \lambda_M h_k^2)} = \frac{1}{D_0}$ and $\mu' = 2\sigma_w \left(\frac{1}{\lambda_0'^2 D_0} - \lambda_M \sum_{k=1}^n \frac{h_k^4}{(1+\lambda_0' \lambda_M h_k^2)} \right)^{-1/2}$.

The performance of the power allocation scheme based on the MSE bound is shown in Fig. 4. From Fig. 4 (a) and (b) it can be seen that the optimal power allocation scheme based on the MSE bound significantly outperforms the uniform power allocation scheme based on exact MSE when the number of sensors in the network n is large or the observation correlation coefficient ρ_d is relatively small or the local SNR quality γ_0 is moderate and high. However, it is seen that (Fig. 4 (b)) for large ρ_d, when n and γ_0 is small, the power allocation scheme based on the MSE bound does not perform well. In those cases, the uniform power allocation scheme based on exact MSE provides less total power consumption.



(a) (b)

Fig. 4. The performance of the power allocation scheme based on MSE bound for $\rho = 1$ and correlated observations. (a). Total power vs. MSE, $n = 20$, $\gamma_0 = 20dB$ (b). Total power vs. number of sensors, $\gamma_0 = 12dB$, $D_0 = 0.08$

V. Synchronization in Sensor Transmissions and the Effect of Synchronization Error on MSE Performance

An important assumption that has been made in the above analysis is perfect synchronization of sensor transmissions. In practice, achieving perfect synchronization among nodes might be a difficult task. In this section we discuss a strategy for achieving synchronization and consider the impact of synchronization errors on the MSE performance. For the analysis given below we assume a network model with i.i.d. observations and $\rho = 1$.

We follow a similar strategy as described in [9] to achieve synchronization in the sensor network. We assume that there is a master-node which broadcasts the carrier and timing signals to the rest of the sensor nodes (slave nodes). Then there are $(n-1)$ slave nodes, each at distance $d_k + \delta_k$ from the master node for $k = 1, 2, \cdots, n-1$ where d_k and δ_k are the nominal distance and the sensor placement error of the k-th node, respectively. The master node broadcasts a carrier signal $\cos(2\pi f_0 t)$ where f_0 is the carrier frequency. The received carrier signal at the k-th slave node is a noisy version of $\cos(2\pi f_0 t + \psi_k + \psi_{ek})$ where $\psi_k = \frac{2\pi f_0 d_k}{c}$ and $\psi_{ek} = \frac{2\pi f_0 \delta_k}{c}$. Each slave node employs a Phase Locked Loop (PLL) to lock onto the carrier. If each slave node precompensates for the difference in their nominal distances d_k, to the master node, by transmitting its modulated and locally processed observation with a proper delay and phase shift ψ_k, then the received signal at the fusion center

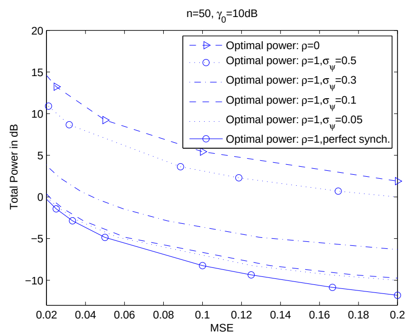


Fig. 5. The effect of the synchronization error for the MSE performance

is corrupted by the timing error and the phase error due to the sensor placement error δ_k. Considering only the phase error due to sensor placement error, the matched filter output at the fusion center is given by $y = \sum_{k=1}^{n} h_k g_k z_k \cos(\psi_{ek}) + w$. To analyze the effect of phase error due to sensor placement error, we assume that the placement error δ_k is distributed as Gaussian with zero mean and the variance σ_δ^2 which is much smaller than the wavelength λ_0. Then the phase error $\psi_{ek} \sim \mathcal{N}(0, \sigma_\psi^2)$ and we assume that σ_ψ^2 is small. To obtain the BLUE estimator, we take the expectation of y with respect to both z_k and ψ_{ek}. i.e. $E(y) = \theta e^{-\frac{\sigma_\psi^2}{2}} \sum_{k=1}^{n} h_k g_k$ assuming the observation noise v_k is i.i.d.. Then the BLUE estimator is $\hat{\theta}_{BLUE}(y) = \frac{y}{e^{-\frac{\sigma_\psi^2}{2}} \sum_{k=1}^{n} h_k g_k}$ and the resulting MSE with the phase error is given by,

$$\begin{aligned} \text{MSE}'(\hat{\theta}) &= \frac{e^{-\sigma_\psi^2} \sigma_v^2 \sum_{k=1}^{n} h_k^2 g_k^2 + \sigma_w^2}{e^{-\sigma_\psi^2} \left(\sum_{k=1}^{n} h_k g_k\right)^2} \\ &= \frac{\sigma_v^2 \sum_{k=1}^{n} h_k^2 g_k^2 + e^{\sigma_\psi^2} \sigma_w^2}{\left(\sum_{k=1}^{n} h_k g_k\right)^2} \end{aligned}$$

which is greater than the MSE with perfect synchronization in (10), showing that the synchronization error causes a degradation of MSE performance at the fusion center. Figure 5 shows the effect of the synchronization error on the MSE performance for i.i.d. observations. It can be seen that when the variance of the phase error σ_ψ^2 is significantly small, the affect of the synchronization error on the MSE performance with perfect non-orthogonal channels ($\rho = 1$), is small. Even for relatively large σ_ψ^2, the use of perfect non-orthogonal channels gives significant performance compared to that of orthogonal channels ($\rho = 0$).

VI. Conclusion

The distributed estimation of a non-random parameter in a bandlimited channel with AF processing at local nodes is addressed in this paper. We consider in general, correlated observations. First, assuming equal power and identical channel gains, asymptotic performance of MSE was analyzed for correlated observations. It was shown that the performance based on the derived asymptotic expression closely matches with the exact MSE performance even for relatively small network sizes. It was also shown that the use of non-orthogonal channels results significant performance over that of the orthogonal channels.

Next, assuming fading channels between sensor nodes and the fusion center, we derived the optimal power allocation schemes with both orthogonal and perfectly correlated channels while keeping the required MSE at the fusion center under a given threshold. In the case of i.i.d. observations, it was shown that the derived optimal power allocation scheme has a distributed implementation with a limited feedback from the fusion center. Also it was shown that the optimal power allocation schemes with both perfectly correlated and orthogonal channels have better performance over corresponding uniform power allocation schemes. For correlated observations with $\rho = 1$, the power allocation scheme was found analytically using the derived bound for the MSE. It was shown that the optimal power allocation scheme based on the MSE bound has a significant performance over the uniform power allocation scheme based on the exact MSE when n is large, γ_0 is high and for relatively small observation correlation coefficient ρ_d.

When the communication between the sensors and the fusion center is non-orthogonal, the coherent gain achieved above is based on the assumption that the sensor transmissions are perfectly synchronized. We also discussed the synchronization of the sensor transmissions and the effect of synchronization errors on the MSE performance. It was shown that, for relatively small synchronization errors, the performance of the power allocation scheme for perfectly correlated channels does not have a significant degradation and it is still better than that of using orthogonal channels. Also it gives an insight on deciding the level of tolerance of the sensor placement errors within which the multiple-access communication has better performance over the orthogonal communication.

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