OPTIMAL SENSOR DEPLOYMENT FOR DISTRIBUTED DETECTION IN THE PRESENCE OF CHANNEL FADING

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ABSTRACT

The problem of optimal (fixed) wireless sensor network (WSN) design for distributed detection of a randomly-located target is addressed. This is an extension of the previous work reported in [1] where the problem was addressed for a one-dimensional (1-D) network assuming wireless channels between sensors and the fusion center undergo only the path-loss attenuation. In this paper we consider both one and two-dimensional (2-D), equi-spaced WSN models in the presence of short-term fading in addition to path-loss attenuation. The target is assumed to be exponentially distributed with a known mean. The optimal inter-node spacing is derived by optimizing the Bhattacharya bound on the error probability of the Bayesian detector. In the presence of fading, it is shown that the optimal node placement depends on the channel SNR, path loss exponent and the mean target location. However, we show that for low channel SNR's, the optimal spacing obtained for no fading case, which is only a function of path-loss exponent and the mean target location, is a good approximation to that with fading. In particular, it is not a function of the channel SNR. It is shown that in many cases the deviation from optimal inter-node spacing can cost significant performance penalty. From numerical results, it is verified that the optimal inter-node spacing obtained based on the Bhattacharya bound holds true if the performance measure were to be the exact fusion error probability.

1. INTRODUCTION

Initial optimization of node locations can have a great impact on the overall performance of a wireless sensor network. Optimal deployment of nodes will allow the nodes to capture the best information regarding a target that it attempts to detect, thereby saving the network resources. In this paper, we formulate and solve the problem of optimal node deployment (the best node placement) in one and two-dimensional (1-D and 2-D) WSNs to detect a randomly-located target.

In recent years WSN's have gained considerable attention in various detection, tracking and monitoring applications due to their versatility and flexibility. Distributed target detection and decision fusion have been researched extensively over the years in various contexts (see [2–6], and references therein). In many cases, distributed wireless sensor networks operate under strict constraints on node power and available communication bandwidth [2, 7, 8]. These inherent resource constraints make judicial system design important in order to extend the lifetime of the network. In this paper, our focus is specifically on the optimal design of wireless sensor networks for such applications.

We address the optimal node deployment problem for 1-D and 2-D wireless sensor networks to detect a randomly-located target. The target is assumed to be randomly distributed with a known parameter. For the 1-D WSN, it is assumed that there are n, equi-spaced nodes, where d denotes the distance between any two adjacent nodes as shown in Fig. 1. For the 2-D WSN model, the X and Y coordinates of the sensor locations are assumed to be equi-spaced with a distance d as shown in Fig. 5. Distributed nodes make independent binary decisions on the presence or absence of a target based on their own observations, and send the local decisions to the fusion center over a wireless channel which is assumed to undergo both path loss attenuation and small-scale fading. Based on the received noisy versions of the local decisions, the fusion center makes a final decision on whether the target is present or not.

The wireless channel suffers from both path loss due to attenuation as well as random fading. The amount of path loss is a function of the distance between a particular node and the fusion center and the path loss index of the wireless channel. In addition, the quality of local decisions from distributed nodes is a function of the relative location of the target at any given time with respect to a specific node. Hence, the fusion error probability performance critically depends on the node locations of the wireless sensor network which are essentially determined by the inter-node spacing parameter d for the assumed equi-spaced WSN models. The goal in this paper is to propose an optimal WSN design by deriving the best d that leads to the minimum achievable Bayesian fusion probability of error.

It should be noted that the inter-node spacing d directly relates to the infrastructure cost of the wireless sensor network as well as the coverage area. If one were to use too small a value of d, more sensor nodes will be required to cover a given area leading to more infrastructure cost. On the other hand, for a fixed number of nodes n it will limit the coverage area. If d were to be too large the sensor network could be sparse leading to poor sensing performance, as well as needless waste of transmit power over long communication distances. The optimal d would provide the correct trade-off between these extremes leading to the best possible performance.

The remainder of this paper is organized as follows: Section 2 details the network model, derives the optimal Bayesian decision fusion strategy in the regime of high local observation SNR and analyzes its error probability. Sections 3 and 4 derive the optimal sensor placements that lead to the minimum Bayesian error performance for 1-D and 2-D target-location models, respectively. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL, OPTIMAL DECISION FUSION RECEIVER AND ITS PERFORMANCE

Let us consider an equi-spaced wireless sensor network with n spatially distributed nodes. The target to be detected is assumed to be randomly located. The k-th sensor observation is given by,

$$\begin{aligned} H_0: &\quad y_k = x_{0,k} + v_k,\ k = 1, \cdots, n \\ H_1: &\quad y_k = x_{1,k} + v_k,\ k = 1, \cdots, n \end{aligned}$$

where H_0 and H_1 denotes target absent and present hypotheses respectively, the observation noise v_k is assumed to be i.i.d. zero mean random variables and $x_{j,k}$ is the signal to be detected under hypothesis j, where $j = 0, 1$. The k-th node makes a binary decision $\delta_k(y_k) \in \{0, 1\}$ based on the likelihood ratio of its own observation and transmits its decision to the fusion center over a wireless channel which undergoes both path-loss attenuation and fading. Let $P_{d_k}(\mathbf{r}_t)$ and $P_{f_k}(\mathbf{r}_t)$ be the false alarm and detection probabilities associated with the local decision $\delta_k(y_k)$ at the k-th node. We assume that these probabilities in general depend on the target location $\mathbf{r}_t$.

The transmitted symbol from the k-th node is given by $u_k = 2\delta_k - 1$ where $u_k \in \{-1, 1\}$. Then the received signal at the fusion center from the k-th node is given by, $z_k = g_k h_k u_k + w_k$, for $k = 1, \cdots, n$, where g_k is the amplifier gain at the k-th node, h_k is the complex channel fading coefficient between the k-th node and the fusion center and w_k is the receiver additive noise which is assumed to be i.i.d. Gaussian with mean zero and variance σ^2. In a spatially distributed wireless network, the received signal power at the fusion center from the k-th node varies according to the distance between the k-th node and the fusion center. If the distance from the k-th node to the fusion center is r_k, the received power from that node is attenuated proportional to r_k^α where $\alpha \geq 2$ is the path loss exponent.

The optimal detection procedures at the fusion center for binary hypothesis testing problem are the likelihood ratio tests (LRT's) based on the received signal vector $\mathbf{z} = [z_1, \cdots, z_n]^T$ [9]. Assuming coherent detection, the required likelihood ratio (LR) at the fusion center is $L(\mathbf{z}|\mathbf{h}) = \prod_{k=1}^n \frac{p(z_k|h_k, H_1)}{p(z_k|h_k, H_0)}$, where $\mathbf{h} = (h_1, \cdots, h_n)^T$. The conditional density $p(z_k|h_k, H_1)$ can be written as:

$$p(z_k|h_k, H_1) = \frac{1}{\sqrt{2\pi}\sigma} \Bigg[\mathbb{E}_{\mathbf{r}_t}\{P_{d_k}(\mathbf{r}_t)\} e^{-\frac{|z_k - g_k h_k|^2}{2\sigma^2}} + (1 - \mathbb{E}_{\mathbf{r}_t}\{P_{d_k}(\mathbf{r}_t)\}) e^{-\frac{|z_k + g_k h_k|^2}{2\sigma^2}} \Bigg], \quad (1)$$

where $\mathbb{E}_{\mathbf{r}_t}\{.\}$ denotes the expectation with respect to the random target location and we have let $p(u_k|\mathbf{r}_t, H_1)$ equals to $P_{d_k}(\mathbf{r}_t)$ if $u_k = 1$ and $1 - P_{d_k}(\mathbf{r}_t)$ if $u_k = -1$, respectively. Similarly,

$$p(z_k|h_k, H_0) = \frac{1}{\sqrt{2\pi}\sigma} \Bigg[\mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\} e^{-\frac{|z_k - g_k h_k|^2}{2\sigma^2}} + (1 - \mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\}) e^{-\frac{|z_k + g_k h_k|^2}{2\sigma^2}} \Bigg] (2)$$

where we have let $p(u_k|\mathbf{r}_t, H_0)$ equals to $P_{f_k}(\mathbf{r}_t)$ if $u_k = 1$ and $1 - P_{f_k}(\mathbf{r}_t)$ if $u_k = -1$, respectively. Then the LR becomes,

$$L(\mathbf{z}|\mathbf{h}) = \prod_{k=1}^n \frac{\bar{P}_{d_k} e^{\frac{g_k Re\{h_k^* z_k\}}{\sigma^2}} + (1 - \bar{P}_{d_k}) e^{-\frac{g_k Re\{h_k^* z_k\}}{\sigma^2}}}{\bar{P}_{f_k} e^{\frac{g_k Re\{h_k^* z_k\}}{\sigma^2}} + (1 - \bar{P}_{f_k}) e^{-\frac{g_k Re\{h_k^* z_k\}}{\sigma^2}}} \quad (3)$$

where $\bar{P}_{f_k} = \mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_r)\}$ and $\bar{P}_{d_k} = \mathbb{E}_{\mathbf{r}_t}\{P_{d_k}(\mathbf{r}_r)\}$.

If the k-th node location is $\mathbf{r}_k$, then assuming a Rayleigh fading channel between the target location and the k-th node, we may reasonably approximate the false-alarm probability, averaged over the Rayleigh distribution, as

$$P_{f_k}(\mathbf{r}_t) \approx \frac{1}{2\left(1 + \frac{\gamma_0}{|\mathbf{r}_k - \mathbf{r}_t|^2}\right)}, \quad (4)$$

where γ_0 denotes the local observation SNR at each distributed node (assumed to be the same for all nodes). Under these conditions it is also reasonable to assume that

$$P_{d_k}(\mathbf{r}_t) \approx 1 - P_{f_k}(\mathbf{r}_t), \quad (5)$$

an approximation that we will make use of throughout the rest of the paper. In order to obtain a useful characterization of the optimal fusion rule that facilitates performance analysis and network design optimization, in the following we investigate the LR (3) in the high observation SNR regime of $\gamma_0 \gg 1$. When local observation SNR γ_0 at distributed nodes is such that $P_{f_k}(\mathbf{r}_t) \ll 1$ and $1 - P_{d_k}(\mathbf{r}_t) \ll 1$, for all $k = 1, \cdots, n$, we may simplify (3) to obtain

$$L(\mathbf{z}|\mathbf{h}) \approx \prod_{k=1}^n \frac{\bar{P}_{d_k}}{1 - \bar{P}_{f_k}} e^{\frac{2 g_k Re\{h_k^* z_k\}}{\sigma^2}}. \quad (6)$$

Using (5), the corresponding log likelihood ratio (LLR) is

$$T(\mathbf{z}|\mathbf{h}) = \log L(\mathbf{z}|\mathbf{h}) \quad = \quad \frac{2}{\sigma^2}\sum_{k=1}^{n} g_k Re\{h_k^* z_k\}. \quad (7)$$

From (7) we note that optimal fusion tests compare the linear decision fusion statistic $T(\mathbf{z}|\mathbf{h})$ to the threshold $\log(\tau)$, where τ is a threshold determined by the particular optimality criteria:

$$\delta(\mathbf{z}) \quad = \quad \begin{cases} 1 & \\ & \text{if} \quad T'(\mathbf{z}) \quad \begin{matrix} \geq \\ < \end{matrix} \quad t \\ 0 & \end{cases}, \quad (8)$$

where we have defined $T'(\mathbf{z}) = \sum_{k=1}^{n} g_k Re\{h_k^* z_k\}$ and $t = \frac{\sigma^2}{2}\log\tau$.

To analyze the performance of the above coherent detector, we assume that in the case of a large sensor network (i.e. large n), the decision statistic T' is a normal random variable under both hypotheses. When local observation quality is good, it can then be shown that (see Appendix) under each hypotheses H_0 and H_1, $T'(\mathbf{z})$ has the following distribution:

$$\begin{aligned} H_0 &: T'(\mathbf{z}) \sim \mathcal{N}\left(\sum_{k=1}^{n} g_k^2|h_k|^2\left(2\bar{P}_{f_k}-1\right), \frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2|h_k|^2\right) \\ H_1 &: T'(\mathbf{z}) \sim \mathcal{N}\left(\sum_{k=1}^{n} g_k^2|h_k|^2\left(1-2\bar{P}_{f_k}\right), \frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2|h_k|^2\right), \end{aligned} \quad (9)$$

where we have used the approximation (5) in obtaining (9). The false-alarm and detection probabilities at the fusion center can then be derived to be,

$$P_F \quad = \quad \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{t+\sum_{k=1}^{n} g_k^2|h_k|^2\left(1-2\bar{P}_{f_k}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2|h_k|^2}}\right)\right\}$$

and

$$P_D \quad = \quad \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{t-\sum_{k=1}^{n} g_k^2|h_k|^2\left(1-2\bar{P}_{f_k}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2|h_k|^2}}\right)\right\},$$

respectively, where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$. Using the fact that $t = 0$, for Bayesian fusion with the minimum probability of fusion error optimality criterion with equal prior probabilities, when local decision quality is good, the Bayesian fusion error probability can be written as

$$P_e = \mathbb{E}_{\mathbf{h}}\left\{Q\left(\frac{\sum_{k=1}^{n} g_k^2|h_k|^2\left(1-2\bar{P}_{f_k}\right)}{\sqrt{\frac{\sigma^2}{2}\sum_{k=1}^{n} g_k^2|h_k|^2}}\right)\right\}. \quad (10)$$

When the channels between sensors and the fusion center undergo fading, we assume that $\mathbb{E}\{|h_k|^2\} = \gamma_k^2$ where γ_k^2 represents the received channel power at the fusion center from the k-th node. Since it is not straightforward to analyze expectation of the fusion error probability in (10) in closed form, in the following we consider the Bhattacharya bound on the Bayesian performance, $P_e \leq \frac{1}{2}e^{\mu_B}$. The Bhattacharya error exponent μ_B is defined as $\mu_B = \ln\mathbb{E}\{e^{\frac{1}{2}\log L(\mathbf{z})}\}$ where $L(\mathbf{z})$ is the likelihood ratio of $\mathbf{z}$ [9].

3. 1-D SENSOR NETWORK WITH RANDOMLY LOCATED TARGET

The 1-D WSN is assumed to be made of n equi-spaced nodes, where d denotes the distance between any two adjacent nodes as shown in Fig. 1 with fusion center at the origin. In this

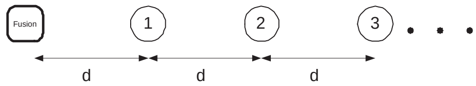

Fig. 1. 1-D Equi-spaced Sensor Network.

case the average received power level at the fusion center due to node k is, $\gamma_k^2 = \frac{\gamma_d^2}{k^\alpha}$, where $\alpha \geq 2$ is the path loss exponent of the wireless channel and γ_d^2 is the average received channel power from the first node when it transmits at a unit power. The average received channel power per node (averaged over all nodes) is

$$\bar{\gamma}_d^2 \quad = \quad \frac{1}{n}\sum_{k=1}^{n}\gamma_k^2 \quad = \quad \frac{\gamma_d^2}{n}\sum_{k=1}^{n}\frac{1}{k^\alpha} \quad \approx \quad \frac{\gamma_d^2}{n}\zeta(\alpha), \quad (11)$$

where $\zeta(\alpha) = \sum_{k=1}^{\infty}\frac{1}{k^\alpha}$, for $\alpha > 1$, is the Riemann-zeta function. Thus the average received power level at the fusion center from the k-th node can be written in terms of the parameter $\bar{\gamma}_d^2$ as

$$\gamma_k^2 \quad = \quad \frac{n\bar{\gamma}_d^2/\zeta(\alpha)}{k^\alpha}. \quad (12)$$

For the analysis of the paper, hereafter, target location is assumed to follow an exponential distribution with a known parameter D_t. But in fact, the analysis can be performed easily as far as first and second order statistics of the target distribution are available as described later in this section.

Assuming the target is distributed as exponentially with the parameter D_t starting from the origin, and letting $r_k = kd$ for $k = 1, \cdots, n$, we can find the average false alarm probability at the k-th node, assuming $\gamma_0 \gg 1$, as,

$$\bar{P}_{f_k} = \mathbb{E}_{\mathbf{r}_t}\{P_{f_k}(\mathbf{r}_t)\} \quad \approx \quad \frac{D_t^2}{2\gamma_0}\left[1+\left(1-\frac{kd}{D_t}\right)^2\right] (13)$$

For the 1-D network model assuming Rayleigh fading, the Bhattacharya bound P_{eb} on the fusion error probability becomes

$$\begin{aligned} P_{eb} &= \frac{1}{2}\mathbb{E}_{\mathbf{h}}\left\{\exp\left(-\frac{1}{\sigma^2}\sum_{k=1}^{n} g_k^2|h_k|^2(1-2\bar{P}_{f_k}) + \frac{1}{4\sigma^2}\sum_{k=1}^{n} g_k^2|h_k|^2\right)\right\}, \\ &= \frac{1}{2}\prod_{k=1}^{n}\frac{1}{\gamma_k^2}\left(\frac{1}{\gamma_k^2}+\gamma_c(1-2\bar{P}_{fk})-\frac{\gamma_c}{4}\right)^{-1}, \end{aligned} \quad (14)$$

using the relationship in (5) and $\gamma_c = \frac{g^2}{\sigma^2}$ is the channel SNR assuming equal power at each node such that $g_k^2 = g^2$ for $k = 1, \cdots, n$. Substituting for γ_k^2 and $\bar{P}_{f_k}$ in (14) we get

$$P_{eb} = \prod_{k=1}^{n}\frac{\zeta(\alpha)}{2n\bar{\gamma}_d^2}k^\alpha\left(\frac{k^\alpha\zeta(\alpha)}{n\bar{\gamma}_d^2}+\gamma_c\left[\frac{3}{4}-\frac{D_t^2}{\gamma_0}-\frac{D_t^2}{\gamma_0}\left(1-\frac{kd}{D_t}\right)^2\right]\right)^{-1}.$$

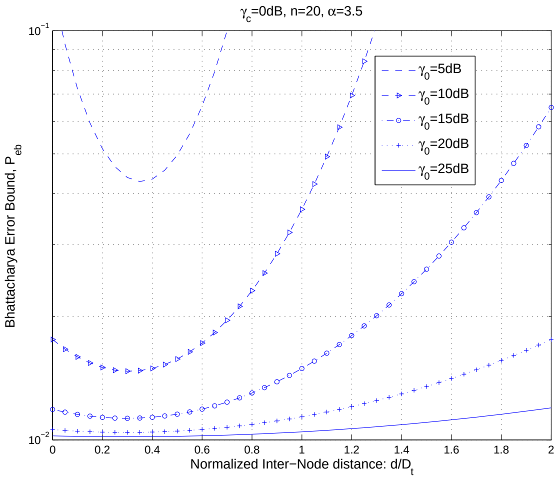

Fig. 2. Bhattacharya Error Exponent Vs the Normalized Inter-node Spacing; $D_t = 1$, $\bar{\gamma}_d^2 = 1$, $\gamma_c = 0dB$, $\alpha = 3.5$, $n = 20$

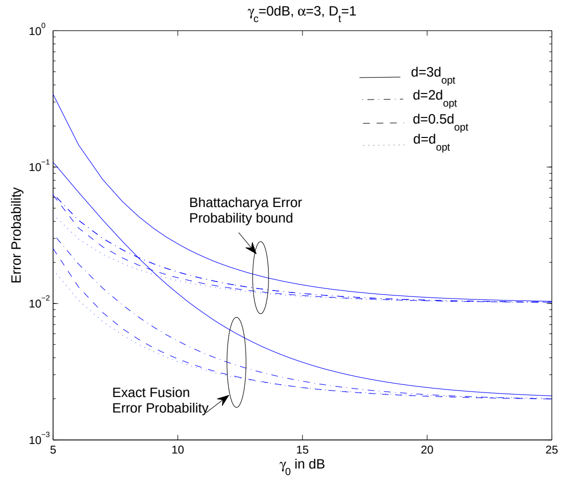

Fig. 3. Bhattacharya Error Bound and the Exact Fusion Error Probability Vs Local Observation SNR γ_0; $D_t = 1$, $\bar{\gamma}_d^2 = 1$, $\gamma_c = 0dB$, $\alpha = 3.5$, $n = 20$

Our goal is to minimize P_{eb} with respect to d to find the optimal inter-node spacing. Since $\log(.)$ is a monotonic function of its argument, minimizing P_{eb} is equivalent to minimizing $\log(P_{eb})$ w.r.t. d. Thus, (after dropping the terms that do not depend on d), the optimal inter-node spacing is given by

$$d_{opt} = \arg\max_{d} \left\{ \sum_{k=1}^{n} \log\left(\frac{k^{\alpha}\zeta(\alpha)}{n\bar{\gamma}_d^2} + \gamma_c\left[\frac{3}{4} - \frac{D_t^2}{\gamma_0} - \frac{D_t^2}{\gamma_0}\left(1-\frac{kd}{D_t}\right)^2\right]\right)\right\}.$$

It can be shown that d_{opt} is given by the solution to the following equation:

$$\sum_{k=1}^{n} \frac{k(1-\frac{kd}{D_t})}{\mu_k} = 0, \tag{15}$$

where $\mu_k = \frac{k^{\alpha}\zeta(\alpha)}{n\bar{\gamma}_d^2} + \gamma_c\left[\frac{3}{4} - \frac{D_t^2}{\gamma_0} - \frac{D_t^2}{\gamma_0}\left(1-\frac{kd}{D_t}\right)^2\right]$. For large γ_0, it is interesting to note that from (15),

$$\lim_{\gamma_0\to\infty} d_{opt} = \frac{\sum_{k=1}^{n}\frac{k}{(\lambda_k+\frac{3}{4}\gamma_c)}}{\sum_{k=1}^{n}\frac{k^2}{(\lambda_k+\frac{3}{4}\gamma_c)}} D_t \tag{16}$$

where $\lambda_k = \frac{k^{\alpha}\zeta(\alpha)}{n\bar{\gamma}_d^2}$. From (16), it is seen that for large γ_0, the optimal inter-node distance d_{opt} depends on the channel SNR in addition to the path loss component and the mean target location. But in the regime of small channel SNR, i.e. when $\gamma_c \to 0$, it can be verified that the optimal inter-node distance is given by, $d_{opt} \approx \frac{\zeta(\alpha-1)}{\zeta(\alpha-2)} D_t$, which is the same as in the case of no fading [1]. Thus, it is seen that the optimal inter-node distance derived for no fading scenario is a good approximation to the optimal inter-node distance with fading, in the region of high γ_0 and relatively small γ_c.

Although, the above derivations are based on the exponential target distribution, in fact it can be shown that the optimal spacing derived above holds for any target distribution as far as its fist and second order statistics are available. If $\mathbb{E}\{r_t^2\}$ and $\mathbb{E}\{r_t^2\}$ are first and second order statistics of any target distribution, it can be shown that the average false alarm probability at the k-th node can be approximated by (for $\gamma_0 \gg 1$),

$$\bar{P}_{f_k} \approx \mathbb{E}_{\mathbf{r}_t}\left\{P_{f_k}(\mathbf{r}_t)\right\} \approx \frac{1}{2\gamma_0}\left[r_k^2 - 2r_k\mathbb{E}\{r_t\} + \mathbb{E}\{r_t^2\}\right].$$

and the corresponding Bhattacharya error bound becomes,

$$P_{eb} = \prod_{k=1}^{n}\frac{\zeta(\alpha)}{2n\bar{\gamma}_d^2}k^{\alpha}\left(\frac{k^{\alpha}\zeta(\alpha)}{n\bar{\gamma}_d^2} + \gamma_c\left[\frac{3}{4} - \frac{1}{\gamma_0}\left(k^2d^2 - 2kd\mathbb{E}\{r_t\} + \mathbb{E}\{r_t^2\}\right)\right]\right)^{-1}.$$

Following a similar procedure, it can be shown that the optimal spacing d_{opt} for large γ_0 region given in (16) for exponential distribution, can be generalized to any target distribution with finite first moment $\mathbb{E}\{r_t\}$, as

$$\lim_{\gamma_0\to\infty} d_{opt} = \frac{\sum_{k=1}^{n}\frac{k}{(\lambda_k+\frac{3}{4}\gamma_c)}}{\sum_{k=1}^{n}\frac{k^2}{(\lambda_k+\frac{3}{4}\gamma_c)}}\mathbb{E}\{r_t\}. \tag{17}$$

Hence, the optimal spacing only depends on the first moment of the target distribution.

Figure 2 shows the Bhattacharya error bound (14) with the normalized inter-node distance for different γ_0 values under Rayleigh fading. It is seen that as γ_0 increases, for a fixed γ_c, the optimal inter-node distance d_0 does not depend on the value of γ_0. It can be verified that that the optimal inter-node distance found in (16) for large γ_0, closely matches with the exact optimal inter-node distance observed in Fig. 2.

In Fig. 3, the Bhattacharya error bound and the exact error probability Vs γ_0 was plotted with the deviations of internode distance d from its optimal value as found in (16), for fixed γ_c. The exact error probability was obtained by averaging (10) over 1000 independent fading samples. Although the

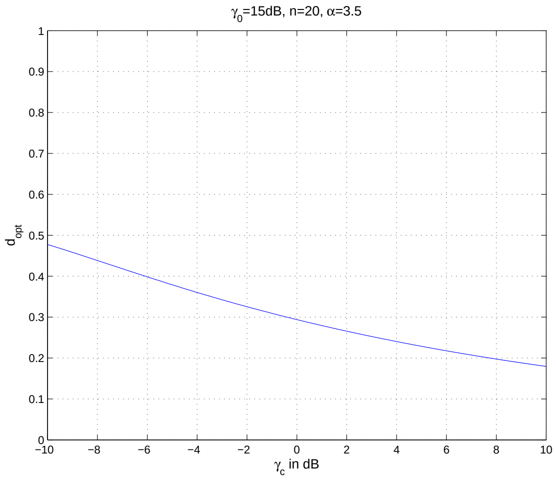

Fig. 4. Optimal Inter-node distance as a function of γ_c: $\bar{\gamma}_d^2 = 1$, $\alpha = 3.5$, $\gamma_0 = 15dB$, $n = 20$, D_t=1

Bhattacharya error bound is not a tight bound for the exact error probability, it seen that with d_{opt} and its deviations, both measures show similar characteristics. In particular, the deviation of the inter-node distance from its optimal value may cause a significant performance loss even for moderately high γ_0 values.

From Fig. 4, it is seen that for moderate values of γ_c the maximum variation of normalized optimal inter-node distance over γ_c is relatively small.

4. 2-D SENSOR NETWORK MODEL WITH A RANDOMLY LOCATED TARGET

Next, we consider a 2-D sensor network grid as shown in Fig. 5. The (k, j)-th node of the grid is assumed to be located at (r_k, r_j) for $k, j = 0, \cdots, M$ (excep (0,0) point) where r_k and r_j are X and Y coordinates of the (k, j)-th node with respect to the origin, where the fusion center is assumed to be located. Also let us assume that the X and Y coordinates of node locations are equi-spaced such that $r_k = kd$ and $r_j = jd$ for $k, j = 0, \cdots, M$ excluding the $(0, 0)$ point. The total number of sensors in the network, $n = M(M + 2)$. We also assume that X and Y coordinates of the target have independent marginal exponential distributions with a known common mean D_t. Then assuming $\gamma_0 \gg 1$, using (4) the average false alarm probability at the (k, j)-th node can be approximated as,

$$\bar{P}_{f_{kj}} = \mathbb{E}_{\mathbf{r}_t}\{P_{f_{k,j}}(\mathbf{r}_t)\} = \frac{D_t^2}{2\gamma_0}\left(2 + \left(1 - \frac{kd}{D_t}\right)^2 + \left(1 - \frac{jd}{D_t}\right)^2\right).$$

Denote by γ_{kj}^2 the average received power level at the fusion center form the (k, j)-th node. Then $\gamma_{k,j}^2$ can be written as, $\gamma_{k,j}^2 = \frac{\gamma_d^2}{(k^2+j^2)^{\frac{\alpha}{2}}}$ where γ_d^2 is the average received power at the fusion center from $(1, 0)$ or $(0, 1)$-th node and $\alpha \geq 2$ is the path loss exponent. The average received power at the fusion center per node is given by $\bar{\gamma}_d^2 = \frac{\gamma_d^2}{n}K_1$ where $K_1 = \sum_k \sum_j \frac{1}{(\sqrt{k^2+j^2})^{\alpha}}$ ($k=j\neq 0$) and $n = M(M + 2)$, as defined above. Then,

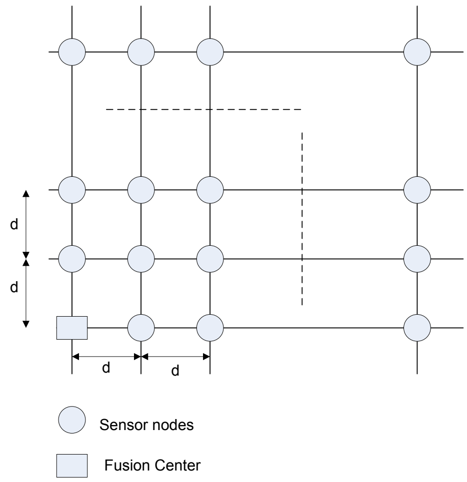

Fig. 5. 2-D Sensor Network Model

$$\gamma_{k,j}^2 = \frac{\bar{\gamma}_d^2 n}{K_1(k^2 + j^2)^{\alpha/2}}. \tag{18}$$

4.1. No Short-Term Fading

With no-short term fading, the fusion error probability (10) for the 2-D network model becomes,

$$P_e = Q\left(\frac{\sqrt{\gamma_c}\sum_{k=j\neq 0}\sum_j \gamma_{k,j}^2(1 - 2\bar{P}_{fkj})}{\sqrt{\frac{1}{2}\sum_{k=j\neq 0}\sum_j \gamma_{k,j}^2}}\right) \tag{19}$$

By minimizing P_e in (19), the optimal spacing d, along both X and Y coordinates can be shown to be, (the details are omitted here due to space limitation) $d_{opt} = \frac{K_2}{K_3}D_t$, where $K_2 = \sum_{k=0, j=k\neq 0}^{M}\sum_{j=0}^{M}\frac{(k+j)}{(k^2+j^2)^{\alpha/2}}$ and $K_3 = \sum_{k=0, j=k\neq 0}^{M}\sum_{j=0}^{M}\frac{(k^2+j^2)}{(k^2+j^2)^{\alpha/2}}$. It is observed that when there is no fading the optimal d is only a function of path loss exponent and the mean target location, as in the 1-D model [1].

4.2. With Short-Term Fading

Similar to 1-D case, in 2-D model with short-term fading the performance measure is taken as Bhattacharya error bound which can be shown to be

$$\begin{aligned} P_{eb} \quad &= \quad \frac{K_1}{n\bar{\gamma}_d^2}\prod_{k,j,k=j\neq 0}(k^2 + j^2)^{\alpha/2}\left(\frac{K_1(k^2 + j^2)^{\alpha/2}}{n\bar{\gamma}_d^2} + \right. \\ &\left. \gamma_c\left[\frac{3}{4} - \frac{2D_t^2}{\gamma_0} - \frac{D_t^2}{\gamma_0}\left(1 - \frac{kd}{D_t}\right)^2 - \frac{D_t^2}{\gamma_0}\left(1 - \frac{jd}{D_t}\right)^2\right]\right)^{-1} \end{aligned} \tag{20}$$

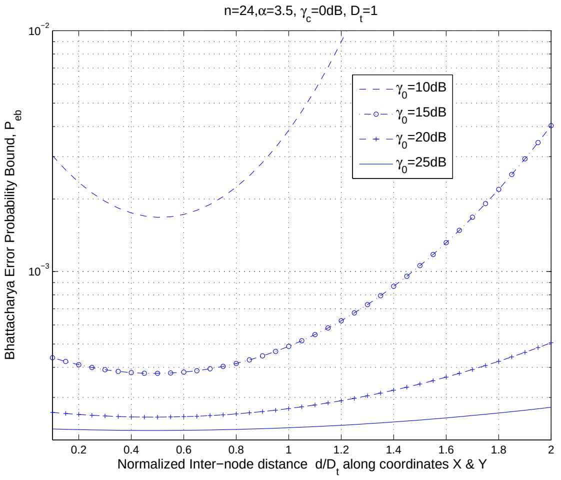

Fig. 6. Bhattacharya Error Probability Bound with the Normalized Inter-node Spacing Along Coordinates X and Y; $D_t = 1$, $\bar{\gamma}_d^2 = 1$, $\gamma_c = 0dB$, $\alpha = 3.5$, $n = 24$

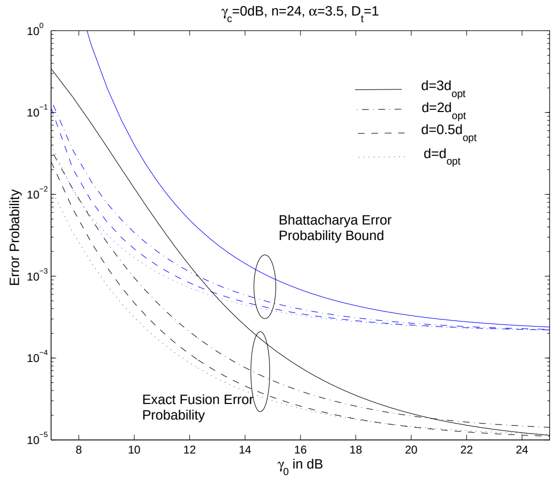

Fig. 7. Bhattacharya Error Bound Vs. Channel SNR (γ_c); $D_t = 1$, $\bar{\gamma}_d^2 = 1$, $\alpha = 3.5$, $n = 24$, $\gamma_c = 0dB$

Following a similar procedure as in Section 3 for 1-D network model, d that optimizes the Bhattacharya error bound in (20) can be found by solving the following non-linear equation:

$$\sum_{k,j,k=j\neq 0} \frac{(k+j) - (k^2+j^2)\frac{d}{D_t}}{\mu_{k,j}} = 0 \qquad (21)$$

where
$\mu_{k,j} = \frac{K_1(k^2+j^2)^{\alpha/2}}{n\bar{\gamma}_d^2} + \gamma_c \left[\frac{3}{4} - \frac{2D_t^2}{\gamma_0} - \frac{D_t^2}{\gamma_0}\left(1-\frac{kd}{D_t}\right)^2 - \frac{D_t^2}{\gamma_0}\left(1-\frac{jd}{D_t}\right)^2\right]$
When γ_0 is large, from (21), we have

$$d_{opt} = \frac{\sum\limits_{k,j,k=j\neq 0} \frac{k+j}{(\lambda_{k,j}+\frac{3}{4}\gamma_c)}}{\sum\limits_{k,j,k=j\neq 0} \frac{k^2+j^2}{(\lambda_{k,j}+\frac{3}{4}\gamma_c)}} D_t, \qquad (22)$$

where $\lambda_{k,j} = \frac{K_1(k^2+j^2)^{\alpha/2}}{n\bar{\gamma}_d^2}$. AS in the 1-D network model, we see that d_{opt} depends on the channel SNR, path loss component and the characteristics of the target distribution, for large γ_0 values. However, in the low channel SNR region it can be verified that, $\lim_{\gamma_c \to 0} d_{opt} = \frac{K_2}{K_3} D_t$, (which is the same as without fading) where K_2 and $K3$ are as defined in subsection 4.1.

Figure (6) shows the Bhattacharya bound P_{eb} (20) with the normalized inter-node distance for various γ_0 values for a fixed γ_c. Similar to the 1-D network model, the figure verifies the analytical result obtained in (22) for the optimal d for moderate to large γ_0 values.

The exact fusion error probability and the Bhattacharya bound as a function of γ_0 for a fixed γ_c is shown in Fig. 7 for deviations of inter-node distance along X and Y coordinates from its optimal value. Similar to the 1-D network model, it is seen that the with the derived optimal spacing, the exact fusion error probability shows a similar behavior as that of the Bhattacharya bound. As observed in 1-D network model we can see that the SNR penalty is severe when $d > d_{opt}$ compared to when $d < d_{opt}$ (by the same factor). Of course, when d is large, a few number of sensors are used to cover the same area which causes the performance penalty seen in the figure. Therefore the optimal d_{opt} provides the correct trade-off between the error performance and the required node density, as in the case of $1-D$ network model.

5. CONCLUSIONS

We considered the problem of optimal design of fixed wireless sensor networks for distributed target detection with decision fusion in the presence of fading. The optimal fusion receiver and its error probability performance were derived assuming high local observation SNR at distributed nodes. We considered both 1-D and 2-D sensor networks with equi-spaced nodes, and the inter-node distance d was optimized to detect an exponentially distributed target with a known mean. In fact the analysis remains similar for general target distribution models as far as the first and second order statistics of the distribution are available. In the presence of fading, the optimal d is obtained by optimizing the Bhattacharya error probability bound, where it was shown that the optimal d is determined by the path loss exponent, mean target location and the channel SNR. In [1], it was shown that with no-fading, the optimal d is only a function of channel path loss exponent and the mean target location. With fading, for relatively low channel SNR, however, it was verified that the optimal inter-node spacing can be approximated to be a function of only the channel path loss exponent and the mean target location even in the presence of fading (same as the optimal spacing with no-fading). These properties of the optimal inter-node spacing simplify the design of optimal WSN's since the optimality is essentially preserved under various network conditions.

Acknowledgement

This research was supported in part by the National Science Foundation under the grant CCF-0830545.

Appendix

Suppose $S_n = X_1 + \cdots + X_n$ is the sum of n independent random variables with $\mathbb{E}\{X_k\} = \eta_k$ and $Var(X_k) = \nu_k^2$ for $k = 1, \cdots, n$. Define $K^2 = \sum_{k=1}^n v_k^2$ and $L^3 = \sum_{k=1}^n \rho_k^3$ where $\rho_k^3 = \mathbb{E}\{|X_k - \mathbb{E}\{X_k\}|^3\}$ is the third absolute central moment of X_k. Then if $\lim_{n\to\infty} \frac{L}{K} \to 0$, the central limit theorem states that the sum S_n converges in distribution to a Gaussian random variable with mean $\sum_{k=1}^n \eta_k$ and the variance $\sum_{k=1}^n \nu_k^2$ as $n \to \infty$ [10]. In the following we show that the decision variable $T'(\mathbf{z})$ can be considered to be a Gaussian random variable as claimed in (9), by verifying the applicability of the above condition.

Let $X_k = g_k Re\{h_k^* z_k\} = g_k^2 |h_k|^2 u_k + g_k Re\{h_k^* w_k\}$ after substituting for z_k. Then $T'(\mathbf{z}) = \sum_{k=1}^n X_k$, a sum of independent random variables.

To apply the CLT for $T'(\mathbf{z})$, we prove the sufficient condition under H_0 and a similar derivation follows under H_1. For high local observation SNR region, the mean, variance and the third absolute moment of X_k under H_0 can be shown as,

$$\begin{aligned} \eta_k &= \mathbb{E}\{X_k\} = g_k^2|h_k|^2(2P_{f_k} - 1) \\ \nu_k^2 &= var(X_k) = \frac{1}{2} g_k^2 |h_k|^2 \sigma^2 + 4 g_k^4 |h_k|^4 P_{f_k}(1 - P_{f_k}) \\ &\approx \frac{1}{2} g_k^2 |h_k|^2 \sigma^2 \\ \rho_k^3 &= \frac{4}{\sqrt{2\pi}} \nu_k^3 = \frac{g_k^3 |h_k|^3}{\sqrt{\pi}} \end{aligned} \tag{23}$$

respectively. We assume that the each node is operated with a finite power such that $g_{min}^2 < g_k^2 < g_{max}^2$ for $k = 1, \cdots, n$. Then the two sums K^2 and L^3 can be bounded as,

$$\begin{aligned} K^2 &= \sum_{k=1}^n \frac{1}{2} g_k^2 |h_k|^2 > \frac{1}{2}\sigma^2 g_{min}^2 \sum_{k=1}^n |h_k|^2 \\ L^3 &= \sum_{k=1}^n \frac{g_k^3 |h_k|^3}{\sqrt{\pi}} < \frac{g_{max}^2}{\sqrt{\pi}} \sum_{k=1}^n |h_k|^3 \end{aligned} \tag{24}$$

Note that we assume that the channels undergo Rayleigh fading, so that $|h_k|$'s are realizations of Rayleigh random variables. Then we may approximate the two sums such that $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^n |h_k|^2 \approx \mathbb{E}\{|h_k|^2\}$ and $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^n |h_k|^3 \approx \mathbb{E}\{|h_k|^3\}$. Then

$$\begin{aligned} \lim_{n\to\infty} \frac{L}{K} &< \frac{\left(\frac{g_{max}^2}{\sqrt{\pi}} \sum_{k=1}^n |h_k|^3\right)^{1/3}}{\left(\frac{1}{2}\sigma^2 g_{min}^2 \sum_{k=1}^n |h_k|^2\right)^{1/2}} \\ &\approx \frac{\left(n \frac{g_{max}^2 \mathbb{E}\{|h_k|^3\}}{\sqrt{\pi}}\right)^{1/3}}{\left(\frac{1}{2} n \sigma^2 g_{min}^2 \mathbb{E}\{|h_k|^2\}\right)^{1/2}} \approx 0 \end{aligned} \tag{25}$$

which completes the proof.

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