Distributed Power Control in CDMA Cellular System

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Abstract

In wireless cellular communication, it is essential to find effective means of power control of signals received from randomly dispersed users within one cell. Effective power control will heavily impact the system capacity. Distributed power control (DPC) is a natural choice for such purpose, because unlike centralized power control, DPC does not require extensive computational power. Distributed power control should be able to adjust the power levels of each transmitted signal using only local measurements, so that in reasonable time all users will maintain the desired signal to interference ratio. In this paper, we review different approaches for power control focusing on CDMA systems. We also introduce state-space methods and linear quadratic power control (LQPC) to solve the power control problem. A simulation environment was developed to compare LQPC with earlier approaches. The results show that LQPC is more effective and is capable of computing the desired transmission power of each mobile station in fewer iterations, as well as being able to accommodate more users in the system.

Keywords

Centralized power control, distributed power control, and linear quadratic control.
I. INTRODUCTION

In any multiple access system, the need for power control is evident. In general, and due to the path loss experienced by electromagnetic waves as they propagate between sources and receivers, a close-in transmitter-receiver pair will have more power transfer than a pair which are further apart. This becomes a problem as many transmitters (as is the case in multiple access systems) are trying to communicate with the same receiver because the closest transmitter will overcome all others. In frequency division multiple access (FDMA) and time division multiple access (TDMA), this problem is dealt with using various design approaches including power control described in this paper. The effects of multiple access however become especially acute in code division multiple access (CDMA).

The CDMA scheme was originally motivated in commercial applications by the need for more system capacity than what the previous schemes, i.e. (TDMA) or (FDMA) could offer. But, this advantage can be hindered in the case of CDMA by the increased interference caused by other users. Since all signals in a CDMA system are sharing the same bandwidth, it is critical to use power control to maintain acceptable signal to interference ratio (SIR) for all users, hence maximizing the system capacity [1].

Another critical problem with CDMA is the near-far problem. This problem also occurs due to the lack of power control: If all mobiles were to transmit at a fixed power the mobile closest to the base station will overpower all others. Yet, another reason for power control is the battery life time: If the mobile station is always transmitting at higher power than that needed to maintain an acceptable Signal to Interference Ratio (SIR), the battery will have a short life time. With power control, each mobile station may transmit using the minimum power needed for maintaining the required signal to interference ratio.
As is well known, the mobile channel is best modeled statistically leading in general to a Rayleigh or Ricean channel [2]. The channel large-scale models are however based on the assumption that electromagnetic waves will experience a path loss inversely proportional to the distance traveled raised to some power. An accurate model of the wireless channel and its state is usually unobtainable. Also, any power control algorithms developed should be able to adjust the power levels of each mobile using local measurements only, so that in reasonable time all users will maintain the desired signal to interference ratio. In this paper, we will review the idea of centralized power control but concentrate on the general class of distributed control algorithms as they seem to be more realistic when the number of mobiles grow. Also, only the uplink (mobile to base) control will be reviewed, but all results may be applied to the downlink (base to mobile) case.

Some of the early work in power control was provided by [3]. In [4], [5], [6] centralized power control was studied, and due to the complexity of the system, centralized power control was suggested to be used only for providing theoretical limits. When all users could be accommodated with acceptable signal to interference ratio, [7] suggested a distributed power control algorithm that will converge and computes the required transmission power of each mobile station. In [8] a second-order constrained power control (CSOPC) algorithm was presented. This approach uses the current and past power values to determine the necessary transmission power of each mobile. CSOPC was compared with the algorithm presented in [7] and was shown to converge at a faster rate. Convergence analysis of distributed power control algorithms is investigated in [9]. In [10] a framework for uplink power control in cellular radio systems was presented. Our review to solving the power control problem will be within such framework. Throughout this paper it is assumed that
all users could be accommodated and therefore removal algorithms (to determine which users should be disconnected in order to maintain the required signal to noise ratio), will not be discussed here.

This paper is organized as follows. Background material on the power control problem is given in Section II. Section III reviews the power control algorithm of [11]. Section IV formulates the power control problem using the link balance approach. Section V reviews the CSOPC approach. Section VI presents our new approach to power control, followed in Section VII by a description of the simulation environment and the simulation results. Our conclusions are given in Section VIII.

II. BACKGROUND

As mentioned earlier, power control is a very critical aspect of CDMA systems. Without power control in such systems, the quality of the transmitted signal will deteriorate, and various problems such as near-far will occur. Power control may be divided into two areas: open-loop and closed-loop power control. In open-loop control, it is assumed that the channel between the mobile station and the base station is completely symmetric. In this ideal case, measuring the power level received by the base station would determine the transmitting power of the mobile station, and thus be able to adjust it at will. This situation however is not realistic since the frequencies for the forward and reverse links are usually different [11], and the wireless channel is constantly varying [2]. To account for the fact that in practice the channel is not symmetric, closed-loop control should be used. Closed-loop control however is costly, if not implemented in a distributed fashion. The focus in this paper will thus be on closed-loop control that can quickly determine the
transmission power for signals that are randomly dispersed.

A. Terms and Definitions

In this section some of the notation used in this paper will be introduced.

*Signal to interference ratio (SIR):* SIR is a measure of the quality of the received signal and will be used to determine the control action that needs to be taken. The SIR, represented as $\gamma$, is defined as \[12\]

$$\gamma = \frac{E_b}{I_0} \frac{R_b}{B_c}$$

where $E_b$ is energy per bit of the received signal in Watts, $I_0$ is interference power in Watts per Hertz, $R_b$ is the bit rate in bits per second, and $B_c$ is the radio channel bandwidth in Hertz.

*Outage Probability:* The probability of failing to achieve adequate reception of the signal due to co-channel interference. It is defined as the ratio of the number of disconnected or handed-over users to that of the total number of users in the system.

*Removal Algorithm:* A removal algorithm is a strategy for removing the minimal required number of users possible in order to minimize the outage probability. Removal algorithms are necessary when it becomes impossible to accommodate all the current users in the system. Removal algorithms are however beyond the scope of this paper and will not be discussed further.
III. Bang-Bang Power Control

The goal of power control is to maintain the desired SIR in an environment of varying propagation loss. Under the control scenario of this section, it is assumed that the changes in the propagation loss are slow enough for the control mechanism can to track them. The control law is also designed to deal with the inherent delays in the system. We then present such a control law as described in [11].

For any particular mobile, let $T(j)$ be the transmitted energy in dB of the $j$th power control measurement period, and $L(j)$, in dB, be the propagation loss during the same period. Then, the received energy over this measurement period is given by

$$E(j) = T(j) - L(j) \quad \text{dB}$$

Due to the propagation time delay, where in this case a delay of one symbol period is assumed, the transmitted power for the $(j+1)$th interval is given by

$$T(j + 1) = T(j) + \Delta \cdot C[(E(j - 1))]$$

where $\Delta$ is a fixed increment by which the transmission power is increased or decreased, and where

$$C(E) = \begin{cases} 
-1 & \text{with probability } P_d^l(E_s/I_0) \\
+1 & \text{with probability } 1 - P_d^l(E_s/I_0)
\end{cases}$$

and $P_d^l$ is the probability that the power will be reduced taking into account the probability of command error, and $E_s$ is the symbol energy and $I_0$ is the interference energy. Equation 3 is a form of bang-bang control [13]. Combining Equation (2) and (4), the following closed-loop equation is obtained

$$E(j + 1) = E(j) + \Delta \cdot C[E(j - 1)] - [L(j + 1) - L(j)] \quad \text{dB}$$
Few things are to be noted about this type of controller. First, the control command can only be increased or decreased by a fixed increment. Second, Equation 5 is a nonlinear difference equation and is not easily solvable without using further assumptions [11], [14].

IV. LINK BALANCE PROBLEM

In this section, the power control is determined based on the link balance problem. In this paper it is assumed that the transmitted signal experiences link gain as \( d^{-4} \), where \( d \) is the distance (in meters) between the mobile station and the base station. Other propagation models [2] could be incorporated just as easily, but they are not discussed here. Figure 1 shows a simplified diagram of the communication link. A mobile \( i \) uses a base station \( A \) which is closest to it for communication purposes. The communication gain between base station \( A \) and mobile station \( i \) is denoted by \( G_{Ai} \). If the transmission power of mobile \( i \) is \( p_i \), the signal to interference ratio for mobile \( i \), represented by \( \gamma_i \) as given by:

\[
\gamma_i = \frac{p_i}{\sum_{j \neq i} Q p_j w_{ij}}
\]

where \( w_{ij} \) is defined as

\[
w_{ij} = \begin{cases} 
\frac{G_{kj}}{G_{ki}} & i \neq j, \\
0 & i = j.
\end{cases}
\]

where \( Q \) is the total number of mobiles in the system, and the transmission power is subject to the following constraint:

\[
0 \leq p_i \leq P_i
\]
where $\bar{p}_i$ is the maximum transmission power of mobile $i$.

Using Equation 6 the Link Balance Problem (LBP) is formulated as follows: Find the power level $p_i$ such that

$$\gamma_i = \frac{p_i}{\sum_{j \neq i} p_j w_{ij}} \geq \gamma^*$$

(9)

where $\gamma^*$ is the desired threshold below which the signal quality is unacceptable. Note that this model does not yet include the noise introduced by the channel. In the next subsections, different approaches to solving LBP will be discussed.

A. Centralized Power Control

Centralized power control assumes that all information about the link gains is available, then, in one step, the maximum achievable SIR level is computed. In fact, let

$$W = \begin{pmatrix}
0 & w_{12} & \cdots & w_{1Q} \\
w_{21} & \ddots & \cdots & w_{2Q} \\
\vdots & \ddots & \ddots & \vdots \\
w_{Q1} & \cdots & \cdots & 0
\end{pmatrix}_{Q \times Q}$$

and

$$P = \begin{pmatrix}
p_1 \\
\vdots \\
p_Q
\end{pmatrix}$$

(11)

The LBP has an analytical solution as follows [3]: The largest achievable SIR level $\bar{\gamma}$ is related to the matrix $W$ as $\bar{\gamma} = 1/\lambda^*$ where $\lambda^*$ is the largest real eigenvalue of matrix $W$. The power vector $P^*$ achieving this maximum level is given by the eigenvector corresponding to $\lambda^*$. Thus the power control problem is reduced to a general eigenvalue problem.
The main limitation with such an approach is exactly the fact that it is centralized: To compute the power for a given mobile station $i$, the data of all other mobile stations has to be available. From a practical point of view, as the number of mobile grows, this approach becomes infeasible, or at least computationally costly. Even if it were possible to obtain all the necessary information, there are no guarantees that $\tilde{\gamma} \geq \gamma^*$. If $\tilde{\gamma} < \gamma^*$ a removal algorithm will be needed. With the centralized power control approach the removal algorithm becomes very computationally expensive, which is another reason for the impracticality of this approach.

**B. Distributed Power Control**

As opposed to centralized power control, distributed power control should be able to adjust iteratively the power levels of each transmitted signal using only local measurements, so that in reasonable time all users will achieve and maintain the desired signal to interference ratio.

If we assume that $\gamma^*$ is the desired signal to interference, and that each mobile station $i$ has receiver noise $n_i$, Equation (9) may be rewritten as

$$\gamma_i = \frac{p_i}{\sum_{j \neq i} p_i w_{ij} + \frac{n_i}{G_{ki}}} \geq \gamma^*$$  \hspace{1cm} (12)

where $n_i$ is the $i^{th}$ receiver noise in Watts.

The goal now is to find the transmission power of mobile $i$ such that the following inequality is satisfied.

$$p_i \geq \psi_i(P) = \gamma^* \left( \sum_{j \neq i} p_i w_{ij} + \frac{n_i}{G_{ki}} \right)$$  \hspace{1cm} (13)

where $\psi(P)$ is known as *interference function* and has the following properties [10]:
• **Positivity:** $\psi_i(P) > 0$.

• **Monotonicity:** if $P > P'$, then $\psi_i(P) > \psi_i(P')$.

• **Scalability:** For all $\alpha > 1$, $\alpha \psi_i(P) > \psi_i(\alpha P)$.

Since it is desired to use the minimum transmission power possible, Inequality (13) becomes an equality and an iterative method for power control could be written as [10]

$$p_i(n + 1) = \psi_i(P(n))$$

(14)

Given the power constraint in Inequality (8), constraint iterative power control algorithm in Equation (14) becomes

$$p_i(n + 1) = \min\{\overline{p}_i, \psi_i(P(n))\} = \min\{\overline{p}_i, \frac{\gamma^*}{\gamma_i(n)} p_i(n)\}$$

(15)

where $\gamma_i(n)$ is the signal to interference ratio of mobile $i$ at iteration $n$. It is important to note that unlike centralized power control, only the total interference is needed to compute the power levels. Convergence of the iterative algorithms given by Equations (14) and (15) is studied in [10]. Different approaches using iterative methods are being designed trying to accomplish a faster convergence rate. In the following two sections two different approaches will be presented. The first is the constrained second order power control (CSOPC)[8], and the second is our linear quadratic power control (LQPC).

**V. Constrained Second Order Power Control**

In this section, the LBP of equation (12) will be converted into a set of linear equations. Then the results presented in [8] will be reviewed.
Equation (12) could be written as a set of linear equations as follows

\[ \mathbf{X}\mathbf{P} = \Xi \]  

(16)

where \( \mathbf{P} \) is defined in Equation (11), and

\[ \mathbf{X} = \mathbf{I} - \mathbf{A} \]  

(17)

\[ \mathbf{A} = \{a_{ij}\}, \quad a_{ij} = \gamma^*w_{ij} \]  

(18)

\[ \Xi = \{\xi_i\}, \quad \xi_i = \gamma^* \frac{N_i}{G_{ik}} \]  

(19)

Thus Equation (12) has been converted to a set of linear equations, that could be iteratively solved for \( \mathbf{P} \) [8].

CSOPC is developed by applying the successive overrelaxation method (SOR) [15] to Equation (16). The CSPOC results in [8] were compared with the distributed constraint power control (DCPC) in [7]. CSOPC was proven to be more effective; consequently, later in this paper the CSOPC algorithm will be used as the comparison benchmark. In this section a brief overview of the CSOPC approach will be given. Through some manipulations the following iterative algorithm was obtained [8].

\[ p_i(n + 1) = \min \left\{ \overline{p}_i, \max \left\{ 0, a(n) \frac{\gamma^*}{\gamma_i(n)}p_i(n) + (1 - a(n))p_i(n - 1) \right\} \right\} \]  

(20)

where as described earlier, \( \overline{p}_i \) is the maximum allowable power for mobile \( i \), \( \gamma_i(n) \) is the signal to interference ratio of mobile \( i \) at iteration \( n \), \( p_i(0) \) is chosen randomly between 0 and \( \overline{p}_i \), and \( a(n) \) is a decreasing sequence such that \( \lim_{n \to \infty} a(n) = 1 \). As an example, the following \( a(n) \) sequence was used in [8]:

\[ a(n) = 1 + \frac{1}{1.5^n}, \quad n = 1, 2, \ldots, l \]  

(21)
where \( l \) is the total number of iterations. Equation (20) determines the necessary power using the current and the past power values, which accounts for the terminology of “second-order”. Note that if \( a(n) = 1 \), Equation (20) reduces to Equation (15) and that the \( \min \) and \( \max \) operators are used to guarantee that the power will be within the allowable range based on Equation (8).

VI. LINEAR QUADRATIC CONTROL

Borrowing on results from modern control theory, we present a state-space formulation and linear quadratic control [13] as a viable design methodology for power control. Our approach is to view each mobile to base station connection as a separate subsystem as described by

\[
s_i(n + 1) = \frac{p_i(n) + u_i(n)}{I_i(n)} = s_i(n) + v_i(n)
\]

(22)

where \( I_i(n) = \sum_{j \neq i} p_i w_{ij} + \frac{n_i}{\sigma^2} \), \( v_i(n) = u_i(n)/I_i(n) \) and by definition \( s_i(n) = p_i(n)/I_i(n) \). The input \( u_i(n) \) to each subsystem should only depend on the total interference produced by the other users. The goal is to find the right control command that will make each \( s_i \) track a desired signal to interference ratio \( \gamma^* \). For simplicity we will assume that \( \gamma^* \) is the same for all mobile stations, although other cases may be easily accommodated.

To accomplish such a task, and to eliminate any steady-state errors [13], a new state is added to the system. This is that of the integrator of the error \( e_i(n) = s_i(n) - \gamma^* \) [16], which in the discrete-time case, is nothing more than a summation of the previous values. Therefore, the new state is \( \zeta_i(n) \) where

\[
\zeta_i(n + 1) = \zeta_i(n) + e_i(n) = \zeta_i(n) + s_i(n) - \gamma^*
\]

(23)
Let us define \( x_i(n) \) as

\[
x_i(n) = \begin{pmatrix} e_i(n) \\ s_i(n) \end{pmatrix}
\]

(24)

Using the above notation, each subsystem can now be expressed as a second-order linear state-space system by

\[
x_i(n + 1) = \begin{pmatrix} e_i(n + 1) \\ s_i(n + 1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x_i(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_i(n)
\]

(25)

\[
y_i(n) = (0 \ 1) x_i(n)
\]

(26)

we then choose the feedback controller

\[
v_i(n) = -[k_\zeta \ \ k_s] x_i(n)
\]

(27)

If we choose the appropriate feedback gains \( k_\zeta \) and \( k_s \), then the closed-loop system corresponding to Equations (25) will be asymptotically stable. Therefore, the steady-state state \( s_i(n) \) will go to \( \gamma^* \).

In order to use LQ control theory, we choose the following quadratic performance measure [13]

\[
J = \sum_{n=0}^{\infty} \{ x'(n)Qx(n) + v'(n)Rv(n) \}
\]

(28)

where \( ()' \) denotes transpose, the term \( x'(n)Qx(n) \) is a weight on the control accuracy, \( v'(n)Rv(n) \) is a measure of control effort, and they are chosen to be

\[
Q = \begin{pmatrix} 200 & 0 \\ 0 & 0 \end{pmatrix}, \quad R = 0.1
\]

(29)
the gain matrix \( K = (k_x \quad k_s) \) is found. \( Q \) and \( R \) are chosen in such a way that the inequality (8) and the properties of the standard interference function are satisfied. Such performance index is a standard one in the design of controllers for linear systems, but has never been used in the power control arena. Once the gain \( K \) is found, the new power command can be computed as follows

\[
p_i(n + 1) = \min \{ \overline{p}_i, s_i(n + 1) \cdot I_i(n) \}
\]

(30)

VII. Results

A. Simulation Environment

A simulation environment is essential in order to be able to test and compare results. This simulation environment has the following components:

- Generation of a hexagonal cell and its 6 neighboring cells.
- Random choice the number of mobiles in each cell.
- Random allocation the mobiles in each cell.
- The user has control over the minimum and maximum number of mobiles in each cell.
- Generation of the path loss based on available models.

A.1 Parameters

Most of the parameters used for the simulation are taken from the IS-95 system [17] and are as follows:

- Desired energy per bit, \( E_b \), to interference power per hertz, \( I_0 \), is 7dB .
- Bit rate, \( R_b \), is 9600 bits per second.
• Radio channel bandwidth, $B_c$, is $1.2288\text{MHz}$.

• Receiver Noise, $n_i = n = 10^{-12}$, $1 \leq i \leq Q$.

\textbf{B. Simulation Results}

The system was simulated with two different maximum transmission power, 1 and 5 watts. The outage probability was used as a measure for comparing the \textit{constrained second order power control} (CSOPC) and the new approach, LQPC, developed in this paper. Outage probability versus the number of iterations and versus the number of mobile stations in each cell was computed and plotted. Since in reality, the users are randomly dispersed, each point on the curves is obtained after simulating the system 100 times, and averaging out the results.

As seen from the simulation results in Figure (3) for $P_i = 1$ watts, the difference between the two approaches is not large. Nevertheless, as shown in Figure 3, for 18 mobile stations per cell, the new approach reaches zero outage probability in 3 iterations versus 5 for CSOPC. For $P_i = 5$ watts, the difference is more noticeable, see Figure (6). With higher maximum level for the transmission power, the system can accommodate more mobile stations. By comparison, the new approach is more effective in handling a larger number of mobile stations in the system. In Figure 5, it can be seen that the outage probability for 26 mobile stations per cell goes to zero in 7 iterations. In 7 iterations the outage probability using CSOPC is approximately 19 percent. This does not mean that CSOPC can not accommodate 26 mobiles, but that rather that more iterations may be needed for CSOPC to converge to the right solution. As mentioned earlier there were no removal algorithms incorporated in either approaches. It is also important to note that the new
approach can handle 26 mobile stations with zero outage probability as opposed to 21 using CSOPC, as shown Figure (6).

VIII. CONCLUSIONS

In this paper we have presented an overview of various power control algorithms for CDMA systems. In addition, a new approach to controlling the transmission power of a mobile station was introduced. A simulation environment was designed to test existing as well as the new techniques developed. A comparison between the new approach and the constrained second order power control introduced in [8] was made. As seen from the simulation results the new approach is faster and can accommodate more mobiles in the system. The advantage of LQPC, besides the fact that it is more effective, is that possibility of adding measurement errors to the model, and then more advanced control theory concepts may be brought to bear to solve the most general power control problem.

There is still however a need for an approach to determine the best $Q$ and $R$ for use in the power control solution. Yet another approach is to incorporate the saturation function to represent the constraint on the transmission power directly and treat the problem as a non-linear control problem.

REFERENCES


Fig. 1. Gain of the communication link
Fig. 2. 7-cell configuration
Fig. 3. Outage probability versus iterations for $\bar{p}_i = 1$ with 18 mobile stations per cell
Fig. 4. Outage probability versus number of mobile stations per cell for $\bar{y}_i = 1$
Fig. 5. Outage probability versus iterations for $\overline{\mu}_i = 5$ with 26 mobile stations per cell

Fig. 6. Outage probability versus number of mobile stations per cell for $\overline{\nu}_i = 5$