Simultaneous Passification and Stabilization of a Class of Nonlinear Minimum Phase systems Via Static Output Feedback

Ali Jadbabaie, Chaouki T. Abdallah
NASA Center for Autonomous Control Engineering(ACE)
Electrical and Computer Engineering Department
University of New Mexico
Albuquerque, NM 87131, USA
{alij, chaouki}@eece.unm.edu

Abstract

In this paper, we extend some recent results on the stabilization of output feedback linearizable nonlinear systems. We show that these systems not only cab be stabilized with a much simpler controller than previously thought, but they can also be rendered passive using the same output feedback controller. Finally, we extend these results to the problem of simultaneous stabilization of a family of systems and present simulations to demonstrate the effectiveness of this method.

Key words: minimum phase, relative degree, output feedback, passivity, simultaneous stabilization.

1. Introduction

For more than three decades, feedback stabilization of nonlinear systems has occupied a major role in the nonlinear system literature. The importance of this issue is due to the fact that feedback stabilization is a preliminary step in achieving additional control objectives, e.g., asymptotic tracking, robustness, etc. It may also bedesirable to render the system passive via feedback. Motivated by the dissipation of energy across resistors in an electric circuit, passivity has been widely used in order to analyze the stability of a general class of interconnected nonlinear systems(see [3, 7]). Thanks to these developments we now know a complete answer to the question: when is a finite dimensional nonlinear system passive, and if is not, how can we render it passive? This question has been answered in an elegant manner in the nonlinear control literature.(see [5, 2] and references there in).

Despite the fact that now we have a fairly complete theory of stabilization and passivity of nonlinear systems using state feedback, there are fewer results considering the same problem but using smooth static output feedback instead. Results in [2] indicate that a passive nonlinear system with positive storage function can be rendered strictly passive, i.e, asymptotically stable using smooth output feedback. The authors in [6] have come up with conditions that make a nonlinear system output-feedback linearizable, and then proposed a control action that guarantees asymptotic stability.

In this paper, we extend the results given in [6] for the output feedback stabilization of minimum phase nonlinear systems of relative-degree one, and prove that they satisfy the conditions of the nonlinear version of Meyer-Kalman-Yakubovich lemma. Our output feedback controller is much simpler, and can be used for simultaneous stabilization and passification of a family of these systems.

The structure of this paper is as follows:
Some known results regarding the necessary and sufficient conditions on output feedback linearization of nonlinear minimum phase systems with relative degree one are presented in section 2. We show that these systems can be rendered passive via output feedback, i.e., that they satisfy the nonlinear version of the celebrated Meyer-Kalman-Yakubovich lemma[7, 2]. We also prove that a simple static gain can be used to stabilize and passify, instead of the controller presented in [6]. In section 3, we use these results to render a family of nonlinear minimum phase systems with relative degree one passive using a single output feedback controller. In section

4, we present a simulation to indicate the effectiveness of this approach. Our conclusions are given in section 5.

2. Output feedback Linearization

In this section, we address the problem of designing output feedback controllers to achieve asymptotic stabilization for of the nonlinear SISO system described by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, u is the control input, and $h(x)$ is a smooth output function with $h(0) = 0$, and f, g are smooth vector fields on $\mathbb{R}^n$. The problem is of practical interest since in many situations the states are not available for measurement. We know that a minimum phase relative degree one LTI system can be rendered strictly passive using pure output feedback [1]. It is logical to ask if the same result holds for nonlinear systems having the same properties. We will show that this is indeed the case, and the same result holds for a class of minimum-phase nonlinear systems with relative degree one which are globally static output feedback linearizable. These systems satisfy the conditions in theorem (6.3.1) of [6], and can be written in the following form:

$$\begin{aligned} \dot{\eta} &= \Gamma\eta + \beta y + \gamma(y)y \\ \dot{y} &= e_1\eta + dy + y\phi(y) + \sigma(y)u \end{aligned} \tag{2}$$

where $\dot{\eta} = \Gamma\eta$ is the zero dynamics, β is a constant vector, d is a scalar $\phi(y)$ is a smooth function of y with $\phi(0) = 0$, $e_1 \in \mathbb{R}^{n-1}$ is the first unit row vector, and $\sigma(y)$ is nonzero, $\eta \in \mathbb{R}^{n-1\times n-1}$, β and $\gamma(y) \in \mathbb{R}^{n-1}$, y, u, d, $\sigma(y)$, and $\phi(y) \in \mathbb{R}$.

Remark 1 *Output feedback linearizable systems that satisfy the conditions of theorem (6.3.1) in [6] are minimum-phase, i.e., they have stable zero dynamics.*

The control input as given in [6] is:

$$\begin{aligned} u &= -\sigma^{-1}(y)y[\phi(y) + d + k \\ &\quad +\gamma^T(y)P^2\gamma(y) + \beta^T P^2\beta] \end{aligned} \tag{3}$$

where k is a positive real constant and P is the positive definite solution of the following Lyapunov equation(Γ is a Hurwitz matrix since the system is minimum phase).

$$\Gamma^T P + P\Gamma = -3I \tag{4}$$

Considering the Lyapunov function candidate

$$V(\eta, y) = \eta^T P\eta + \frac{1}{2}y^2 \tag{5}$$

it was shown in [6] that the time derivative of this Lyapunov function along the trajectory of the system is negative definite. If we have a reference input w, then the control input becomes

$$v = u + \sigma^{-1}(y)w \tag{6}$$

which leads to the closed-loop system given by

$$\begin{bmatrix} \dot{\eta} \\ \dot{y} \end{bmatrix} = \bar{f}(\eta, y) + \bar{g}(\eta, y)w \tag{7}$$

where

$$\begin{aligned} \bar{g}(\eta, y) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \bar{f}(\eta, y) &= \begin{bmatrix} \Gamma\eta + \beta y + \gamma(y)y \\ e_1 y + ky + \gamma^T(y)P^2\gamma(y)y + \beta^T P^2\beta y \end{bmatrix} \end{aligned} \tag{8}$$

A straightforward calculation shows that

$$\begin{aligned} L_{\bar{f}}V(\eta, y) &< 0 \\ L_{\bar{g}}V(\eta, y) &= y \end{aligned} \tag{9}$$

which are the required conditions for the nonlinear version of the MKY lemma. Therefore, we have the following theorem:

Theorem 1 *The system (2) can be rendered passive via static output feedback using the u given in 3.*

Remark 2 *This result guarantees more than just stability and is a simple generalization of results in [2] which guarantee passivity using state feedback.*

Remark 3 *If we apply this result to the case of MIMO LTI systems, we obtain the same results as in [1]. In other words the results in this paper are a generalization of output feedback SPR design in [1] to the case of output feedback linearizable nonlinear systems.*

Although the controller in (3) guarantees the negative definiteness of the Lyapunov function candidate in (5), it is not easy to implement. In the following theorem we propose a simpler controller that will also guarantee passivity of the closed-loop system.

Theorem 2 *The output feedback linearizable system (2) can be rendered passive using the following output feedback controller if $\gamma(y)$ is a bounded function.*

$$u = -\sigma^{-1}(y)y[\phi(y) + d + k + r] \tag{10}$$

where r is the reference input.

Proof: we use the same Lyapunov function candidate as in (5), written in its matrix form

$$V(\eta, y) = \begin{bmatrix} \eta^T & y^T \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \eta \\ y \end{bmatrix} \tag{11}$$

using the control input in (10), we can rewrite the closed-loop system as

$$\begin{aligned} \begin{bmatrix} \dot{\eta} \\ \dot{y} \end{bmatrix} &= A(\eta, y) \begin{bmatrix} \eta \\ y \end{bmatrix} + \bar{g}(\eta, y)r \\ &= \begin{bmatrix} \Gamma & \beta + \gamma(y) \\ e_1 & -k \end{bmatrix} \begin{bmatrix} \eta \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \\ y &= \begin{bmatrix} 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \eta \\ y \end{bmatrix} \end{aligned} \tag{12}$$

To guarantee the stability of the unforced system, the time derivative of Lyapunov function along the system trajectory has to be negative definite, i.e.,

$$\begin{aligned} \dot{V} &= \begin{bmatrix} \eta^T & y^T \end{bmatrix} \left(\begin{bmatrix} \Gamma & \beta + \gamma(y) \\ e_1 & -k \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Gamma & \beta + \gamma(y) \\ e_1 & -k \end{bmatrix} \right) \begin{bmatrix} \eta \\ y \end{bmatrix} < 0 \end{aligned} \tag{13}$$

for this to be true for all values of η and y, we should have

$$\begin{bmatrix} \Gamma^T P + P\Gamma & e_1^T + P(\beta + \gamma(y)) \\ e_1 + (\beta + \gamma(y))^T P & -2k \end{bmatrix} < 0 \tag{14}$$

using the LMI lemma [4], the above inequality is true if and only if the following two conditions hold:

$$\begin{aligned} 0 &> \Gamma^T P + P\Gamma \\ 0 &> \Gamma^T P + P\Gamma + \frac{1}{2k}[e_1^T + P(\beta + \gamma(y))] \\ &\quad [e_1^T + P(\beta + \gamma(y))]^T \end{aligned} \tag{15}$$

the first condition is already satisfied, since we know that the system is minimum phase. For the second condition to be true, we first find $P > 0$ such that

$$\Gamma^T P + P\Gamma = -\frac{1}{2}I \tag{16}$$

then replacing (16) in (15) and multiplying both sides of the inequality by $2k$ ($k > 0$)

$$-kI + z(y)z(y)^T < 0 \tag{17}$$

where z is defined as

$$z(y) = [e_1^T + P(\beta + \gamma(y))] \tag{18}$$

for (17) to be true we have to make sure that k is large enough so that

$$k > \max_y \{eig[z(y)z(y)^T]\} = \max_y z(y)^T z(y) \tag{19}$$

given that $\gamma(y)$ is bounded. If (19) is satisfied, (13) will be guaranteed and therefore the system in (12) is asymptotically stable. Using the same argument as before the system also satisfies the conditions in (9) and therefore by the nonlinear version of the MKY lemma, it is passive.

3. Passivity and Simultaneous Output Feedback Stabilization

Next, we consider the problem of stabilization and passivity of a family of the nonlinear output feedback linearizable systems. For simplicity we assume that all of the systems are already in the form of equation (2). Consider the following family of plants

$$\begin{aligned} \dot{\eta} &= \Gamma_i \eta + \beta_i y + \gamma_i(y) y \\ \dot{y} &= e_1 \eta + d_i y + y\phi_i(y) + \sigma(y) u \end{aligned} \tag{20}$$

where $i = 1, \ldots, p$ and p is the number of plants, and $\sigma(y)$ is the same for all p systems.

Theorem 3 *(Simultaneous output feedback stabilization): Consider the p plants in (20) where $\gamma_i(y)$ are bounded functions. Then, there exists a single control action u which makes the whole family of systems passive and stable.*

Proof: consider the following control input

$$u = -\sigma^{-1}(y)y[\phi_j(y) + d_j + k]$$

where j is the index of the plant chosen to be fed back to all others. Let V_i be the Lyapunov function candidate for each system as follows

$$V_i(\eta, y) = \begin{bmatrix} \eta^T & y^T \end{bmatrix} \begin{bmatrix} P_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \eta \\ y \end{bmatrix} \tag{21}$$

where each P_i satisfies the following Lyapunov equations:

$$\Gamma_i^T P_i + P_i \Gamma_i = -\frac{1}{2} I \tag{22}$$

the closed-loop systems become

$$\begin{aligned} \dot{\eta} &= \Gamma_i \eta + \beta_i y + \gamma_i(y) y \\ \dot{y} &= e_1 \eta - [(d_i - d_j) + (\phi_i(y) - \phi_j(y)) + k]y + r \end{aligned} \tag{23}$$

If we pick k large enough such that

$$[(d_i - d_j) + (\phi_i(y) - \phi_j(y)) + k] = k' > 0 \tag{24}$$

then using the same argument as in proof of theorem (2), to guarantee that the derivative of (21) is negative definite, we have to satisfy the following conditions

$$\begin{aligned} -kI + z_i(y) z_i(y)^T &< 0 \ \forall i = 1, ..., p \\ z_i(y) &= \{e_1^T + P_i[\beta_i + \gamma_i(y)]\} \end{aligned} \tag{25}$$

which results in

$$k > \max_i \max_y z_i^T(y) z_i(y) \quad \forall i = 1, ..., p \tag{26}$$

so if (24) and (26) are satisfied, using the proof in theorem (2), all of the p plants will be asymptotically stable. Moreover since (9) is satisfied for every subsystem, they are also passive.

4. Simulation Results

To show the effectiveness of our method, we present a simple numerical example. Consider the system (where θ is an unknown parameter)

$$\begin{aligned} \dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= \theta x_1^3 + u \\ y &= x_1 \end{aligned} \tag{27}$$

it can be shown [6] that this system satisfies the conditions in theorem (6.3.1) given in [6],therefore after transforming it to the normal form we have:

$$\begin{aligned} \dot{\eta} &= -\eta - y + \theta y^3 \\ \dot{y} &= \eta + y + u \end{aligned} \tag{28}$$

assuming the following initial conditions:
$\begin{bmatrix} \eta(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} x_2(0) - x_1(0) \\ x_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
the control action

$$u = -2.5y$$

will stabilize the system for all values of θ in the interval [-10,5.5]. When θ is zero, the system turns into a linear one. For $\theta > 5.5$, the condition in (19) is violated and a larger control action is needed, but for any range of θ , there exists an output feedback gain that would stabilize the system.

The initial condition response is plotted in figures (1) and (2) for different values of θ.

5. Conclusion

In this note we extended some recent results in [2, 6] concerning the stabilization and passification of nonlinear minimum-phase systems with relative degree one. The controller that we propose is much simpler than the one in [6]. We have also shown that this controller makes the system passive with a positive definite storage function. Simulation results were presented to illustrate this method.

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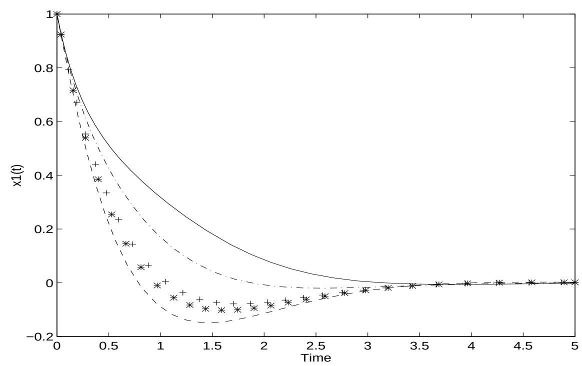

Figure 1: Time response of $x_1(t)$ for different values of θ in the interval $[-10, 5.5]$

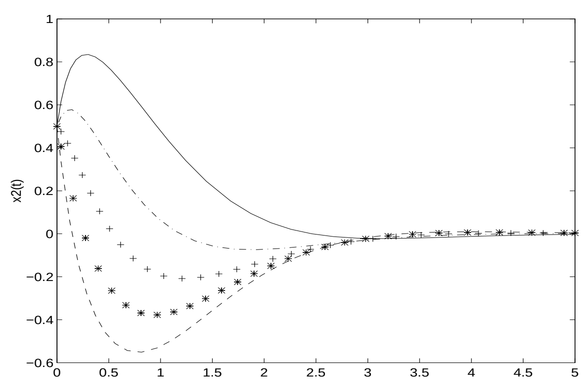

Figure 2: Time response of $x_2(t)$ for different values of θ in the interval $[-10, 5.5]$

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