F.T. of convolution of two signals

$$h(t) = f(t) * g(t)$$

$$H(\omega) = ?$$

$$h(t) = \int f(\tau)\, g(t-\tau)\, d\tau$$

$$H(\omega) = \int \int f(\tau)\, g(t-\tau)\, d\tau \; e^{-j\omega t}\, dt$$

$$= \int f(\tau) \int g(t-\tau)\, e^{-j\omega t}\, dt \;\, d\tau \quad \text{(reverse integrals)}$$

$$q = t - \tau$$
$$t = q + \tau$$
$$dt = dq$$

$$= \int f(\tau) \int g(q)\, e^{-j\omega (q+\tau)}\, dq\, d\tau$$

$$= \int f(\tau) \underbrace{\left[\int g(q)\, e^{-j\omega q}\, dq \right]} e^{-j\omega \tau}\, d\tau$$

$$= \int f(\tau)\, G(\omega)\, e^{-j\omega \tau}\, d\tau$$

$$= G(\omega) \underbrace{\int f(\tau)\, e^{-j\omega \tau}\, d\tau}$$

$$H(\omega) = G(\omega) F(\omega) \;/\!\!/$$

Properties of F.T.

$f(at) \overset{\mathcal{F}}{\Longleftrightarrow}$ $\qquad = \int f(at) e^{-j\omega t}\, dt$

$$q = at \Rightarrow \frac{q}{a} = t$$
$$dq = a\,dt \Rightarrow dt = \frac{dq}{a}$$

$$\frac{1}{a}\int f(q)\, e^{-\frac{j\omega q}{a}}\, dq$$

$$\sigma = \frac{\omega}{a}$$

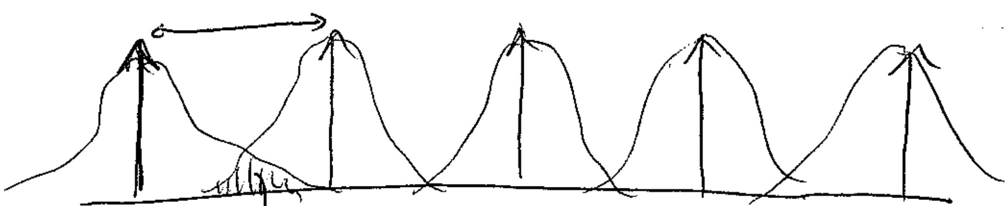
$$\mathcal{F} = \frac{1}{a}F(\sigma) = \frac{1}{a}F\left(\frac{\omega}{a}\right)$$

Aliasing

$g(t) = f(t)\,\text{Ш}(at)$ ← sample a continuous signal

$\Downarrow \mathcal{F}$

$G(\omega) = \frac{1}{a} F(\omega) * \text{Ш}\left(\frac{\omega}{a}\right)$ ← this means replicas of $F(\omega)$ at each spike



aliasing!

if $F(\omega)$ has max frequency f_{max}

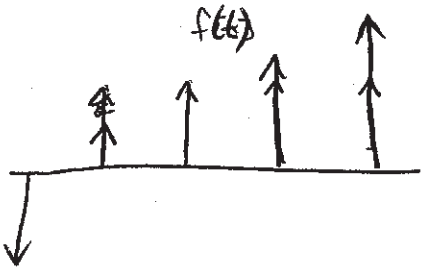
then the sampling rate $> 2 f_{max}$ to avoid aliasing!

Down sampling – Reduce the number of samples in the image (decrease its <u>resolution</u>)

Up sampling – Increase the number of samples in the image (increase the resolution)

Image Resampling

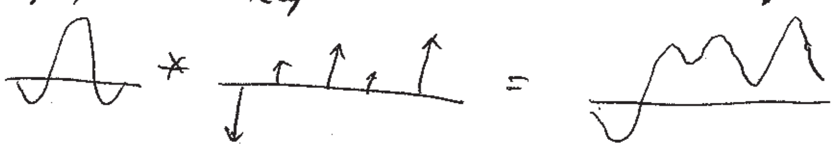
① $f(t)$



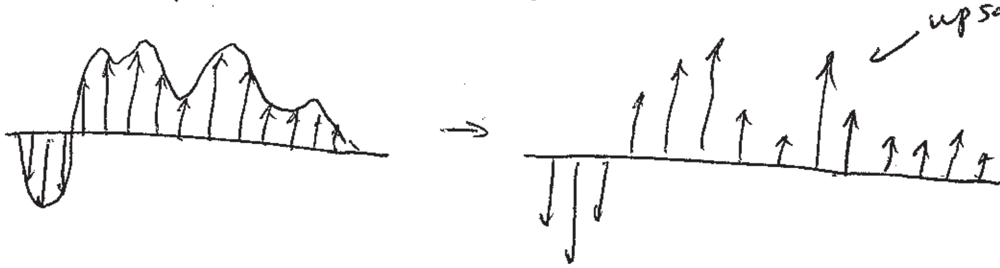
original signal

② Apply a reconstruction filter

$g(t)$ $f(t)$

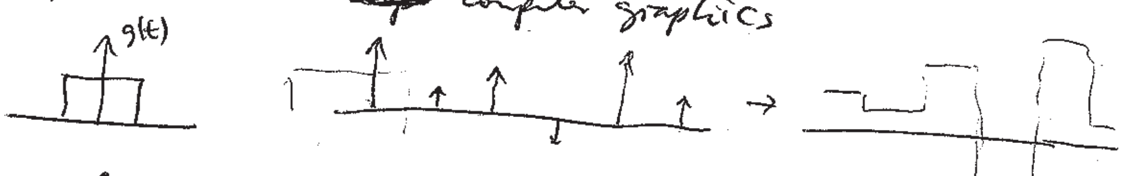


③ Resample the continuous signal

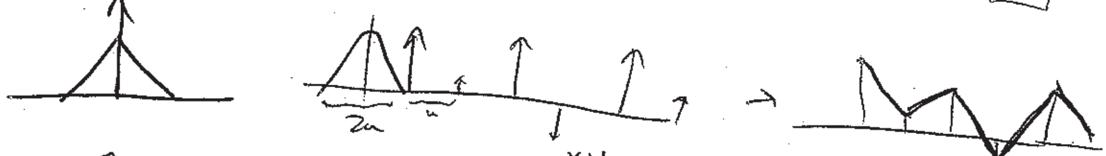


. common reconstruction filters used in computer graphics

Real time { Nearest neighbor

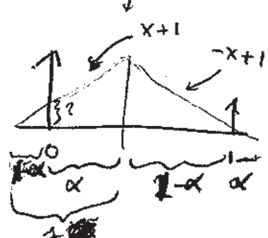


Triangle (Linear)



so linear blend: A B

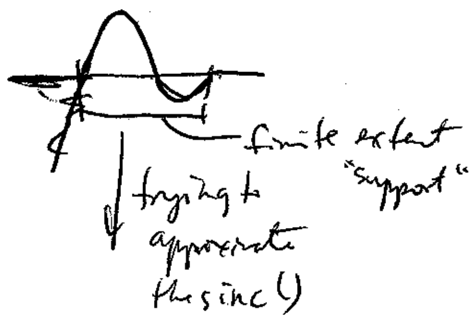
$t = (1-\alpha)A + \alpha B$ is a line

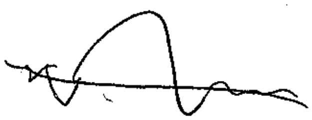


$\frac{1-\alpha}{q_1} = \frac{1}{1} \Rightarrow 1-\alpha = q_1$

$\frac{\alpha}{q_2} = 1 \Rightarrow \alpha = q_2$

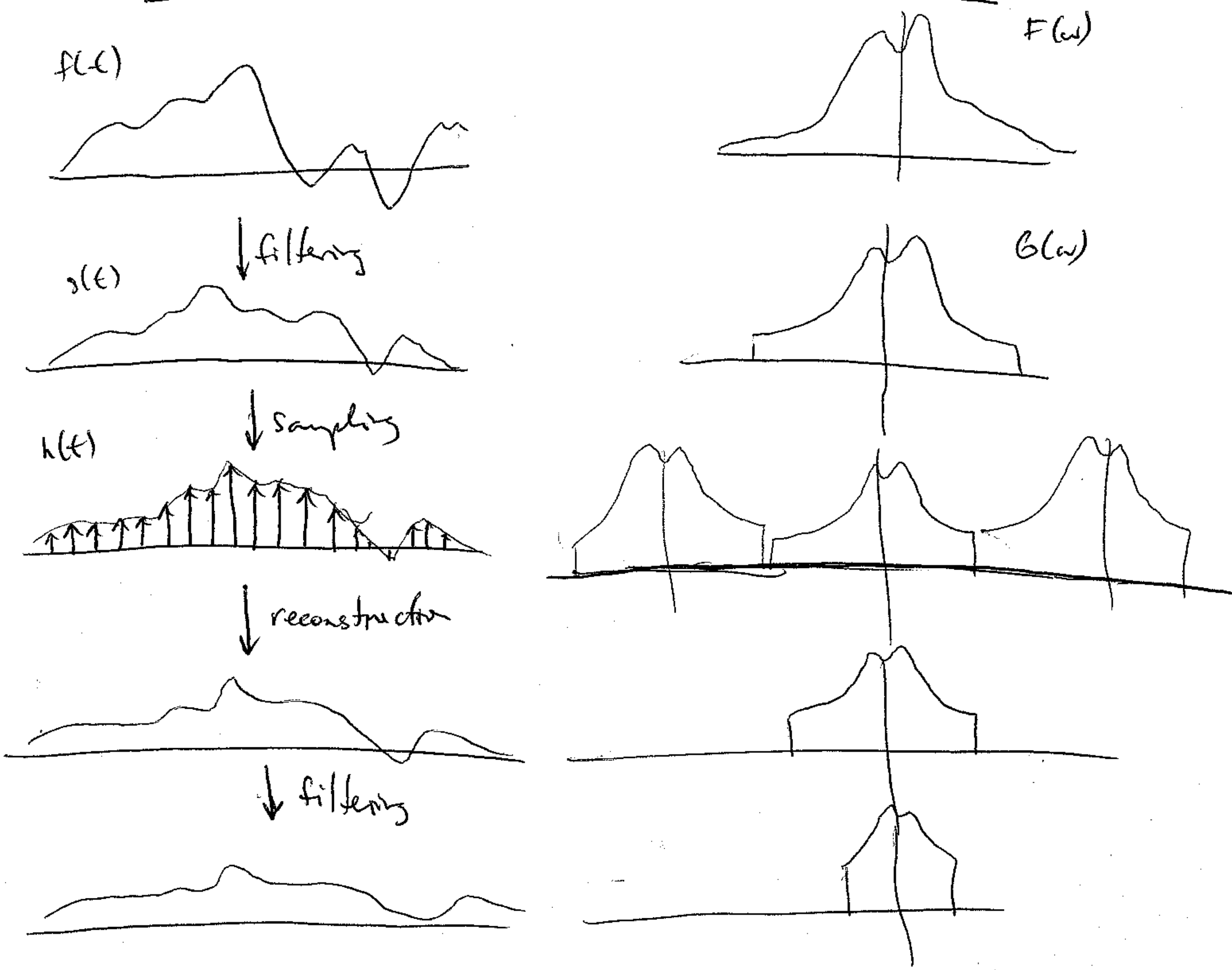
Bicubic



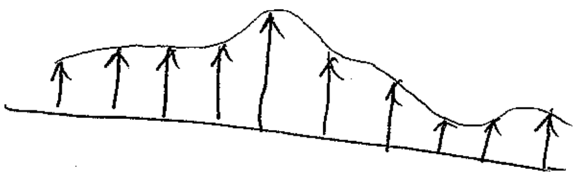


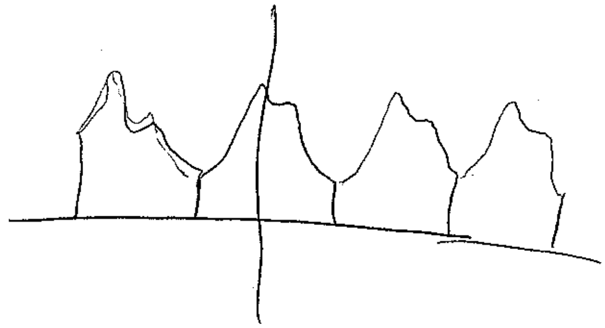
Sampling and Resampling

SPATIAL DOMAIN $\xrightarrow{\mathcal{F}}$ FREQUENCY DOMAIN



↓ resampling





In Rendering, we don't always have an analytical form for the expression $f(t)$.

How do you apply a filter so that you can sample?

<u>Super sampling</u>! 1. Take a lot more samples
2. Average them together to imitate a discrete filter.