Parametric curves (surfaces)

The boundary of the curve is defined by parametric functions.

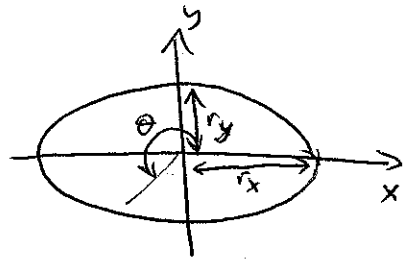
line between two points → $P(t) = P_1 + (P_2 - P_1)t \Rightarrow$ $x = P_{1x} * (1-t) + P_{2x} t$, $y = P_{1y} * (1-t) + P_{2y} t$

(t ↑ parametric variable)

Ex: Ellipse

$x = r_x \cos\theta$

$y = r_y \sin\theta$



Extension

$x = f_x(u)$, $y = f_y(u)$, $z = f_z(u)$ or $x = f_x(u,v)$, $y = f_y(u,v)$, $z = f_z(u,v)$

So to define arbitrary curves:

$$x(u) = \sum_{i=0}^{n} B_i(u) * V_{ix}$$

$$y(u) = \sum_{i=0}^{n} B_i(u) * V_{iy}$$

$B_i(u)$ are functions that will be used to "blend" the points together.

Ex: let $B_0(u) = 1-u$

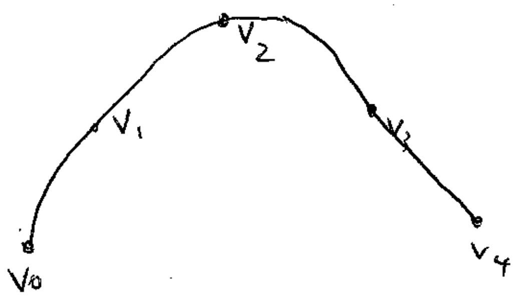
$B_1(u) = u$

$B_2(u) = 0 \ldots$

$x(u) = (1-u)V_{0x} + (u)V_{1x}$

$y(u) = (1-u)V_{0y} + (u)V_{1y}$

} same as parametric line!



Use polynomials to blend:

$$B_i(u) = \sum_{j=0}^{m} a_{ij}\, u^j$$

} polynomials of order m
← · Easy to compute
· Infinitely continuous

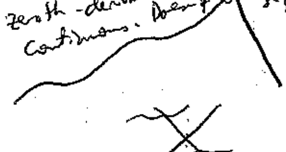
Splines – curves defined piecewise by polynomials

Cubic splines – Defined by degree 3 polynomials.

Cubic B-spline (B = basis):
1. Four control vertices control each pt. (local control)
2. Cubic polynomials
3. C^2 continuity

Continuity

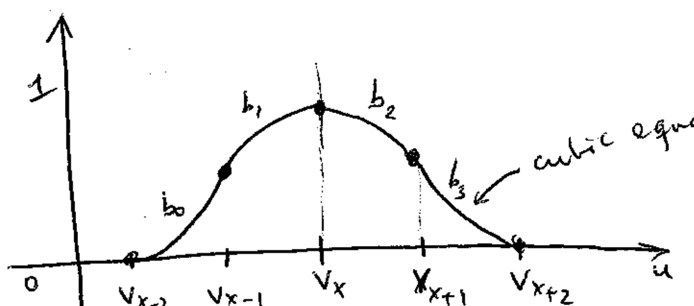
C_0 zeroth-derivative (value) is continuous. Doesn't change suddenly



C_1 first derivative is continuous (derivative doesn't change suddenly)

C_2 2nd derivative is continuous

⋮



4 poly equations ← 16 variables

$$b_0(u) = a_0 u^3 + b_0 u^2 + c_0 u + d_0$$

$$b_1(u) = a_1 u^3 + b_1 u^2 + c_1 u + d_1$$

$$\vdots$$

$$b_3(u) = a_3 u^3 + b_3 u^3 + c_3 u + d_3$$

C_2 continuity means 15 constraints:

- Position
- First derivative
- Second derivative

} all have to match

$$0 = b_0(0)$$
$$b_0(1) = b_1(0)$$
$$b_1(1) = b_2(0)$$
$$b_2(1) = b_3(0)$$
$$b_3(1) = 0$$

position

$$0 = \dot{b}_0(0)$$
$$\dot{b}_0(1) = \dot{b}_1(0)$$
$$\dot{b}_1(1) = \dot{b}_2(0)$$
$$\dot{b}_2(1) = \dot{b}_3(0)$$
$$\dot{b}_3(1) = 0$$

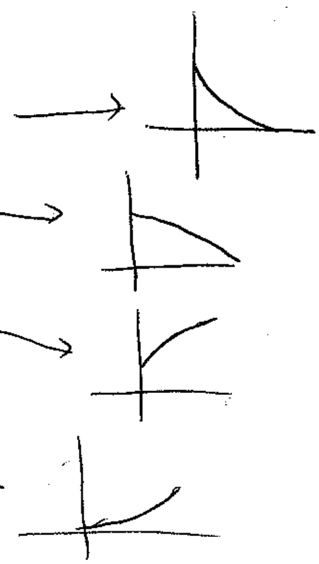
$$0 = \ddot{b}_0(0)$$
$$\ddot{b}_0(1) = \ddot{b}_1(0)$$
$$\ddot{b}_1(1) = \ddot{b}_2(0)$$
$$\ddot{b}_2(1) = \ddot{b}_3(0)$$
$$\ddot{b}_3(1) = 0$$

Normalize:

$$b_0(0) + b_1(0) + b_2(0) + b_3(0) = 1$$

Solve:

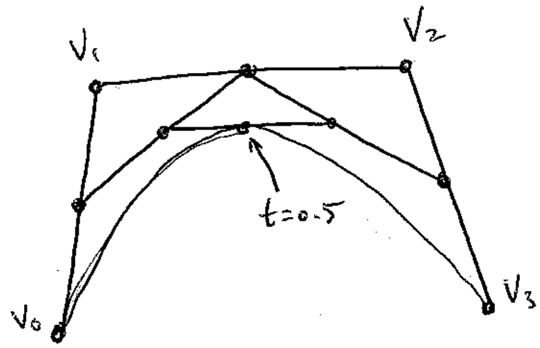
$$b_3(u) = -\frac{1}{6}u^3 + \frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{6}$$
$$b_2(u) = \frac{1}{2}u^3 - u^2 + \frac{2}{3}$$
$$b_1(u) = -\frac{1}{2}u^3 + \frac{1}{2}u^2 + \frac{1}{2}u + \frac{1}{6}$$
$$b_0(u) = \frac{1}{6}u^3$$



Cubic Bezier Curves

$$P(u) = (1-u)^3 V_0 + 3u(1-u)^2 V_1 + 3u^2(1-u) V_2 + u^3 V_3 \qquad u \in [0,1]$$

- controlled by 4 points (local)
- C^1 continuity



t = 0.5