Affinity by Distance:

$$\mathrm{aff}(x,y) = \exp\left\{ - \left(\overbrace{(x-y)^t (x-y)}^{\text{effectively difference squared added together}} / 2\sigma_d^2 \right) \right\}$$

Affinity by Intensity:

$$\mathrm{aff}(x,y) = \exp\left\{ - \left((I(x) - I(y))^t (I(x) - I(y)) / 2\sigma_e^2 \right) \right\}$$

Affinity by Color:

$$\mathrm{aff}(x,y) = \exp\left\{ - \left(\mathrm{dist}(c(x), c(y)) \right)^2 / 2\sigma_c^2) \right\}$$

Affinity by texture:

$$\mathrm{aff}(x,y) = \exp\left\{ - \left((\overset{\text{output of filters } f_1, \ldots, f_n}{f(x)} - f(y))^t (f(x) - f(y)) / 2\sigma_e^2 \right) \right\}$$

K-Means clustering

Objective function:

$$\Phi(\text{clusters}, \text{data}) = \underbrace{\sum_{i \in \text{clusters}}}_{\substack{\text{added up} \\ \text{over all} \\ \text{clusters}}} \underbrace{\left\{ \sum_{j \in i^{th} \text{ cluster}} (x_j - c_i)^T \overbrace{(x_j - c_i)}^{\substack{\text{distance between} \\ \text{point and cluster center}}} \right\}}_{\text{"cost of cluster"}}$$

Segmentation using Eigenvectors

A - affinity matrix

objective function:

maximize $w_n^T A w_n$, subject to $\underbrace{w_n^T w = 1}_{\text{normalization}}$

which is the sum:

$$\sum_{i,j} (\text{association of element } i \text{ with cluster } n) \times (\text{affinity between } i \text{ and } j) \times (\text{association of element } j \text{ with cluster } n)$$

Lagrangian:

$$w_n^T A w_n + \lambda (w_n^T w_n - 1)$$

Differentiating:

$$A w_n = \lambda w_n$$

hence w_n is the eigenvector of the affinity matrix A.

$V \rightarrow A, B$

Normalized cuts

$$\frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

sum of all the edges on V that have one end in A and the other in B

sum of all edges that have one end in A

have y be a vector of elements $\{+1, -b\}$

some constant

indicates which component the element belongs to

Degree matrix :

$$D_{ii} = \sum_j A_{ij}$$

Minimization :

$$\frac{y^T(D-A)y}{y^T D y}$$

$$(D-A)y = \lambda D y$$

if $z = D^{1/2} y$

$$\underbrace{D^{-1/2}(D-A)D^{-1/2}}_{\text{find eigenvectors of this matrix}} z = \lambda z$$

find eigenvectors of this matrix

those will be z.

then $z = D^{1/2} y$.