Objective function:

$$w_n^T A w_n$$

association of element i with cluster n

affinity between i,j

association of element j with cluster n

constraint $w_n^T w_n = 1$

Method of Lagrange multipliers

$$w_n^T A w_n + \lambda(w_n^T w_n - 1)$$

Differentiating, dropping factors:

$$A w_n = \lambda w_n$$

eigenvector!

Aside Eigenvectors

$$Ax = \lambda x$$

matrix, vector, scalar, vector

Solving for eigenvectors:

$$Ax = \lambda I x$$

$$Ax - \lambda I x = 0$$

$$(A - \lambda I)x = 0$$

want to solve for x!

solution given by

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

$$\lambda_{12} = \frac{1}{2}\left[(a_{11} + a_{22}) \pm \sqrt{4a_{12}a_{21} + (a_{11} - a_{22})^2}\right]$$

Ex:

$$A = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda & 0 \\ -\frac{1}{2} & 1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\lambda_{12} = 1$

so $\begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ a \end{bmatrix} = 0$

null space of matrix

$$\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 0 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ a \end{bmatrix}$$

you get the same vector back! (multiplied by $\lambda = 1$)

Normalized Cuts

$$\text{Ncut}(A,B) = \frac{\overbrace{\text{cut}(A,B)}^{\text{sum of all edges with one end in } A \text{ and the other in } B}}{\underbrace{\text{assoc}(A,V)}_{\text{sum of all edges that have one end in } A}} + \frac{\text{cut}(A,B)}{\text{assoc}(B,V)}$$

Degree matrix D ← diagonal matrix

if n elements it is $n \times n$

$$D_{ii} = \sum_j A_{ij}, \quad D_{ij} = 0 \text{ where } i \neq j$$

diagonal elements are the sum of the weights coming into each node.

write as:

$$\text{Ncut}(A,B) = \frac{y^T(D-A)y}{y^T D y} \quad \text{with } y_i \in \{1, -b\}$$

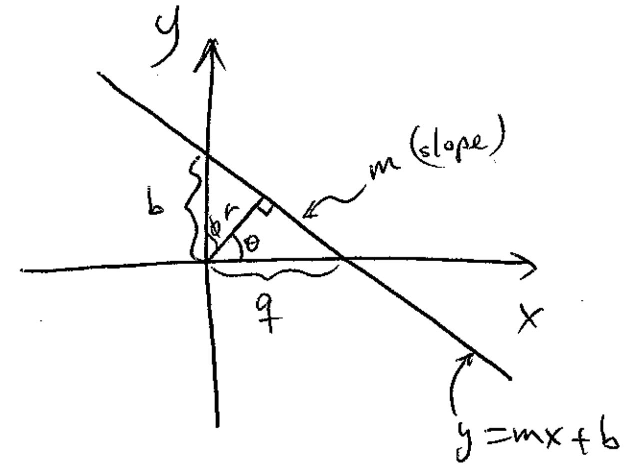
Relaxing y:

$$(D-A)y = \lambda D y$$

Solved with eigenvalues:

$$D^{-\frac{1}{2}}(D-A)D^{-\frac{1}{2}} z = \lambda z \quad \text{where } z = D^{\frac{1}{2}} y$$

Hough Transform



$$\cos\phi = \frac{r}{b} \Rightarrow b = \frac{r}{\cos\phi} = \frac{r}{\sin\theta}$$

$$90 = \phi + \theta$$
$$90 - \phi = \theta$$
$$\cos\phi = \sin\theta$$

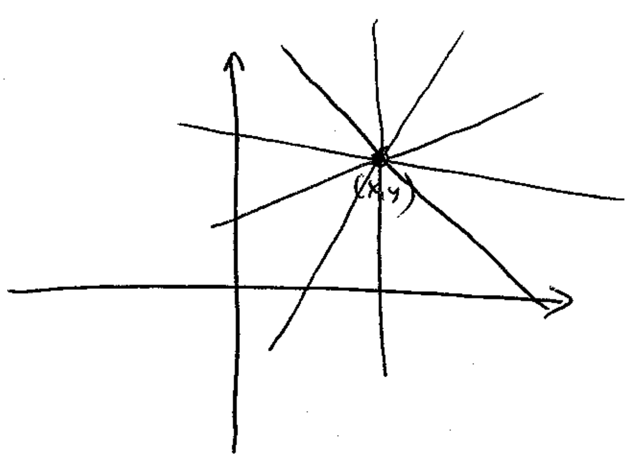
$$\cos\theta = \frac{r}{q} \Rightarrow q = \frac{r}{\cos\theta}$$

$$m = -\frac{\text{rise}}{\text{run}} = -\frac{b}{q} = -\frac{\frac{r}{\sin\theta}}{\frac{r}{\cos\theta}} = -\frac{\cos\theta}{\sin\theta}$$

$$y = \left(-\frac{\cos\theta}{\sin\theta}\right)x + \left(\frac{r}{\sin\theta}\right)$$

∴ you can express a line uniquely by the pair (r, θ)

Through a given point, there are ∞ # of lines:



Line space