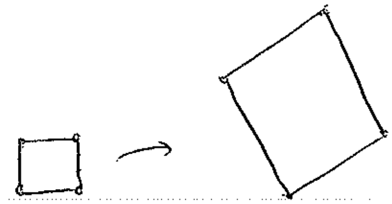
4/9/08 (1)

Similarity transform:

scale factor

$$\begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \leftarrow \text{4 DOF}$$

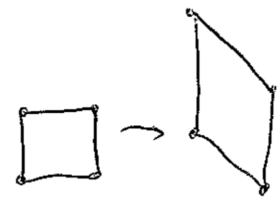


Affine transform:

6 DOF

non-singular matrix

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$$



Note that a point at ∞ stays at ∞:

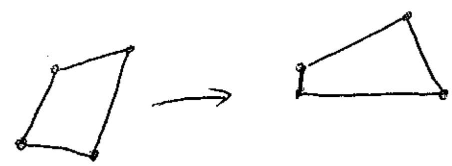
point at ∞

$$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix} \leftarrow \text{also a point at } \infty$$

Projective transform:

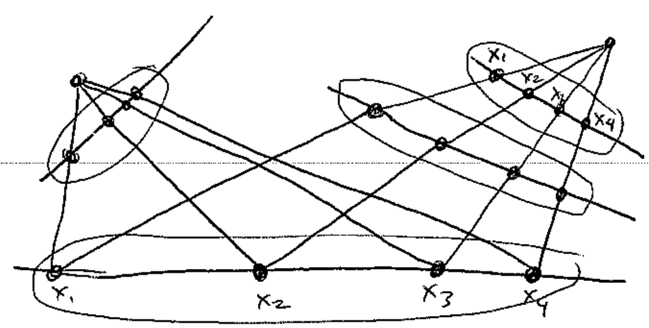
$$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$

8 DOF



invariant: cross-ratio

In 1D:



4 sets of colinear points

Cross-ratio is the same for all points.

$$\begin{vmatrix} x_{1_1} & x_{2_1} \\ x_{1_2} & x_{2_2} \end{vmatrix}$$

Cross ratio: $$\text{Cross}(x_1, x_2, x_3, x_4) = \frac{|x_1\, x_2|\,|x_3\, x_4|}{|x_1\, x_3|\,|x_2\, x_4|}$$

if homogeneous points are finite with $w = 1$, then

$$\frac{(x_1 - x_2)(x_3 - x_4)}{(x_1 - x_3)(x_2 - x_4)}$$

Transformation of the line at ∞ for affine transformations:

Transformation of line: $l' = H^{-T} l$

what is H^{-T} for an affine transformation $H = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$?

$$H^{-1} = \begin{bmatrix} A^{-1} & -A^{-1}t \\ 0^T & 1 \end{bmatrix}$$

$$H \cdot H^{-1} = I = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} Q & v \\ 0^T & 1 \end{bmatrix} = I$$

$$Q = A^{-1}$$

$$Av + t = 0$$

$$Av = -t$$

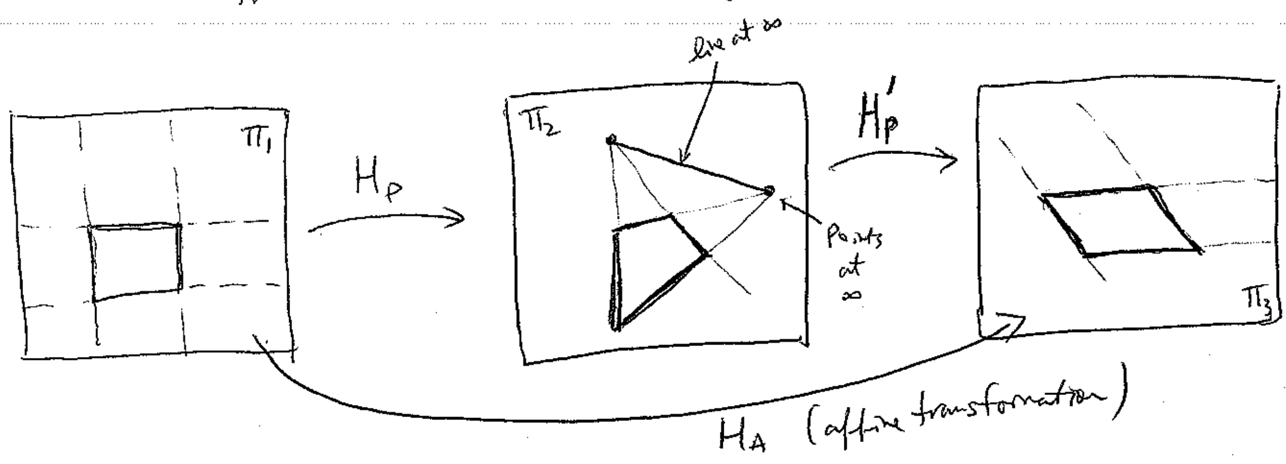
$$A^{-1}A v = -A^{-1}t$$

$$v = -A^{-1}t$$

$$(H^{-1})^T = \begin{bmatrix} A^{-T} & 0 \\ -t^T A^{-T} & 1 \end{bmatrix}$$

$$l' = \begin{bmatrix} A^{-T} & 0 \\ -t^T A^{-T} & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{line at } \infty} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{stays at } \infty}$$

So once we know where the line at infinity maps to in a projection, we can now make affine measurements in original plane.



If we develop a transform H_p' that transform l_∞ back to $(0,0,1)^T$ then we have "affinely" rectified the image.

$$H_p' = H_A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

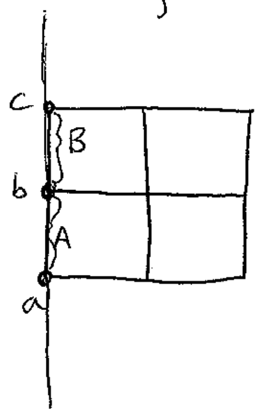
line at infinity $(l_1, l_2, l_3)^T$

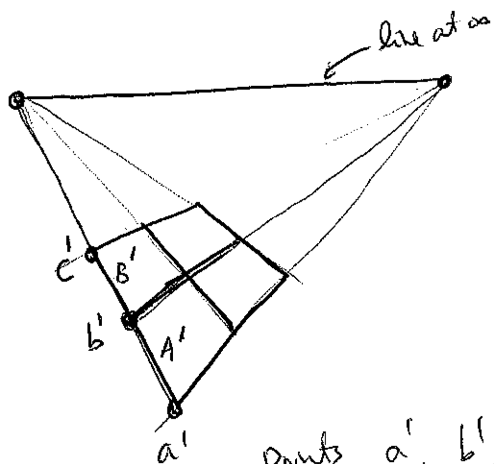
Exercise to you: verify $(H_p')^{-T}\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ Affine properties can be recovered by only specifying a line. (2 DOF)

Computing a vanishing point from a length ratio

Given two intervals on a line w/ known length ratio, the point at infinity on the line may be determined.





points a, b, c have coordinates on this line $\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} A \\ 1 \end{pmatrix}, \begin{pmatrix} A+B \\ 1 \end{pmatrix}$

points a', b', c' have coords on this line $\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} A' \\ 1 \end{pmatrix}, \begin{pmatrix} A'+B' \\ 1 \end{pmatrix}$

- Find the $H_{2\times 2}$ that maps $a \mapsto a'$, $b \mapsto b'$, $c \mapsto c'$
- Then apply $H_{2\times 2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to give you the intersection of the line with the line at ∞.

point at infinity