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A couple of loose ends from last time:

1. Claim:

$$\cos\theta = \frac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* l)(m^T C_\infty^* m)}}$$

conic dual to circular points

is invariant under projective transform

projective

$$\text{point} \left\{ x' = H x \right.$$

$$\text{lines} \begin{cases} m' = H^{-T} m \\ l' = H^{-T} l \end{cases}$$

$$l^T C_\infty^* m \xmapsto{H} (l^T \cancel{H^{-1}})(\cancel{H} C_\infty^* \cancel{H^T})(\cancel{H^{-T}} m)$$

$$= l^T C_\infty^* m$$

$$\begin{matrix}\text{dual} \\ \text{conic}\end{matrix} \left\{ C_\infty^{*\prime} = H C_\infty^* H^T \right.$$

Same with $l^T C_\infty^* m$, $l^T C_\infty^* l$ and $m^T C_\infty^* m$

$\therefore$ invariant under projection transformation

2.

$$C_\infty^{*\prime} = (H_P H_A H_S) C_\infty^* (H_S^T H_A^T H_P^T)$$

$$= (H_P H_A)(H_S C_\infty^* H_S^T)(H_A^T H_P^T)$$

$$= (H_P H_A)(C_\infty^*)(H_A^T H_P^T)$$

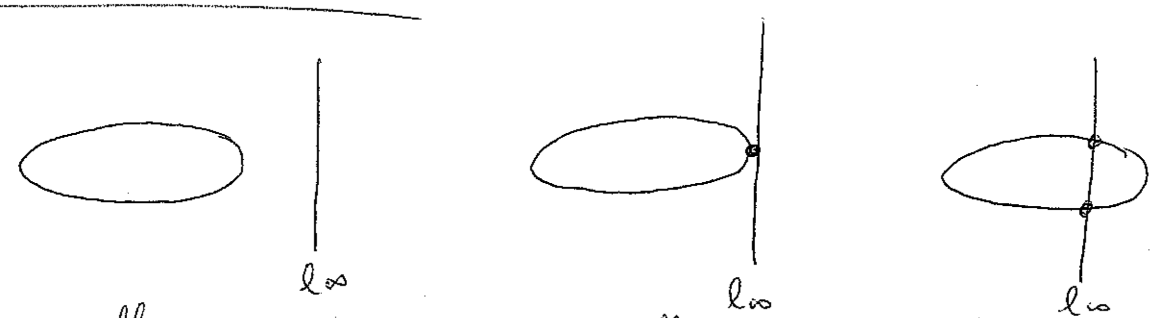
$$= \begin{bmatrix} KK^T & KK^T v \\ v^T KK^T & v^T KK^T v \end{bmatrix}$$

$$H_P = \begin{bmatrix} I & 0 \\ v^T & v \end{bmatrix}$$

$$H_A = \begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix}$$

Suppose we are mapping from an affinely rectified image back to world. There have been no v^T, so $v = 0$

Affine classification of conics:



ellipse parabolla hyperbola

In 3D space $\mathbb{P}^3$

- 4D homogeneous coordinates

$\mathbb{P}^2$	$\mathbb{P}^3$
conic	Quadric
$x^T C x = 0$	$x^T Q x = 0$

Quadric:

$$a_1 x^2 + a_2 xy + a_3 y^2 + a_4 yz + a_5 z^2$$
$$+ a_6 xz + a_7 x + a_8 y + a_9 z + a_{10} = 0$$

Quadrics:

$$Q = \begin{bmatrix} a_1 & \frac{1}{2}a_2 & \frac{1}{2}a_6 & \frac{1}{2}a_7 \\ \frac{1}{2}a_2 & a_3 & \frac{1}{2}a_4 & \frac{1}{2}a_8 \\ \frac{1}{2}a_6 & \frac{1}{2}a_4 & a_5 & \frac{1}{2}a_9 \\ \frac{1}{2}a_7 & \frac{1}{2}a_8 & \frac{1}{2}a_9 & a_{10} \end{bmatrix}$$

Under transformation

$$X' = HX \Rightarrow Q' = H^{-T} Q H^{-1}$$

$\mathbb{P}^2$		$\mathbb{P}^3$
line at ∞ (l_∞)	$\rightarrow$	plane at ∞ (Π_∞)

$\leftarrow$ the plane at ∞ is fixed (as a set) by an affine transformation.

Absolute conic Ω_∞

Point conic on Π_∞:

$$(x_1, x_2, x_3)\ \underset{\substack{\uparrow \\ C}}{I}\ (x_1, x_2, x_3)^T = 0$$

Points satisfy:

$$x_1^2 + x_2^2 + x_3^2 = 0$$
$$x_4 = 0 \quad \text{on } \Pi_\infty$$

So absolute conic corresponds to a conic with matrix I

Thm. Absolute conic is fixed under the projection transform H iff H is a similarity transform.

Proof Outline

1. The transformation H must be affine because $\Pi_\infty \overset{H}{\mapsto} \Pi_\infty$

also $C = A^{-T} C A^{-1} \Rightarrow I = A^{-T} I A^{-1}$

$$A^T = I A^{-1}$$

$$A^T A = I$$

A is orthogonal matrix, hence H is similarity transform.

Multiple View Geometry

Epipolar geometry