CH. 7

2/11/08 ①

Averaging filter:

$$R_{ij} = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{u=i+k} \sum_{v=j-k}^{v=j+k} F_{uv}$$

tells you how to compute a single pixel of final image

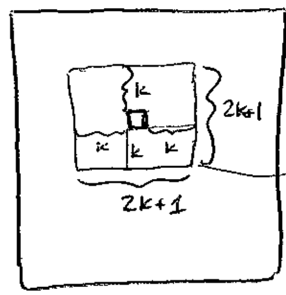


image $\mathcal{F}$

filter kernel area $(2k+1)^2$

to average, we divide by $\frac{1}{(2k+1)^2}$

Convolution: 1D: $h(t) = \int f(\tau) g(t-\tau)\, d\tau$

$$\underbrace{R_{ij}}_{\text{resulting image}} = \sum_{u,v} \underbrace{H_{i-u,j-v}}_{\text{filter kernel}} \underbrace{F_{uv}}_{\text{image to filter}}$$

So if each side of image/filter kernel is $O(n)$, this operation is $O(n^4)$.

Common kernels:

Gaussian function: $G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$

standard deviation

To generate a $(2k+1) \times (2k+1)$ kernel:

$$H_{ij} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{((i-k-1)^2+(j-k-1)^2)}{2\sigma^2}\right)$$

Finite differences

$$\frac{\partial h}{\partial x} = h_{i+1,j} - h_{i-1,j}$$

$$\mathcal{H} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Shift-Invariant Linear systems

Linearity
$$\begin{cases} \text{Superposition: } & R(f+g) = R(f) + R(g) \\ \text{Scaling: } & R(kf) = k\,R(f) \end{cases}$$

Shift invariance:

$$R(f)_{delay} = R(f_{delay})$$

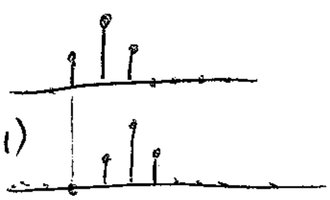
Ex: input vector f

basis: $e_0 = \ldots 0,0,0,1\,0\,0\,0\,0$

$\text{shift}(f,i)$ takes f and "shifts" it i places to the right:

f

shift(f, 1)



So you can write:

$$f = \sum_i \underbrace{f_i}_{\text{coefficients}}\ \underbrace{\text{shift}(e_0, i)}_{\text{basis function}}$$

$$\begin{aligned} R(f) &= R\Big(\sum_i f_i \text{ shift}(e_0, i)\Big) \\ &= \sum_i R(f_i \text{ shift}(e_0,i)) && \text{by Linearity of } R \text{ (superposition)} \\ &= \sum_i f_i\, R(\text{shift}(e_0,i)) && \text{by Linearity (scaling)} \\ &= \sum_i f_i \text{ shift}(\underbrace{R(e_0)}_{\text{"impulse response"}}, i) && \text{by shift-invariance of } R \end{aligned}$$

$$R(f) = g * f$$

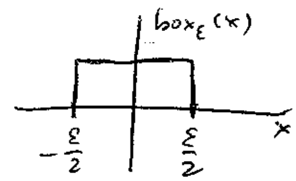
$$R_j = \sum_i f_i \underbrace{g_{j-i}}_{g \text{ signal shifted by } i}$$

<u>Continuous domain</u>

$$\text{Shift}(f, c) = f(u-c)$$

shift invariance: $R(\text{Shift}(f, c)) = \text{Shift}(R(f), c)$

$$\text{box}_\epsilon(x) = \begin{cases} 0 & \text{abs}(x) > \frac{\epsilon}{2} \\ 1 & \text{abs}(x) < \frac{\epsilon}{2} \end{cases}$$



Same as before:

$$R(f) = R\Big(\underbrace{\sum_i \overbrace{f_i}^{f(x_i)} \text{Shift}(\text{box}_\epsilon, x_i)}_{f}\Big) = \sum_i f_i \, \text{Shift}(R(\text{box}_\epsilon), x_i)$$

$$= \sum_i f_i \, \text{Shift}\Big(R\Big(\frac{\text{box}_\epsilon}{\epsilon}\Big), x_i\Big) \, \epsilon$$

as $\epsilon \to 0$

$$\delta(x) = \lim_{\epsilon \to 0} \frac{\text{box}_\epsilon(x)}{\epsilon}$$

$$R(f) = \int f(x') \underbrace{R_\delta(u-x')}_{\text{impulse response of } R} dx'$$

$$h(t) = \int f(\tau) \overbrace{g}^{g}(t-\tau) \, d\tau = f * g$$