Why Gaussians?

Gaussian ** Gaussian = Gaussian

$$G_{\sigma_1} ** G_{\sigma_2} = G_{\sqrt{\sigma_1^2+\sigma_2^2}}$$

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

$$= \underbrace{\left(\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)\right)}_{\text{1-D Gaussian in } x} \underbrace{\left(\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)\right)}_{\text{1-D Gaussian in } y}$$

Derivative of Gaussian Filtered Image

$$\frac{\partial I}{\partial x} = \underbrace{K_{\left(\frac{\partial}{\partial x}\right)}}_{\substack{\text{derivative}\\\text{operator}\\\text{(linear,}\\\text{shift}\\\text{invariant)}}} ** \overbrace{I}^{\text{image}}$$

$$\left(K_{\left(\frac{\partial}{\partial x}\right)} ** (\overbrace{S}^{\text{arbitrary LTI filter}} ** I)\right) = \left(K_{\left(\frac{\partial}{\partial x}\right)} ** S\right) ** I$$

$$= \frac{\partial S}{\partial x} ** I$$

if $S = \overbrace{G_\sigma}^{\text{Gaussian}}$

$$\frac{\partial (G_\sigma ** I)}{\partial x} = \underbrace{\left(\frac{\partial G_\sigma}{\partial x}\right)}_{\substack{\text{derivative}\\\text{of the Gaussian}}} ** I$$

Laplacian Operator

$$(\nabla^2 f)(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$ ← can write this as a filter K_{∇^2}

$$\underbrace{K_{\nabla^2}}_{\substack{\text{Laplacian}\\\text{kernel}}} ** \underbrace{(G_\sigma ** I)}_{\substack{\text{smoothed}\\\text{image}}} = \underbrace{(K_{\nabla^2} ** G_\sigma)}_{\substack{\text{Laplacian of}\\\text{Gaussian}}} ** I$$

$$= (\nabla^2 G_\sigma) ** I$$

Gradient

"del" operator → $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$ ← this is a vector!

∇f ← gradient of f

$|\nabla f|$ ← magnitude of gradient of f

Sobel operator

−1	0	+1
−2	0	+2
−1	0	−1

G_x

+1	+2	+1
0	0	0
−1	−2	−1

G_y

$$|G| = |G_x| + |G_y|$$

$$\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$