Table 1

sequence	generating function	closed form
$< 1, 0, 0, 0, 0, 0, \dots >$	$\sum_{n\geq 0}[n=0]z^n$	1
$< 0, \dots, 0, 1, 0, 0, \dots >$	$\sum_{n\geq 0}[n=m]z^n$	z^m
$< 1, 1, 1, 1, 1, 1, \dots >$	$\sum_{n\geq 0} z^n$	$\frac{1}{1-z}$
$< 1, -1, 1, -1, 1, -1, \dots >$	$\sum_{n\geq 0}(-1)^n z^n$	$\frac{1}{1+z}$
$< 1, 0, 1, 0, 1, 0, \dots >$	$\sum_{n\geq 0}[2 \setminus n]z^n$	$\frac{1}{1-z^2}$
$< 1, 0, \dots, 0, 1, 0, \dots, 0, 1, 0, \dots >$	$\sum_{n\geq 0}[m \setminus n]z^n$	$\frac{1}{1-z^m}$
$< 1, 2, 3, 4, 5, 6, \dots >$	$\sum_{n\geq 0}(n+1)z^n$	$\frac{1}{(1-z)^2}$
$< 1, 2, 4, 8, 16, 32 \dots >$	$\sum_{n\geq 0} 2^n z^n$	$\frac{1}{1-2z}$
$< 1, 4, 6, 4, 1, 0, 0, \dots >$	$\sum_{n\geq 0} \binom{4}{n} z^n$	$(1+z)^4$
$< 1, c, \binom{c}{2}, \binom{c}{3}, \binom{c}{4}, \dots >$	$\sum_{n\geq 0} \binom{c}{n} z^n$	$(1+z)^c$
$< 1, c, \binom{c+1}{2}, \binom{c+2}{3}, \binom{c+3}{4}, \dots >$	$\sum_{n\geq 0} \binom{c+n-1}{n} z^n$	$\frac{1}{(1-z)^c}$
$< 1, c, c^2, c^3, c^4, c^5, c^6, \dots >$	$\sum_{n\geq 0} c^n z^n$	$\frac{1}{1-cz}$
$< 1, \binom{m+1}{m}, \binom{m+2}{m}, \binom{m+3}{m}, \binom{m+4}{m}, \dots >$	$\sum_{n\geq 0} \binom{m+n}{m} z^n$	$\frac{1}{(1-z)^{m+1}}$
$< 0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots >$	$\sum_{n\geq 0} \frac{1}{n} z^n$	$\ln \frac{1}{1-z}$
$< 0, 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots >$	$\sum_{n\geq 0} \frac{(-1)^{n+1}}{n} z^n$	$\ln(1+z)$
$< 1, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \dots >$	$\sum_{n\geq 0} \frac{1}{n!} z^n$	e^z

Table 2

Generating function manipulations

$$
\begin{aligned}
G(z) &= \sum_n g_n z^n \\
\alpha F(z) + \beta G(z) &= \sum_n \left(\alpha f_n + \beta g_n\right) z^n \\
z^m G(z) &= \sum_n g_{n-m} z^n, \qquad \text{integer } m \geq 0 \\
\frac{G(z) - g_0 - g_1 z - \cdots - g_{m-1} z^{m-1}}{z^m} &= \sum_n g_{n+m} z^n, \qquad \text{integer } m \geq 0 \\
G(cz) &= \sum_n c^n g_n z^n \\
G'(z) &= \sum_n (n+1) g_{n+1} z^n \\
zG'(z) &= \sum_n n g_n z^n \\
\int_0^z G(t)dt &= \sum_{n \geq 1} \frac{1}{n} g_{n-1} z^n \\
F(z)G(z) &= \sum_n \left(\sum_k f_k g_{n-k} \right) z^n \\
\frac{1}{1-z} G(z) &= \sum_n \left(\sum_{k \leq n} g_k \right) z^n
\end{aligned}
$$