Uniform Variable Delay with Positive Feedback Can Stabilize Second Order Systems

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Abstract

Taking the analysis developed in [1] as a base, now we extended it to uniform variable delay with positive feedback showing the stability for oscillatory systems.

Introduction

The delay that exists between the components in a network, like actuators, sensors and controllers, is never fixed, rather, it is time-varying or random.

We always think that the effect of time delay in a control system affects its dynamic behavior because it reduces the phase margin, yielding a reduced damping ratio for the closed-loop system and a more oscillatory response. Also, it reduces the gain margin which moves the system to instability [2]. But in this case, the Uniform Random Delay can reduce the oscillatory behavior.

Taking a second order as our oscillatory system and stabilizing it using positive, delay output feedback only, can save many sensors. Recapturing the closed loop stability analysis in [1], we completed the stability regions with negative gain and applied it with uniform variable delay.

Analysis of Positive Stability Regions

Consider the following plant:

$$G(s) = \frac{1}{s^2 + w_n^2} \tag{1}$$

And the time delay compensator given by:

$$C(s) = Ke^{-s\tau} \tag{2}$$

The transfer function in a positive feedback loop is given by

$$\frac{Y(s)}{R(s)} = \frac{Ke^{-s\tau}}{s^2 + w_n^2 - Ke^{-s\tau}} \tag{3}$$

The study of the Nyquist plot of the open loop must consider two conditions for stability $\tau > 0$ and $k < w_n^2$. Basing the analysis in

$$G(jw)C(jw) = \frac{Ke^{-jw\tau}}{w_n^2 - w^2} \tag{4}$$

equation [4] gives three regions. The magnitude of the Nyquist plot is given by

$$\begin{aligned} |G(jw)C(jw)| &= \frac{K}{w_n^2 - w^2},\ \mathit{for}\ 0 \le w < w_n \\ |G(jw)C(jw)| &= \frac{K}{w^2 - w_n^2},\ \mathit{for}\ w > w_n \end{aligned} \tag{5}$$

and the phase given by

$$\begin{aligned} \theta(w) &= -\pi - w\tau; \mathit{for}\ 0 \le w < w_n \\ \theta(w) &= -w\tau; \mathit{for}\ w > w_n \end{aligned} \tag{6}$$

The intersections of the polar plot with the negative real axis take place at the frequencies w_c

$$\begin{aligned} w_c &= 2n\pi/\tau,\ \textit{for}\ 0 \leq w_c < w_n \\ w_c &= (2n+1)\pi/\tau,\ \textit{for}\ w_c > w_n \end{aligned} \tag{7}$$

To guarantee that the magnitude $|G(jw)C(jw)|$ evaluated at w_c is less than 1, for no encirclements of the -1 point take place, we must have

$$\begin{aligned} \frac{k}{w_n^2 - (2n\pi)^2/\tau^2} &< 1,\ \textit{for}\ 0 \leq 2n\pi/\tau < w_n \\ \frac{k}{((2n+1)\pi)^2/\tau^2 - w_n^2} &< 1,\ \textit{for}\ (2n+1)\pi/\tau > w_n \end{aligned} \tag{8}$$

Combining the last two conditions we find the lower and upper bounds of the stability region for positive gain

$$\frac{2n\pi}{\sqrt{w_n^2 - k}} < \tau < \frac{(2n+1)\pi}{\sqrt{w_n^2 + k}}$$
$$0 < k \leq \frac{1+4n}{1+4n+8n^2} w_n^2 \tag{9}$$

Complete Analysis of Stability Regions

Following the analysis developed before, we found the complete stability region for positive and negative gain. The upper and lower bounds for negative gain are given by

$$\frac{2n\pi}{\sqrt{w_n^2 - k}} > \tau > \frac{(2n-1)\pi}{\sqrt{w_n^2 + k}}$$
$$0 > k \geq \frac{1+4n}{1+4n+8n^2} w_n^2 \tag{10}$$

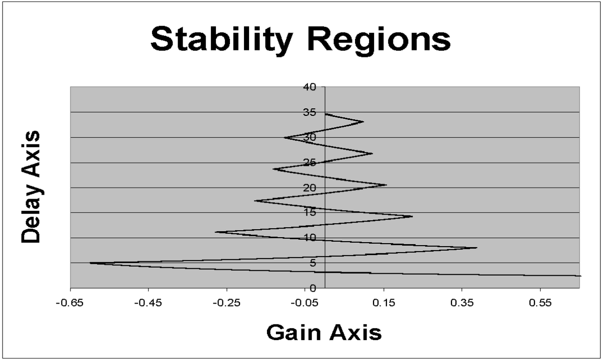

Figure: Stability Regions for $w_n^2 = 1$

Application of Uniform Variable Delay

As we expected, both regions of stabilizing gain get decreased as delay is increased. We can apply the regions of stability in order to make stable a control system that presents uniform variable delay, in agreement to both conditions of stability. Consider a simple unitary and positive feedback loop shown in Figure 2.

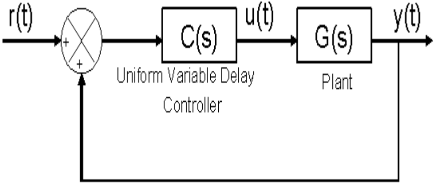

Figure: Block Diagram of Second Order System with Uniform Variable Delay, Positive Feedback

Example

For example, consider a maximum delay of 3 seconds for the variable transport delay with a uniform random time delay. The maximum uniformly distributed random signal has an amplitude of 10. And let the plant transfer function be

$$G(s) = \frac{1}{s^2 + 1} \tag{11}$$

The initial conditions are $y(0)=\dot{y}(0)=0.1$, and the simulation is illustrated in Figure 3.

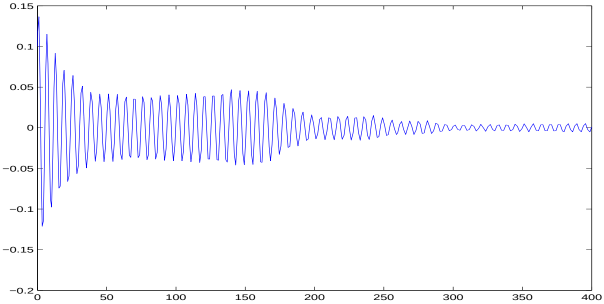

Figure: Simulation of Second Order System with Uniform Variable Delay, Positive Feedback

In order to decrease the magnitude of the gain in the controller, we choose a tenth of the estimated gain, being closer of the delay axis and not so at the edge of stability.

Conclusions

The oscillatory behavior of a second order system is decreased with a minimum steady stare error. The due adjustment of gain value inside the limit of stability damped the oscillation. Hereby, we have an instantaneously adaptable controller in function of delay variation.

Future Work

In the same way as this work had spread in the negative gain axis of the stability region, we can spread to a negative delay too. Moving on the negative delay axis, we can anticipate to the actions and control them with help of the Stability Regions on Figure 4.

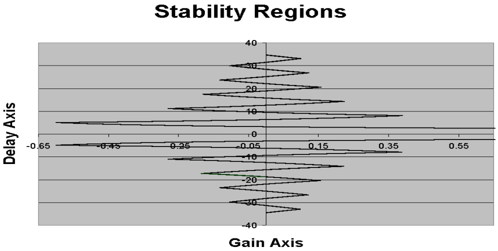

Figure: Stability Regions with Positive and Negative Delay

Bibliography

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