ECE-314: Signals and Systems

Term: Fall 2009

Instructor: Balu Santhanam

HW1 - Solution

P1.42

a) $x(t) = \cos^2(2\pi t) = \frac{1+\cos(4\pi t)}{2}$

$x(t+T_0) = \cos^2(2\pi(t+T_0)) = \frac{1+\cos(4\pi(t+T_0))}{2}$

$x(t) = x(t+T_0)$

iff $\cos(4\pi t) = \cos(4\pi(t+T_0))$

$\Longrightarrow 4\pi t = 4\pi t + 4\pi T_0 + 2k\pi$ or $4\pi t = -4\pi t - 4\pi T_0 + 2k\pi$

① $-4\pi T_0 = 2k\pi$ or ② $8\pi t - 2k\pi = -4\pi T_0$

② $\Longrightarrow$ is time dependent we can not find T_0 independent of "t" from ②

① $\Longrightarrow T_0 = -\frac{k}{2}$

for $k=-1$ $\Longrightarrow$ $\boxed{T_0 = \frac{1}{2} \text{ sec.}}$

e) $x[n] = (-1)^{n^2}$

$x[n+N_0] = (-1)^{(n+N_0)^2} = (-1)^{n^2}(-1)^{N_0^2}(-1)^{2N_0 n}$

$x[n+N_0] = x[n](-1)^{N_0^2}(-1)^{2N_0 n}$

if $\left[(-1)^{N_0^2}(-1)^{2N_0 n}\right] = 1$ we will have $x[n+N_0] = x[n]$

if $N_0 = 2 \Longrightarrow (-1)^4(-1)^{4n} = 1$ for all n

So the Fundamental period is $\boxed{N_0 = 2}$

P.1.44

$$P(t) = \frac{1}{T}\int_0^T A^2\cos^2(\omega t+\phi)\,dt \quad \text{where } T = \frac{2\pi}{\omega}$$

$$= \frac{A^2}{2T}\int_0^T \left(1+\cos(2\omega t+\phi)\right)dt$$

$$= \frac{A^2}{2T}\left[[T] + \underbrace{\int_0^T \cos(2\omega t+\phi)\,dt}_{0}\right]$$

$$\boxed{P(t) = \frac{A^2}{2}}$$

1.46:

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega}(\cos^2\omega t + 2\cos\omega t + 1)\,dt$$

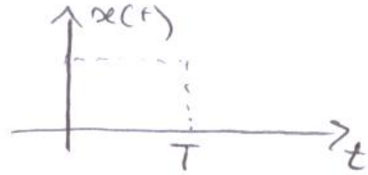
$$= \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega}\left(\frac{1+\cos(2\omega t)}{2}\right)dt + \frac{1}{4}\sin(\omega t)\Big|_{-\pi/\omega}^{\pi/\omega} + \frac{t}{4}\Big|_{-\pi/\omega}^{\pi/\omega}$$

$$= \frac{1}{8}t\Big|_{-\pi/\omega}^{\pi/\omega} + \sin(2\omega t)\Big|_{-\pi/\omega}^{\pi/\omega} \quad 0 + \frac{2\pi}{4\omega}$$

$$= \frac{2\pi}{8\omega} + 0 + \frac{\pi}{2\omega} = \boxed{\frac{3\pi}{4\omega} = E}$$

P.1.49

$$x(t) = \begin{cases} A & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$



$$y(t) = \int_{0^-}^{t} x(\tau)\, d\tau$$

for $0 \leq t \leq T$ $\quad y(t) = \int_{0^-}^{t} A\, d\tau = At$

for $t \geq T$ $\quad y(t) = \int_{0^-}^{T} A\, dt = AT$

for $t < 0$ $\quad y(t) = \int_{0^-}^{0} x(\tau)\, d\tau = 0$

$$y(t) = \begin{cases} 0 & t < 0 \\ At & 0 \leq t \leq T \\ AT & t \geq T \end{cases}$$

$$E_y = \int_{-\infty}^{\infty} y^2(t)\, dt = \int_{0}^{T} A^2 t^2\, dt + \int_{T}^{+\infty} A^2 T^2\, dt$$

$$= A^2\left[\frac{T^3}{3} - 0\right] + A^2 T^2 \left[t\right]_{T}^{+\infty}$$

$$= \frac{A^2 T^3}{3} + \infty$$

$$= +\infty$$

1.57:

d) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \, \delta(t-2k)$

$$x(t+T_0) = \sum_{k=-\infty}^{\infty} (-1)^k \, \delta(t+T_0-2k)$$

$$= \sum_{p=-\infty}^{\infty} (-1)^p \, \delta(t+T_0-2p)$$

let us force the equality

$$\delta(t+T_0-2p) = \delta(t-2k)$$

$$\Rightarrow t+T_0-2p = t-2k$$

$$T_0 - 2p = \cancel{t} - 2k$$

$$p = k + \frac{T_0}{2}$$

$$x(t+T_0) = \sum_{k+\frac{T_0}{2}=-\infty}^{\infty} (-1)^{k+T_0/2} \, \delta(t \cancel{+} 2k)$$

$$= (-1)^{T_0/2} \underbrace{\sum_{k=-\infty}^{\infty} (-1)^k \, \delta(t-2k)}$$ when T_0 is even

$$= (-1)^{T_0/2} \, x(t)$$

$$\Rightarrow \boxed{T_0 = 4}$$ to force $(-1)^{T_0/2} = 1$

f) $x(t) = \cos(t)\, u(t)$

$x(t+T_0) = \cos(t+T_0)\, u(t+T_0)$

This two signals can not be equal unless $\boxed{T_0 = 0}$ so $x(t)$ is not periodic

1.78:

a) $x_e(t) = \dfrac{x(t) + x(-t)}{2}$ $\qquad x_o(t) = \dfrac{x(t) - x(-t)}{2}$

$$\int x_e^2(t)\,dt + \int x_o^2(t)\,dt = \frac{1}{4}\int x^2(t) + x^2(-t) + 2x(t)x(-t)\,dt + \frac{1}{4}\int x^2(t) + x^2(-t) - 2x(t)x(-t)\,dt$$

$$= \frac{1}{2}\int x^2(t) + x^2(-t)\,dt$$

but $\int x^2(t) = \int x^2(-t)$

$$\Longrightarrow \quad = \int \frac{2x^2(t)}{2}\,dt = \int x^2(t)\,dt$$

b) $x_e[n] = \dfrac{x[n] + x[-n]}{2}$

$x_o[n] = \dfrac{x[n] - x[-n]}{2}$

$$\sum_{n=-\infty}^{\infty}\left(x_e^2[n] + x_o^2[n]\right) = \sum_{-\infty}^{\infty} \frac{x^2[n] + x^2[-n] + \cancel{2x[n]x[-n]}}{4}$$

$$+ \sum_{-\infty}^{\infty} \frac{x^2[n] + x^2[-n] - \cancel{2x[n]x[-n]}}{4}$$

$$= \sum_{-\infty}^{\infty} \frac{x^2[n] + x^2[-n]}{2}$$

But $\sum_{-\infty}^{\infty} x^2[n] = \sum_{-\infty}^{\infty} x^2[-n]$ (change of variable)

$$= \sum_{n=-\infty}^{\infty} x^2[n]$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
%   Homework 1   Problem 2
%
%   Fall 2009  ECE 314
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

Lmax=200;

% Part A
[y, Fs, nbits]=wavread('vowi');
wavplay(y,Fs)

N=Fs*nbits
% Part B

DF=zeros(Lmax,1);

for L=1:Lmax;
    for m=1:N-Lmax
        DF(L)=DF(L)+(y(m)-y(m+L))^2;
    end
end



>>
```

```
The value of L that minimize DF(n,L) is:

L =

    80

the corresponding pitch frequency is (Fs/L):

Pfreq =

   125


ans =

the "vowi.wav" voive correspond to a male voice

>>
```