ECE314: Signals and Systems Fall 2009

Solution: Homework 3

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Problem 2.33

(g) $y[n] = \beta^n u[n] * u[n-3], \quad |\beta| < 1$

$$\text{for } n-3 < 0 \qquad n < 3$$
$$y[n] = 0$$
$$\text{for } n-3 \geq 0 \qquad n \geq 3$$

$$y[n] = \sum_{k=0}^{n-3} \beta^k$$
$$y[n] = \left(\frac{1-\beta^{n-2}}{1-\beta}\right)$$
$$y[n] = \begin{cases} \left(\frac{1-\beta^{n-2}}{1-\beta}\right) & n \geq 3 \\ 0 & n < 3 \end{cases}$$

(h) $y[n] = \beta^n u[n] * \alpha^n u[n-10], \quad |\beta| < 1, \quad |\alpha| < 1$

$$\text{for } n-10 < 0 \qquad n < 10$$

$$y[n] = 0$$
$$\text{for } n-10 \geq 0 \qquad n \geq 10$$
$$y[n] = \sum_{k=0}^{n-10} \left(\frac{\beta}{\alpha}\right)^k \alpha^n$$
$$y[n] = \begin{cases} \alpha^n \left(\frac{1-\left(\frac{\beta}{\alpha}\right)^{n-9}}{1-\frac{\beta}{\alpha}}\right) & \alpha \neq \beta \\ \alpha(n-9) & \alpha = \beta \end{cases}$$

Problem 2.34

(b) $m[n] = x[n] * y[n]$

$$\text{for } n+5 < -3 \qquad n < -8$$

$$m[n] = 0$$

$$\text{for } n+5 < 1 \qquad -8 \le n < -4$$

$$m[n] = \sum_{k=-3}^{n+5} 1 = n+9$$

$$\text{for } n-1 < -2 \qquad -4 \le n < -1$$

$$m[n] = \sum_{k=-3}^{0} 1 - \sum_{k=1}^{n+5} 1 = -n-1$$

$$\text{for } n-1 < 1 \qquad -1 \le n < 2$$

$$m[n] = \sum_{k=n-1}^{0} 1 - \sum_{k=1}^{4} 1 = -n-2$$

$$\text{for } n-1 < 5 \qquad 2 \le n < 6$$

$$m[n] = -\sum_{k=n-1}^{4} 1 = n-6$$

$$\text{for } n-1 \ge 5 \qquad n \ge 6$$

$$m[n] = 0$$

$$m[n] = \begin{cases} 0 & n < -8 \\ n+9 & -8 \le n < -4 \\ -n-1 & -4 \le n < -1 \\ -n-2 & -1 \le n < 2 \\ n-6 & 2 \le n < 6 \\ 0 & n \ge 6 \end{cases}$$

Problem 2.38

(b) $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

$$y(t) = h(t-1) + h(t-2) + h(t-3)$$

Problem 2.39

(g) $y(t) = \cos(\pi t)(u(t+1) - u(t-1)) * (u(t+1) - u(t-1))$

$$\begin{aligned}
\text{for } t+1 < -1 \qquad & t < -2 \\
& y(t) = 0 \\
\text{for } t+1 < 1 \qquad & -2 \leq t < 0 \\
& y(t) = \int_{-1}^{t+1} \cos(\pi t) d\tau = \frac{1}{\pi} \sin(\pi(t+1))
\end{aligned}$$

$$\begin{aligned}
\text{for } t-1 < 1 \qquad & 0 \leq t < 2 \\
& y(t) = \int_{t-1}^{1} \cos(\pi t) d\tau = -\frac{1}{\pi} \sin(\pi(t-1)) \\
\text{for } t-1 \geq 1 \qquad & t \geq 2 \\
& y(t) = 0
\end{aligned}$$

$$y(t) = \begin{cases} 0 & t < -2 \\ \frac{1}{\pi} \sin(\pi(t+1)) & -2 \leq t < 0 \\ -\frac{1}{\pi} \sin(\pi(t-1)) & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

(h) $y(t) = \cos(2\pi t)(u(t+1) - u(t-1)) * e^{-t}u(t)$

for $t < -1$

$$y(t) = 0$$

for $t < 1$ $\quad -1 \leq t < 1$

$$y(t) = \int_{-1}^{t} e^{-(t-\tau)} \cos(2\pi t) d\tau$$

$$y(t) = e^{-t} \left[\frac{e^{\tau}}{1+4\pi^2} \left(\cos(2\pi\tau) + 2\pi \sin(2\pi\tau)\right) \Big|_{-1}^{t} \right]$$

$$y(t) = \frac{\cos(2\pi t) + 2\pi \sin(2\pi t) - e^{-(t+1)}}{1+4\pi^2}$$

for $t \geq 1$ $\quad t \geq 1$

$$y(t) = \int_{-1}^{1} e^{-(t-\tau)} \cos(2\pi t) d\tau$$

$$y(t) = e^{-t} \left[\frac{e^{\tau}}{1+4\pi^2} \left(\cos(2\pi\tau) + 2\pi \sin(2\pi\tau)\right) \Big|_{-1}^{1} \right]$$

$$y(t) = \frac{e^{-(t-1)} - e^{-(t+1)}}{1+4\pi^2}$$

$$y(t) = \begin{cases} 0 & t < -1 \\ \frac{\cos(2\pi t) + 2\pi \sin(2\pi t) - e^{-(t+1)}}{1+4\pi^2} & -1 \leq t < -1 \\ \frac{e^{-(t-1)} - e^{-(t+1)}}{1+4\pi^2} & t \geq 1 \end{cases}$$

Problem 2.40

(n) $m(t) = f(t) * g(t)$

for $t < -1$

$$m(t) = 0$$

for $t < 0$ $\quad -1 \leq t < 0$

$$m(t) = -\int_{-1}^{t} \tau e^{-(t-\tau)} d\tau = t - 1 + 2e^{-(t+1)}$$

for $t < 1$ $\quad 0 \leq t < 1$

$$m(t) = \int_{t-1}^{t} \tau e^{-(t-\tau)} d\tau = t - 1 - (t-2)e^{-1}$$

for $t - 1 < 1$ $\quad 1 \leq t < 2$

$$m(t) = \int_{t-1}^{1} \tau e^{-(t-\tau)} d\tau = -e^{-1}(t-2)$$

for $t - 1 \geq 1$ $\quad t \geq 2$

$$m(t) = 0$$

$$m(t) = \begin{cases} 0 & t < -1 \\ t - 1 + 2e^{-(t+1)} & -1 \leq t < 0 \\ t - 1 - (t-2)e^{-1} & 0 \leq t < 1 \\ -e^{-1}(t-2) & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

Problem 2.49

a)$h(t) = cos(\pi t)$

i The system is *not memoryless* because the impulse response in not of the form $c\delta(t)$.

ii The system in *not causal* because $h(t) \neq 0$ for $t < 0$.

iii The system in *not BIBO* because:

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |cos(\pi t)|\, dt = \infty$$

a)$h(t) = u(t+1)$

i The system is *not memoryless* because the impulse response in not of the form $c\delta(t)$.

ii The system in *not causal* because $h(t) \neq 0$ for $t < 0$.

iii The system in *not BIBO* because:

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |u(t+1)|\, dt = \int_{-1}^{\infty} 1 dt = \infty.$$

a)$h[n] = (-1)^n u[-n]$

i The system is *not memoryless* because the impulse response in not of the form $c\delta(t)$.

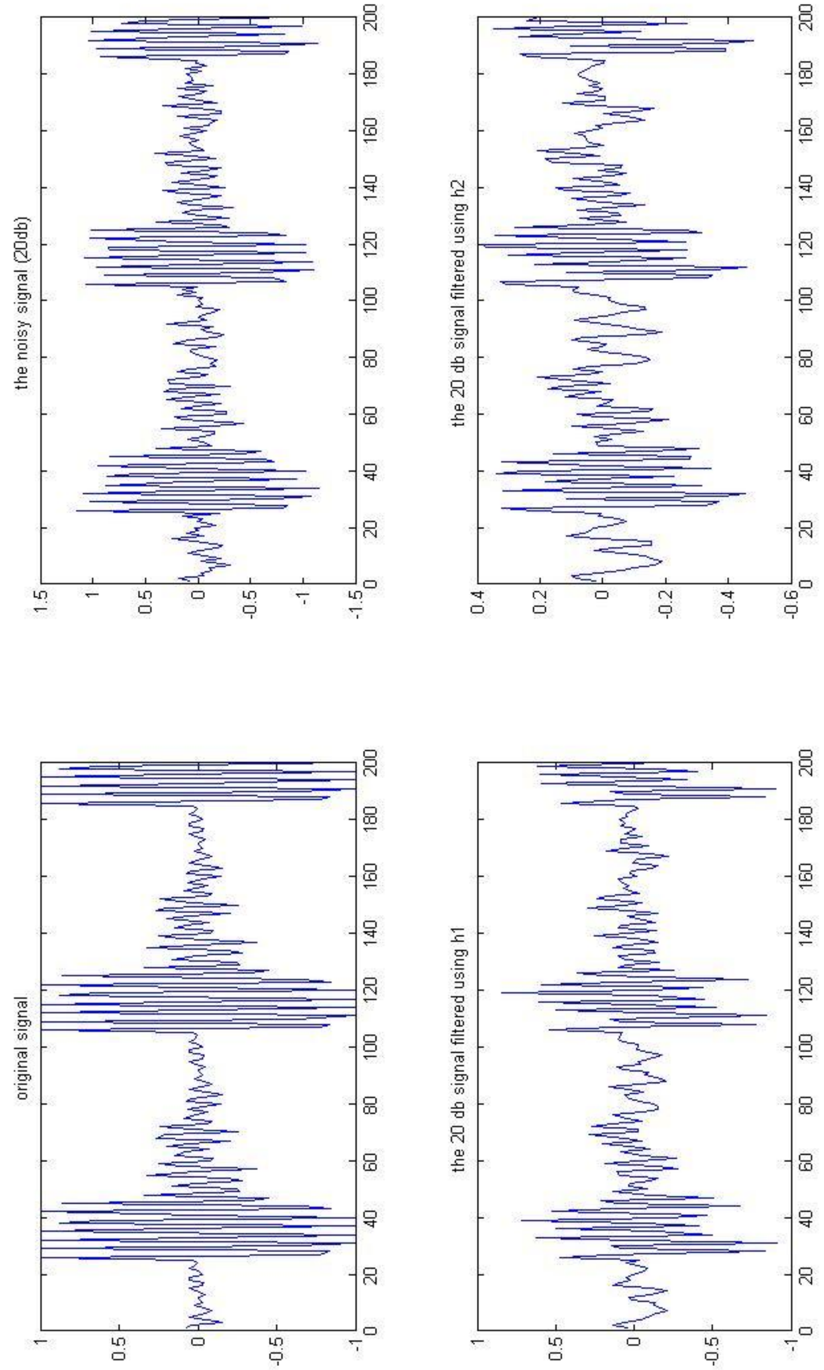
ii The system in *not causal* because $h(t) \neq 0$ for $t < 0$.

iii The system in *not BIBO* because:

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \sum_{-\infty}^{\infty} |(-1)^n u[-n]|\, dt = \sum_{-\infty}^{0} 1dt = \infty.$$

Matlab Problem

```
[y, f]=wavread('vowi');

%adding noise of a)40db b)20db c)0db
y_40=awgn(y,40);
y_20=awgn(y,20);
y_00=awgn(y,0);

% play the sounds
wavplay(y,f)
wavplay(y_40,f)
wavplay(y_20,f)
wavplay(y_00,f)

%transfer functions of the filters
h1=[0.5 0.5];
h2=[0.25 0.5 0.25];

%filtering the signals using h1
y_40_h1=conv(y_40,h1);
y_20_h1=conv(y_20,h1);
y_00_h1=conv(y_00,h1);

%filtering the signals using h2
y_40_h2=conv(y_40,h2);
y_20_h2=conv(y_20,h2);
y_00_h2=conv(y_00,h2);

%plotting the signals for a 20db SNR
subplot(2,2,1),plot(y(1:200)),title('original signal');
subplot(2,2,2),plot(y_20(1:200)),title('the noisy signal (20db)');
subplot(2,2,3),plot(y_20_h1(1:200)),title('the 20 db signal filtered using h1');
subplot(2,2,4),plot(y_20_h2(1:200)),title('the 20 db signal filtered using h2');
```


original signal
the noisy signal (20db)
the 20 db signal filtered using h1
the 20 db signal filtered using h2