Midterm Exam II, Fall 2008

Signals and Systems
University of New Mexico
Instructor: Balu Santhanam
Date Assigned: 11/25/2008
Duration: 2:00 - 3:15 PM

Instructions

1. Write clearly and legibly
2. Provide steps to obtain partial credit
3. You are allowed to bring 2 sheets of Fourier transform formulas.
4. It is assumed that you are aware of the UNM academic honesty policy. Needless to say copying will be dealt with seriously.

Problem # 1.0

A discrete–time LTI system is characterized by the following input-output difference equation:

$$y[n] = ay[n-1] + x[n-1] - ax[n],\ a \in \mathbf{R},\ 0 < a < 1.$$

For the system described here:

1. Calculate the frequency response $H(e^{j\Omega})$ of this system by taking the DTFT of both sides of the difference equation.

2. Plot the magnitude and phase response of this system versus the digital frequency variable $\Omega \in [-\pi, \pi]$

3. Calculate the impulse response $h[n]$ of this system by computing the inverse DTFT of $H(e^{j\Omega})$.

4. Is this system BIBO stable? is it causal? justify your answer properly.

Problem # 2.0

Compute the spectrum of the following signals using Fourier transform tables:

1. $x(t) = \frac{1}{1+t^2}, \quad t \in \mathbf{R}$

2. $x(t) = \frac{1}{\pi t}, \quad t \in \mathbf{R}$

3. $x(t) = \exp(-|\tau|) \cos \omega_o t$

In each case, specify the properties that you are using to calculate spectrum.

Problem # 3.0

A unit amplitude, 50% duty-cycle, rectangular pulse train $x(t)$ is the input to lowpass R-C filter with output $y(t)$. For this system:

1. What are Fourier series coefficients of the output signal $y(t)$.

2. Compute the spectrum $Y(j\omega)$ of the output signal $y(t)$.

3. How would the results change, if the input was a unit amplitude triangular pulse train with the same duty-cycle?