PS #1 , Spring 2012
Digital Signal Processing, ECE-539
Instructor: Balu Santhanam
Date Assigned: 01/25/202
Date Due: 02/01/2012

Problem # 1.0

In class we looked at the Haar wavelet and the scaling function defined by:

$$\phi(t) = \begin{cases} 1 & t \in (0,1) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\psi(t) = \begin{cases} 1 & t \in (0,0.5) \\ -1 & t \in (0.5,1) \end{cases}$$

In this problem we will look at the frequency content of the Haar wavelet and scaling function.

1. Calculate the Fourier transform of the Haar wavelet and scaling function using properties of the Fourier transform.
2. Plot the magnitude of the Fourier transform of both and comment on the frequency content of both.
3. Show that the scaling function and the wavelet satisfy the following time-domain multiscale relation:

$$\begin{aligned} \phi(t) &= \phi(2t) + \phi(2t-1), \\ \psi(t) &= \psi(2t) - \psi(2t-1). \end{aligned}$$

4. Take the Fourier transform on both sides of the result from the previous part to develop a multiscale relation in the frequency domain.

Problem # 2.0

Consider the following sequences of functions:

$$\begin{aligned} \phi_n(x) &= \sin(nx), \quad n = 0, 1, 2, \ldots \infty \\ \psi_m(x) &= \cos(mx), \quad m = 0, 1, 2, \ldots \infty. \end{aligned}$$

Show that: (a) these sequences are both individually orthogonal sequences of functions by calculating the inner product between distinct elements of the sequence, and (b) that the sequences are also mutually orthogonal.

Problem # 3.0

Consider the sequence of polynomials given by:

$$x_i(t) = t^i, \quad i = 0, 1, 2, \ldots \infty.$$

In this problem we will explore the connection between the Gram-Schmidt algorithm described in class and standard orthogonal polynomials.

1. Using the Gram-Schmidt algorithm discussed in the class determine a class of orthogonal polynomials using the weight function $w(t) = 1$ and the weighted inner product:

   $$< x_i(t), x_j(t) >= \int_a^b w(t) x_i(t) x_j(t) dt,$$

   with $a = -1, b = 1$. Can you relate this to a standard system of polynomials.

2. Repeat the procedure using $a = 0, b = \infty$ and the weight function $w(t) = e^{-t}$. Can you relate this set of orthogonal polynomials to a standard set of polynomials.