Midterm Exam, Fall 2010

Probability Theory and Stochastic Processes, ECE-541

University of New Mexico

Instructor: Balu Santhanam

Date Assigned: 10/25/2010, 3:30 PM

Date Due: 10/26/2010, 3:30 PM in class

Instructions

1. Write clearly and legibly. Chicken scratch will promptly and seriously be discarded as hazardous to the instructor.
2. Provide steps and show your working to obtain partial credit.
3. Make use of Fourier transform or Z transform tables as needed. You are allowed to use MATLAB where needed.
4. It is assumed that you are aware of the UNM academic honesty policy. Needless to say copying will be dealt with seriously.

Problem # 1.0 (10 points)

Suppose $X_n(\omega), n \in \mathcal{I}$ and $Y_n(\omega), n \in \mathcal{I}$ are two sequences of random variables defined on $(\Omega, \mathcal{F}, P)$ and they converge to the random variables $X(\omega)$ and $Y(\omega)$ in probability. With this arrangement:

1. Show that the sequence $X_n(\omega) + Y_n(\omega)$ converges in probability to the random variable $X(\omega) + Y(\omega)$.

2. Show that the sequence $cX_n(\omega)$ converges in probability to the random variable $cX(\omega)$.

3. If $g(x)$ is a uniformly continuous function then show that $g(X_n(\omega))$ converges in probability to the random variable $g(X(\omega))$.

4. Show that the sequence $g(X_n(\omega)) + g(Y_n(\omega))$ converges in probability to the random variable $g(X(\omega)) + g(Y(\omega))$.

Problem # 2.0 (10 points)

Consider a causal and stable discrete-time LTI system whose system function is given by:

$$H(z) = \frac{1}{(1 - az^{-1})(1 - bz^{-1})}, \quad a > 0, b > 0, a \neq b.$$

The input to this system is a zero-mean, unit variance, white noise sequence $X[n]$. The corresponding output of the system for this input is the random sequence $Y[n]$. For this environment:

1. calculate the quantities $r_{yx}[k]$ and $r_{yy}[k]$,
2. calculate the power spectral densities $P_{yx}(e^{j\omega})$ and $P_{yy}(e^{j\omega})$. Use of DTFT tables as you see fit.
3. using analytic continuation calculate $P_{yy}(z)$ and its corresponding *region of convergence* (ROC).

Problem # 3.0 (10 points)

Show that if $X(t)$ is a mean-square differentiable stochastic process then it is also mean-square continuous. Also show that if the stochastic process $X(t)$ is mean-square continuous then it is also mean-squared integrable.

Problem # 4.0 (10 points)

A stochastic process $X(t)$ is defined through:

$$X(t) = \exp\left(-|At|^2\right),$$

where A is uniformly distributed on $[-1, 1]$. For this process:

1. draw a few of the sample or member functions.
2. Calculate the mean $\mu_x(t)$ and the variance $\sigma_x^2(t)$.
3. Calculate the ACF $R_{xx}(t_1, t_2)$ and the normalized correlation $\rho_x(t_1, t_2)$.
4. What forms of stationarity apply to this process? Justify your answer properly.