Algebraic Separation Applied to Concurrent Vowel Separation and ECG Signal Separation

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Abstract

Separation of a mixture of periodic or quasi-periodic signals into its constituent components is an important problem that finds applications in sound detection, biomedical signal processing where it appears in the fetal ECG separation problem, spectrum estimation where it finds application in sinusoidal frequency estimation, communications where it finds application in the interference cancelation area, and speech processing, where it finds application in the area of concurrent vowel and speech separation. In this paper, the recently introduced algebraic separation approach is applied to the problem of separating mixtures of real and synthetic vowels and mixtures of ECG signals.

1. Introduction

Separating an additive mixture of periodic or quasi-periodic signals into its constituent components, where one/both components contain useful information, hereafter referred to as the *separation of periodic mixtures* (SPM) problem, is an important signal processing and detection task encountered: (a) when dealing with the recovery of multiple sinusoids in noise, (b) in biomedical signal processing problems such as separating a fetal ECG signal from a composite ECG signal which also contains the maternal ECG signal [3], (c) when dealing with interference rejection in narrowband communication systems [5, 6], and (d) in the area of concurrent vowel and speech separation [1].

Matrix algebraic separation (MAS) of periodic signal mixtures was introduced in [4], where the components of an additive mixture of two periodic signals were separated using their periodicity and samples of the composite signal in a matrix framework. In [5, 6], the MAS algorithm was extended to more than two components and was used in conjunction with the *periodic algebraic separation energy demodulation* (PASED) algorithm for the separation and demodulation of cochannel multicomponent AM–FM signals. In [5], this MAS algorithm was analyzed in the frequency-domain where extra matrix constraints were appended to the basic separation system to facilitate harmonic reassignment at the common harmonics. In this paper, we apply the MAS approach to the problem of separating a mixture of synthetic and real vowels and to the problem of separating two ECG signals [3].

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2. Separation of Periodic Mixtures

The basic idea underlying the MAS algorithm is the modeling of the components of the sum $x[n]$ as periodic signals with fundamental period N_i:

$$x_i[n] = x_i[n+N_i] \iff x_i[n] - x_i[n-N_i] = 0. \quad (1)$$

Component periodicity implies a discrete spectrum with impulses at multiples of $2\pi/N_i$:

$$X_i(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} a_{im}\delta\left(\Omega - \frac{2\pi m}{N_i}\right) \quad , \quad i = 1,2.$$

Since $x[n]$ is also periodic, with a repetition period $P = \text{lcm}(N_1, N_2)$:

$$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} b_m\delta\left(\Omega - \frac{2\pi m}{P}\right).$$

The component separation task is therefore equivalent to the task of obtaining the sequences $\{a_{im}\}$ from the sequence $\{b_m\}$, i.e., reassignment of the composite signal harmonics to the components. If $R = \gcd(N_1, N_2) = 1$, then the harmonics of the component fundamental frequencies are

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distinct[1]. When $R > 1$, the components share harmonics and confusion arises regarding spectral reassignment at :

$$\Omega_k = \left(\frac{2\pi}{R}\right) k\ ,\ k = 0\ ,\ 1\ ,\ \ldots\ ,\ (R-1). \qquad (2)$$

At $\Omega = \Omega_k$, the component spectral impulse amplitudes are superimposed:

$$A_{1k} + A_{2k} = B_k,\ k = 0, 1, \ldots, (R-1), \qquad (3)$$

where $\{A_{ik}\}$ and $\{B_k\}$ are the spectral impulse amplitude sequences of $x_i[n]$ and $x[n]$ at $\Omega = \Omega_k$. This superposition results in a loss of information at these common frequencies. The four basic spectral reassignment options are [5]: (a) Give it entirely to $x_2[n]$ (**OP1**), (b) Give it entirely to $x_1[n]$ (**OP2**), (c) To both components (**OP3**), (d) To neither of the components (**OP4**):

3. The MAS Algorithm

Consider a two-component periodic signal $x[n]$, where the component fundamental periods are N_1 and N_2 samples, respectively. Relating N samples of the composite signal to the samples of the components yields:

$$\underbrace{\begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} x_1[0] \\ \vdots \\ x_1[N_1-1] \\ x_2[0] \\ \vdots \\ x_2[N_2-1] \end{pmatrix}}_{\mathbf{z}},$$

where $\mathbf{I}_{N_1}$ denotes the identity matrix of order N_1 and the rank of the separation matrix $\mathbf{S}$ is $r(\mathbf{S}) = N_1 + N_2 - R$ [4]. This information deficiency at $\Omega = \Omega_k$ translates into the rank deficiency of $\mathbf{S}$. The extra equation/constraint needed to complete this system when $R = 1$ is:

$$\sum_{n=0}^{N_1-1} x_1[n] = 0. \qquad (4)$$

When $R = \gcd(N_1, N_2) > 1$ the constraints[2] needed are obtained via:

$$\sum_{j=0}^{(N_1/R)-1} x_1[Rj + i] = 0\ ,\ i = 0, 1, \ldots, (R-1). \qquad (5)$$

The components are then obtained as the least-squares solution to the *augmented separation system*:

$$\underbrace{\begin{pmatrix} \mathbf{S} \\ \mathbf{C} \end{pmatrix}}_{\tilde{\mathbf{S}}} \mathbf{z} = \underbrace{\begin{pmatrix} \mathbf{x} \\ \mathbf{0} \end{pmatrix}}_{\tilde{\mathbf{x}}}, \qquad (6)$$

where the homogeneous dc value constraints at the scale of R form the constraint matrix $\mathbf{C}$. The solution to this augmented system is of the form [6] :

$$\hat{\mathbf{z}} = \left(\mathbf{S}^T\mathbf{S} + \mathbf{C}^T\mathbf{C}\right)^{-1} \mathbf{S}^T\mathbf{x}. \qquad (7)$$

The component periods are estimated using the *double difference function* (DDF) algorithm [6], i.e., by finding integers $\hat{N}_1, \hat{N}_2$ that minimize the mean absolute error

$$\mathrm{DDF}[n, \hat{N}_1, \hat{N}_2] = \sum_{m=0}^{L-1} |x[n+m] - x[n+m+\hat{N}_1]$$
$$-x[n+m+\hat{N}_2] + x[n+m+\hat{N}_1+\hat{N}_2]|, \qquad (8)$$

where L is the duration and n is the origin of the analysis frame. The modular nature of the DDF algorithm allows easy extension to more components.

4. Common Harmonic Reassignment

The constraints in Eq. (4), (5) can be written as:

$$\mathbf{C}_1\mathbf{x}_1 = 0 \text{ or } \mathbf{C}_2\mathbf{x}_2 = 0, \qquad (9)$$

where $\{\mathbf{C}_i\}_{lm} = \delta[(m-l)_{\mathrm{mod}\ R}]$ and $\delta[n]$ is the discrete-time unit pulse function which has unit amplitude for $n = 0$ and is zero elsewhere. Multiplying Eq. (9) by the DFT matrix $\mathbf{W}_R$, where $\{\mathbf{W}_R\}_{lm} = \exp\left(-j\frac{2\pi lm}{R}\right)$ we have [5]:

$$\{\mathbf{W}_R\mathbf{C}_i\}_{lm} = \exp\left(-j\frac{2\pi lm}{R}\right). \qquad (10)$$

The constraints on $x_i[n]$ are then equivalent to:

$$\mathbf{W}_R\mathbf{C}_i\mathbf{x}_i = X_i(\Omega_k) = 0. \qquad (11)$$

So the time-domain constraints in Eq. (5) are equivalent to forcing spectral nulls on $x_i[n]$ at the common harmonics corresponding to options **OP1** and **OP2**.

Distributive spectral reassignment at the common harmonics between the components can be accomplished via:

$$\mathbf{W}_R\mathbf{C}_1\mathbf{x}_1 = \lambda\mathbf{W}_R\mathbf{C}_2\mathbf{x}_2 \leftrightarrow X_1(\Omega_k) = \lambda X_2(\Omega_k). \qquad (12)$$

This option corresponds to identical spectral reassignment at the common harmonics using a single ratio parameter λ.

The fourth option corresponds to the case where the constraints are applied on both components

$$X_1(\Omega_k) = X_2(\Omega_k) = X(\Omega_k) = 0 \qquad (13)$$

corresponding to the reassignment option **OP4**.

[1]For component periodicities of N_1, N_2, $N = \mathrm{lcm}(N_1, N_2)$ is a repetition period of the sum but not necessarily the fundamental period. The dc frequency, $\Omega = 0$, is always a shared harmonic between the components for any R.

[2]Downsampling the components by the factor R transforms them into components with coprime periods.

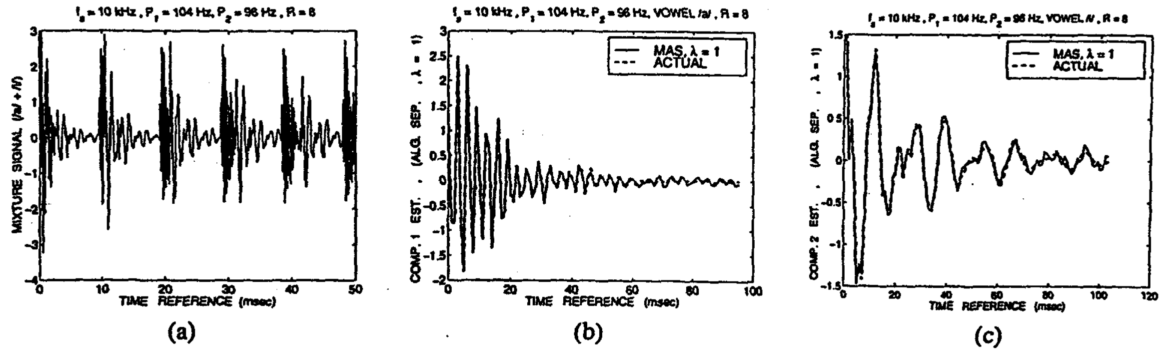

Figure 1. Synthetic vowel separation: Mixture of synthetic vowels /a/ & /i/. The formant frequencies and bandwidths of the vowels are adopted from [7]. The pitch periods of the vowels are $N_1 = 96, N_2 = 104$ samples respectively. (a) Composite signal, (b,c) MAS algorithm estimates of the vowels.

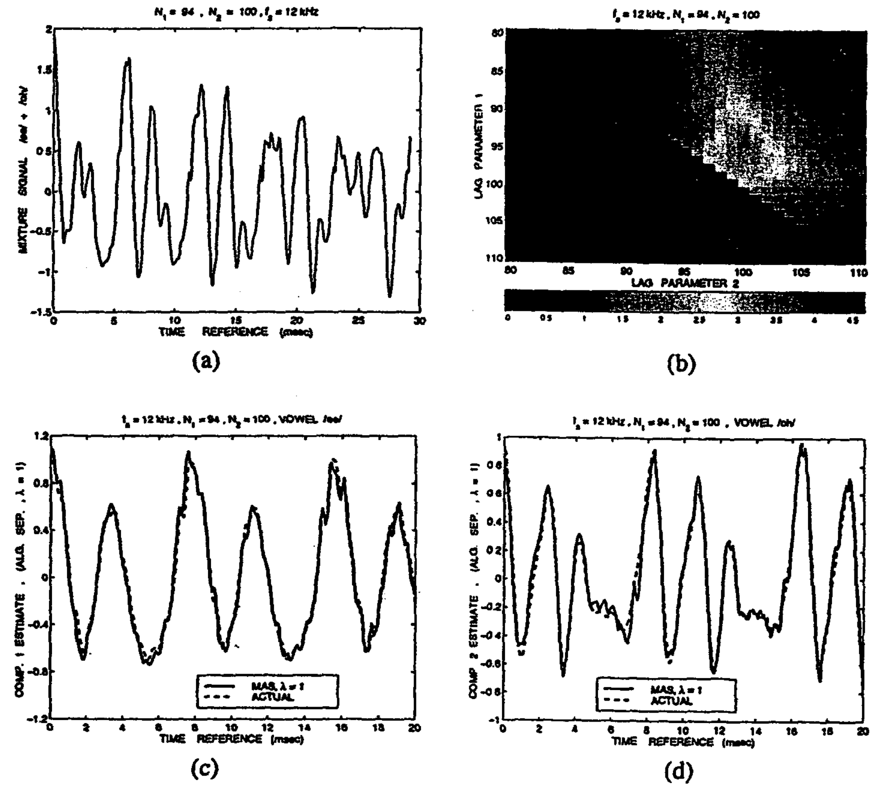

Figure 2. Real vowel separation: Mixture of quasi-periodic segments of real vowels /ee/ & /oh/. The segments are approximately periodic with: $N_1 = 94, N_2 = 100$ samples respectively. (a) Composite signal, (b) DDF period estimation, (c,d) MAS algorithm estimates of the components.

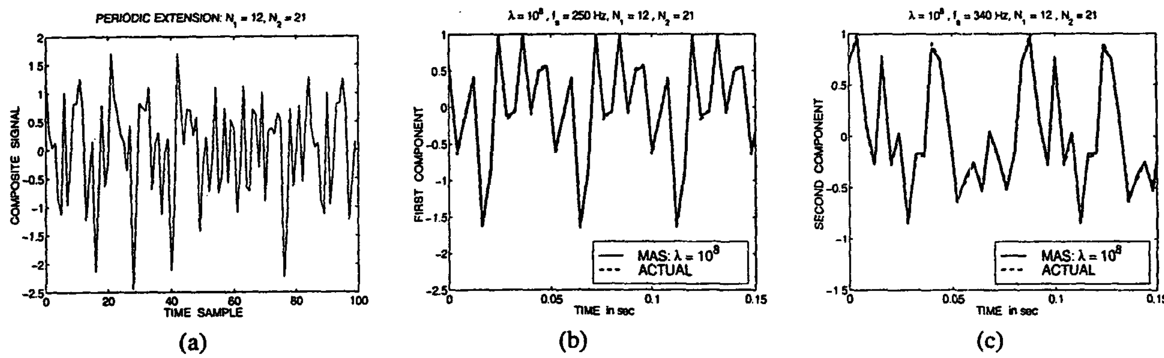

Figure 3. ECG separation: Mixture of periodically extended segments of two ECG signals. The extension periods of the segments are : $N_1 = 12, N_2 = 21$ samples. (a) Composite signal, (b,c) MAS algorithm estimates of the ECG signals.

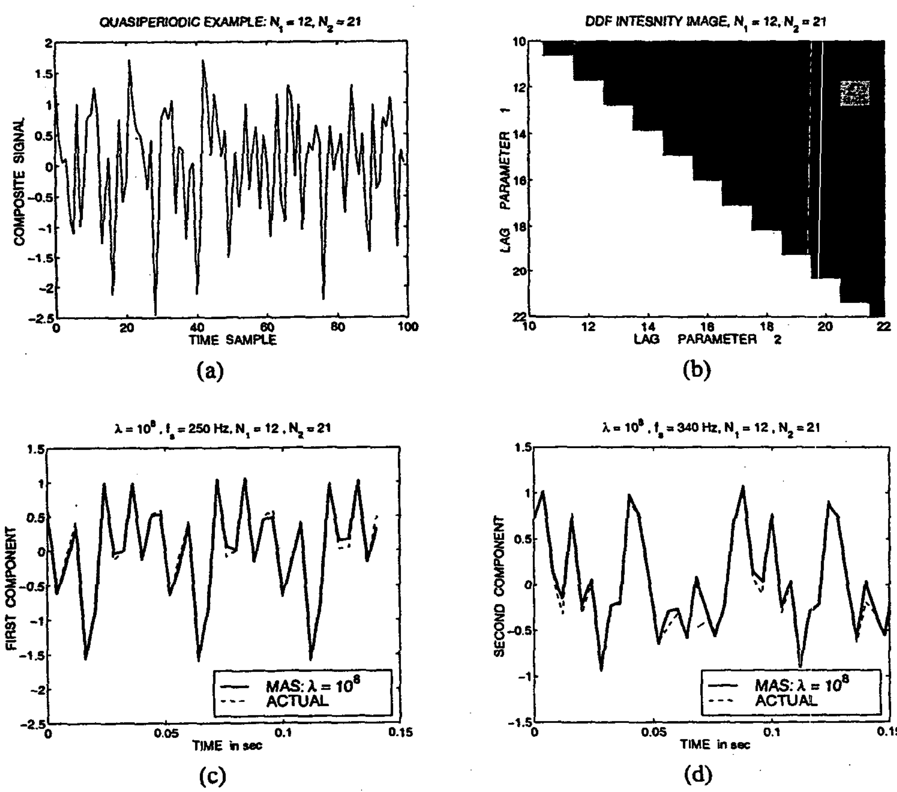

Figure 4. Separation of quasi-periodic segments of two ECG signals: (a) the composite signal, (b) periods of $N_1 = 12$ and $N_2 = 21$ samples estimated via the DDF algorithm, (c,d) MAS component estimates.

5. Experimental Results

5.1. Experiments with Synthetic Vowels

In this section, the MAS algorithm described in the earlier section was applied to the problem of separating a mixture of two synthetic vowels: /a/, /i/. The formant frequencies and bandwidths are adopted from [7]. The sampling frequency for each of the vowels is 10 kHz. The pitch frequencies of the vowels are : $P_1 = 104$ Hz, $P_2 = 96$ Hz. The corresponding pitch periods are 96, 104 samples respectively with the components of the sum sharing $R = 8$ harmonics. The composite signal of the example is shown in Fig. 1(a). The component estimates via the MAS algorithm ($\lambda = 1$) are shown in Fig. 1(c,d) with RMS component separation errors of 9, 11%.

5.2. Experiments with Real Vowels

In this section, the MAS algorithm was applied to the problem of separating quasi-periodic segments of two real vowels. The vowel segment /ee/ was obtained via segmentation from the TIMIT stressed speech database file: `steer.pcm`. The vowel /oh/ was obtained from the same database from the file `oh.pcm`. The sampling frequency was $f_s = 12$ kHz. The composite signal for the example is described in Fig. 2(a) and was obtained by mixing quasi-periodic segments of the vowels with periods of $N_1 = 94, N_2 = 100$ samples. The DDF intensity image used for estimating the periods is shown in Fig. 2(b)). The estimated components are shown in Fig. 2(c,d) with RMS component separation errors of 11, 13%.

5.3. Results with ECG Signals: Periodic Extension

In this section, the MAS algorithm was applied to a mixture of periodically extended segments of ECG signals. The ECG signals were obtained from samples of the MIT-BIH polysomnographic, MGH/MF ECG databases available at `http://ecg.mit.edu/dbsamples.html`. The signal from the MIT-BIH database has a sampling frequency of $f_s = 340$ Hz and the signal from the MGH/MF database has a sampling frequency of $f_s = 250$Hz. The extension periods of the signals were $N_1 = 12$ and $N_2 = 21$ samples respectively with the components sharing $R = 3$ harmonics. The composite signal for the example is shown in Fig.(3)(a). The separated components along with the originals are shown in Fig.(3)(b,c) with RMS component separation errors are 5, 7%.

5.4. Results with Quasi-periodic ECG Signals

In this section, the MAS algorithm was applied to a mixture of quasi-periodic segments of the ECG data obtained from the signals in the previous section. The ECG data used in this example is approximately periodic with periods $N_1 = 12$ and $N_2 = 21$ samples which were estimated using the DDF algorithm. The composite signal for the example is shown in Fig. (4)(a). The DDF intensity image used to estimate the periods is shown in Fig. (4)(b). The component estimates ($\lambda = 10^8$) are shown in Fig. (4)(c,d) with RMS component separation errors are 10, 13%.

5.5. Conclusions and Future Work

In this paper, the recently proposed MAS algorithm was applied to the problem of separating mixtures of concurrent vowels and ECG signals. The MAS algorithm is able to effectively accomplish component separation via the restoration options even when there is significant harmonic overlap. Future research directions include development of fast algorithms for least-squares inversion exploiting the sparse and binary structure of the separation matrix, incorporating better quasi-periodicity models into the MAS method, development of faster methods for period estimation, component enumeration/detection.

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