Adaptive Linear Predictive Frequency Tracking and CPM Demodulation

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***Abstract*- CPM signals find widespread use in wireless communication systems due to their constant modulus property and spectral efficiency. Frequency discrimination based CPM demodulation approaches require efficient instantaneous frequency tracking. Prior attempts at frequency tracking via adaptive linear prediction have invoked the use of the fixed step-size LMS algorithm. In this paper, we present an efficient algorithm that combines aspects of adaptive linear prediction, frequency tracking and frequency transformations based on multirate operations for CPM demodulation using both the adaptive step-size LMS and adaptive forgetting factor RLS algorithms. Simulation results indicate that these algorithms offer a significant reduction in the associated demodulation errors over the conventional LMS/RLS algorithms.**

I. Introduction

Continuous phase modulation (CPM) belongs to a class of non linearly modulated signals with constant envelope, where the information is carried in the phase of the transmitted signal. High spectral efficiency and suitability to non linear class C amplifiers used in mobile radio applications make CPM a popular modulation choice. A specific form of CPM namely Gaussian minimum shift keying (GMSK) has been adopted in the *Global System for Mobile communications* (GSM) [3], [2]. The optimum receiver structure for CPM demodulation employs the Maximum Likelihood (ML) detector based on the Viterbi algorithm [1]. This receiver structure, however has significant computational complexity which grows exponentially with increase in the number of phase states. A simpler suboptimal detector based on differential frequency estimation, decision feedback and correlation operations was proposed in [4]. In recent work [7], CPM signals and associated digital modulation schemes were cast into the framework of AM-FM signal models and a suboptimal approach that uses energy demodulation methods was proposed. Further work into the demodulation of large frequency deviation FM signals or wideband FM signals was recently explored in [8], where frequency transformations derived from multirate operations and heterodyning were shown to produce significant reduction in the associated frequency demodulation errors.

Prior attempts at adaptive linear prediction based instantaneous frequency tracking have typically relied on the conventional LMS algorithm for tracking the *instantaneous frequency* (IF) of digital signals with narrow-band, rapidly time varying spectrum [9]. Efforts to directly track the frequency of a sinusoidal signal via the LMS algorithm have also been recently pursued in [10]. For the IF tracking application, specifically, the choice of the step size parameter is critical, and in turn depends on the rate of variation of the statistics of the input signal and the ambient channel noise both of which could exhibit significant variations in a dynamic SNR environment.

In this paper, we combine aspects of adaptive linear prediction based IF tracking and wideband to narrowband frequency transformations [8] to develop a novel approach for CPM demodulation. Specifically, we apply the *adaptive step-size* based LMS (AS-LMS) algorithm and the *adaptive forgetting factor* based RLS (AF-RLS) algorithm [11], [6] that adapt the step size and memory parameters to enable more efficient tracking of the IF. Simulation results will show that this approach is more suited for IF tracking in a dynamic SNR environment and can provide significant reduction in the demodulation errors in comparison with the standard LMS and RLS algorithms towards the CPM demodulation problem.

II. CPM Signal Model

In general a CPM signal at time t can be expressed as

$$y(t) = A\cos\left(\int_{-\infty}^{t} \omega_i(\tau)d\tau \ + \theta_o\right).$$

where A is the amplitude of the transmitted signal and $\omega_i(\tau)$ and θ_o are the IF and unknown phase offset of the signal. $\omega_i(\tau)$ can further be represented as

$$\omega_i(t) = \omega_c + 2\pi h \sum_{k=-\infty}^{\infty} a[k]p(t - kT_b),$$

where ω_c is the carrier frequency, h is the modulation index, $a[k] \in \{+1, -1\}$ is the binary modulated data, $p(t)$ is some frequency shaping function and T_b is the signaling interval.

The phase deviation from the carrier phase is given by:

$$\phi_{\text{dev}}(t; \boldsymbol{a}) = 2\pi h \sum_{k=-\infty}^{\infty} a[k]q(t - kT_b),$$

where $q(t) = \int_0^t p(\tau)d\tau$ corresponds to the phase pulse shaping function that describes how the underlying phase change

$2\pi h a[k]$ evolves with time, the modulation index determines the rate of change of frequency in the signalling interval. Memory is introduced into the CPM signal by the virtue of the continuity of the phase, additional memory into the modulation scheme can be introduced by adopting frequency pulse of length L, larger than a symbol interval (LREC-CPM), i.e. partial response signalling. In this paper, we will focus our attention on the case with $L = 1$, i.e., (1REC-CPM), i.e. the full response signalling. It is however noted that all the CPM schemes are partial response when viewed as phase modulated signal because of the infinite duration of the phase pulse. Specifically CPM with a rectangular pulse of one symbol duration (1-REC-CPM) is equivalent to continuous phase *frequency shift keying* (CPFSK). MSK is equivalent to 1-REC-CPM with a modulation index of $h = 0.5$, while GMSK can also be put into the CPM framework with a Gaussian frequency pulse shaping function [1]. In this paper CPM signals with a modulation index $h > 1$ will be classified under large deviations CPM signal.

III. Adaptive Linear Predictive IF tracking

The optimal values of the coefficients in a linear predictor $\{g_i\}_{i=1}^{L_p}$ are obtained via the Wiener-Hopf equations [6]:

$$\boldsymbol{G}^{\text{opt}} = \boldsymbol{R}_{xx}^{-1}\boldsymbol{P}_x,$$

where $\boldsymbol{R}_{xx}$ is the data correlation matrix, $\boldsymbol{P}_x$ is the cross-correlation vector and $\boldsymbol{G}^{\text{opt}}$ is the optimal weight vector. The prediction error filter corresponding to this optimal predictor is given by:

$$E(z) = 1 - \sum_{i=1}^{L_p} g_i z^{-i}$$

The IF of the signal of interest is then estimated by first computing the coefficients of the instantaneous prediction error filter, rooting the instantaneous prediction error polynomial and then computing the argument of the complex conjugate pole locations as described in [9], where the standard LMS algorithm was used to update the predictor coefficients.

One of the goals in this paper is to incorporate the use of the AS-LMS and the AF-RLS algorithms into this adaptive linear prediction framework because they are more suited for a dynamic SNR environment and the IF tracking/CPM demodulation application than the conventional LMS/RLS algorithms. The AS-LMS algorithm for the adaptive linear predictor coefficients is summarized via [6]:

$$\begin{aligned}
\boldsymbol{G}_{n+1} &= \boldsymbol{G}_n + \mu_n \boldsymbol{x}(n) f_L^*(n) \\
f_L(n) &= x(n) - \sum_{i=1}^{L} g_{n,i} x(n-i) \\
\mu_{n+1} &= \left[\mu_n + \alpha \Re[\boldsymbol{\Psi}^H(n)\boldsymbol{x}(n) f_L^*(n)]\right]_{\mu_-}^{\mu_+} \\
\boldsymbol{\Psi}(n+1) &= \left[\boldsymbol{I} - \mu_n \boldsymbol{x}(n)\boldsymbol{x}^H(n)\right]\boldsymbol{\Psi}(n) + \boldsymbol{x}(n) f_L^*(n),
\end{aligned}$$

where $\Re$ denotes the real part, $\alpha > 0$ is a small number representing the learning rate of the step size adaptation, $\boldsymbol{\Psi}^H(n)$ denotes the gradient of the weight vector with respect to the step size,

$$\boldsymbol{\Psi}(n) = \frac{\partial \boldsymbol{G}_n}{\partial \mu}|_{\mu=\mu_n}$$

and the notation $\mu \in [\mu_-, \mu_+]$ denotes truncation of the step size to this interval, properly chosen in order to prevent divergence. It is shown in [11] that μ_- plays a relatively insensitive role in the convergence of the step size, whereas μ_+, the upper level of truncation is highly crucial for good convergence behavior.

In a similar vein, the standard RLS algorithm can be generalized to incorporate adaptive memory via the AF-RLS algorithm: [6]:

$$\begin{aligned}
\boldsymbol{k}(n) &= \frac{\lambda_{n-1}^{-1}\boldsymbol{P}(n-1)\boldsymbol{x}(n)}{1 + \lambda_{n-1}^{-1}\boldsymbol{x}^H(n)\boldsymbol{P}(n-1)\boldsymbol{x}(n)} \\
f_L(n) &= x(n) - \sum_{i=1}^{L} g_{n,i} x(n-i) \\
\boldsymbol{G}_n &= \boldsymbol{G}_{n-1} + \boldsymbol{k}(n) f_L^*(n) \\
\boldsymbol{P}(n) &= \lambda_{n-1}^{-1}\boldsymbol{P}(n-1) - \lambda_{n-1}^{-1}\boldsymbol{k}(n)\boldsymbol{x}^H(n)\boldsymbol{P}(n-1) \\
\lambda_n &= \left[\lambda_{n-1} + \alpha \Re[\boldsymbol{\Psi}^H(n-1)\boldsymbol{x}(n) f_L^*(n)]\right]_{\lambda_-}^{\lambda_+} \\
\boldsymbol{S}(n) &= \lambda_n^{-1}\left[I - \boldsymbol{k}(n)\boldsymbol{x}^H(n)\right]\boldsymbol{S}(n-1)\left[I - \boldsymbol{x}(n)\boldsymbol{k}(n)\right] \\
&+ \lambda_n^{-1}\boldsymbol{k}(n)\boldsymbol{k}^H(n) - \lambda_n^{-1}\boldsymbol{P}(n) \\
\boldsymbol{\Psi}(n) &= \left[I - \boldsymbol{k}(n)\boldsymbol{x}^H(n)\right]\boldsymbol{\Psi}(n-1) + \boldsymbol{S}(n)\boldsymbol{x}(n) f_L^*(n),
\end{aligned}$$

where $\boldsymbol{S}(n)$ denotes the gradient of the inverse matrix $\boldsymbol{P}(n)$ with respect to λ:

$$\boldsymbol{S}(n) = \frac{\partial \boldsymbol{P}(n)}{\partial \lambda},$$

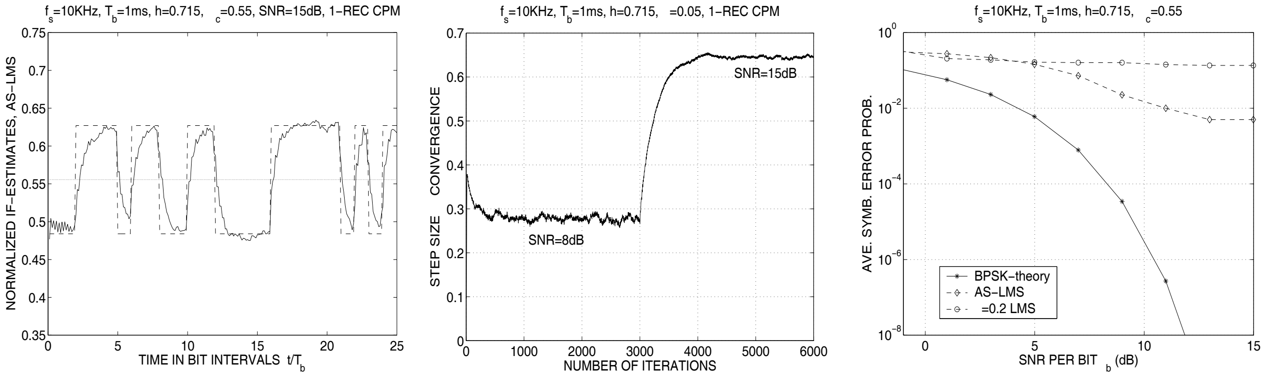
$\boldsymbol{\Psi}(n)$ denotes the gradient of the weight vector with respect to λ:

$$\boldsymbol{\Psi}(n) = \frac{\partial \boldsymbol{G}}{\partial \lambda}|_{\lambda=\lambda_n}$$

and α is the learning rate associated with the forgetting factor update. Similar to the AS-LMS approach, we truncate the forgetting factor to the interval $\lambda \in [\lambda_-, \lambda_+]$. As noted in [11], the lower limit of the truncation λ_- plays a more important role and the value has to be determined through experimentation. For small modulation indices, the CPM signal is narrowband and the IF's are slowly time-varying signals that can be smoothed using simple median and binomial filtering to remove spikes and noise.

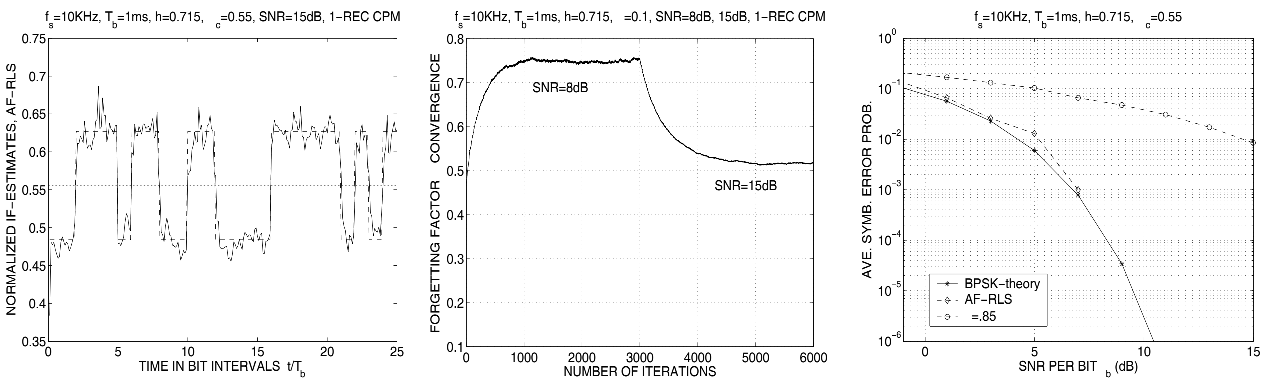
IV. Wideband to Narrowband Conversion

The adaptive linear prediction based IF tracking approaches described in the previous sections are based on the assumption that the signal of interest has narrowband spectral content. For the demodulation of large deviation CPM signals, these approaches will incur more error and this will result in a loss of tracking. Towards improving the tracking capabilities of these algorithms in wideband environments we employ



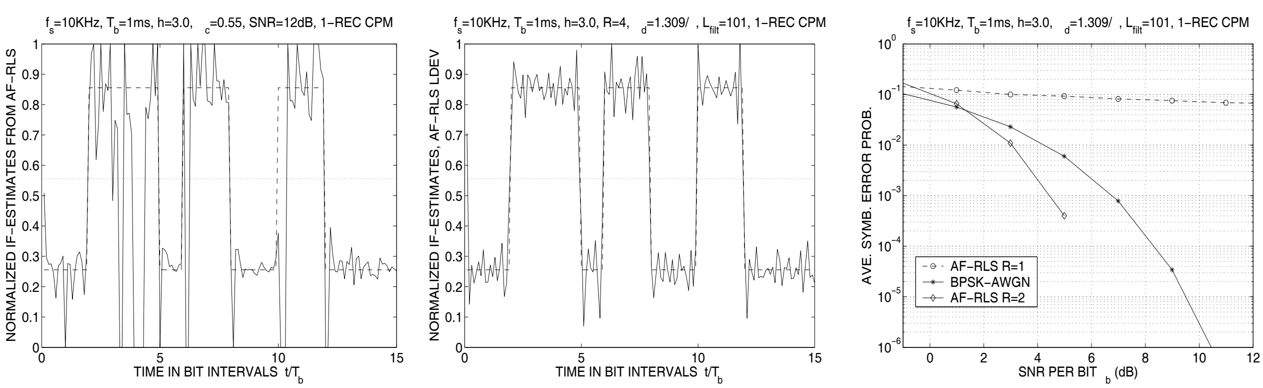
(a) (b) (c)

Fig. 1. CPM demodulation in AWGN with the AS-LMS algorithm: (a) normalized IF estimates derived from the linear predictive IF tracking in 1-REC CPM with the AS-LMS algorithm. (b) step size trajectory for 1-REC CPM, (c) performance comparison of CPM demodulation with fixed step size LMS to that of adaptive step size LMS.



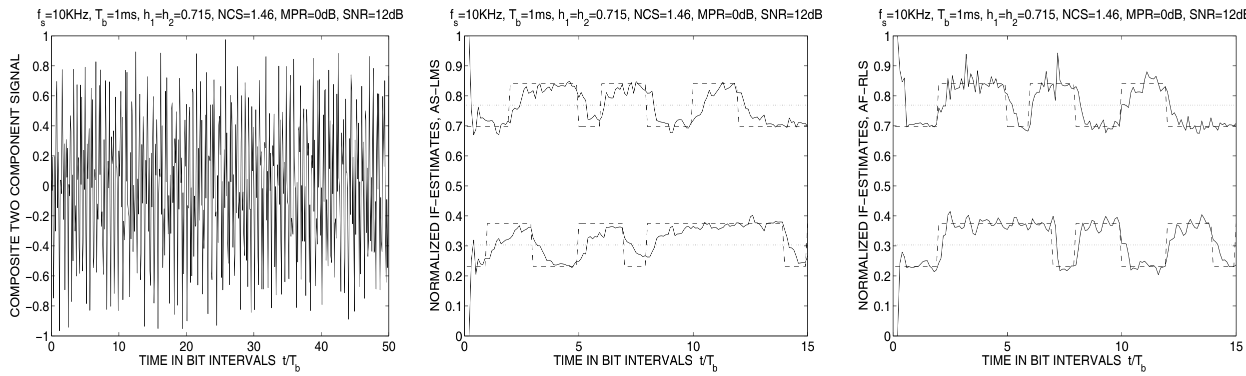
(a) (b) (c)

Fig. 2. CPM demodulation in AWGN with the AF-RLS algorithm: (a) normalized IF estimates derived from the linear predictive IF tracking for 1-REC CPM with the AF-RLS algorithm, (b) forgetting factor trajectory for 1-REC CPM, note that SNR is changed at iteration 3000, (c) performance comparison of fixed forgetting factor RLS to adaptive forgetting factor RLS.



(a) (b) (c)

Fig. 3. CPM demodulation in AWGN for large frequency deviations: (a) IF estimates for 1-REC-CPM without frequency transformations, (b) IF estimates for 1-REC CPM after frequency transformations, (c) effect of including frequency transformations on the average symbol error probability.



(a) (b) (c)

Fig. 4. Two Component CPFSK: (a) Composite CPFSK signal over 10-bit intervals, (b) normalized IF estimates derived from the AS-LMS algorithm, predictor order L_p=4, the estimates can further be smoothed using median smoothing filters, (c) corresponding normalized IF estimates derived from the AF-RLS algorithm, with a predictor order of L_p=4.

frequency transformations derived from multirate operations such as interpolation and decimation and heterodyning as described in [8]. Frequency compression by a factor R serves the purpose of reducing the frequency deviation and the message bandwidth of the original signal by a factor of R and compressing the IF while still retaining the continuous phase of the signal. Frequency upshifting or heterodyning by a factor ω_d serves the purpose of increasing the carrier frequency of the interpolated signal by ω_d so that the parameters of the signal are transformed to regimes where the conventional monocomponent demodulation algorithms perform well. Specifically the frequency compression/expansion operations are implemented in discrete–time via the multirate operations of interpolation and decimation. The decimation and interpolation operations are further implemented efficiently using a polyphase decomposition for the filters [5], [8]. These wideband to narrowband conversion operations in conjunction with the regular demodulation algorithm were shown to provide efficient noise shaping and a significant reduction of the normalized frequency demodulation errors [8].

V. CPM Demodulation Via Adaptive Frequency Tracking

The optimal demodulation approach for CPM signals is of-course the maximum likelihood approach as embodied in the Viterbi algorithm [2], but the computational complexity of this method in terms of the number of phase states is pM^{L-1}, where M is the alphabet size of $a[k]$ and L is the length of the frequency pulse. Our goal here is to demonstrate via simulations that the CPM demodulation scheme described before employing adaptive linear prediction based IF tracking combined with wideband to narrowband frequency transformations provides, albeit suboptimal, a computationally simpler approach to the CPM demodulation problem.

Consider the example in Fig. (1) where we apply the AS-LMS algorithm to the CPM demodulation problem. Fig. (1) (a) describes the normalized IF estimates of the AS-LMS algorithm indicating that the algorithm is able to track the IF of the input signal. Fig. (1) (b) describes the adaptation of the step size. Specifically it can be seen that with larger SNR the step size takes on a larger value allowing for faster convergence, whereas with lower SNR the step size assumes a lesser value. This automatic updating of the step size parameters removes the uncertainty involved with the selection of the optimum value of the parameter. As a figure of merit we choose the average *symbol error probability* (SEP) in the problem of CPM demodulation since for this application, our interest is in the capability to detect the correct bits. The detector used subtracts the carrier frequency estimate from the IF estimate and performs matched filtering with sign detection on the carrier unbiased IF estimate. We compare the average probability of symbol error averaged over 100 experiments obtained via the use of the AS-LMS with that of the detection error for binary antipodal modulation in AWGN as given by [1]:

$$\Pr(\epsilon) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right),$$

where $Q(.)$ is the standard normal tail probability[1], E_b is the energy per bit of the input signal and N_o is the noise spectral density. In Fig. (1) (c) we compare the SEP obtained from the fixed step size LMS based IF tracking algorithm to the AS-LMS algorithm. We observe a performance gain in the AS-LMS case, that is solely due to the better IF-tracking capability in a dynamic SNR environment that is afforded by the step size adaptation.

Consider the example in Fig. (2) where the AF-RLS algorithm has been applied to the CPM demodulation task. The IF estimates of the AF-RLS algorithm are described in Fig. (2) (a) and are descriptive of the superior IF tracking achieved by the AF-RLS algorithm and a significant improvement in the performance of AF-RLS over the AS-LMS algorithm. Fig. (2) (b) describes the trajectory of the forgetting factor for different

[1]This performance metric is used in an effort to study the efficiency of the demodulator in inverting the CPM modulation

SNR's. Note that the forgetting factor takes lesser value with larger SNR, i.e., the data in the distant past is weighted less to enable efficient tracking. In the presence of larger noise the forgetting factor increases and the past data more is weighted more since present data is noisy. In Fig. (2) (c) we evaluate the performance CPM demodulation with AF-RLS for 1-REC and 1-RAC CPM. Note that there is a significant performance gain while using the AF-RLS in comparison to the fixed memory RLS. Simulation results also indicate that after a SNR of 6-8 dB the algorithm completely inverts the effect of CPM modulation and there are no errors in the demodulation process.

Consider the example in Fig. 3 that illustrates the benefits of wideband to narrow frequency transformations, where the modulation index of the signal is $h = 3.0$. The IF estimates of the AF-RLS algorithm are described in Fig. (3) (a). The actual normalized IF occupies most of the entire interval $\omega \in [0, 1]$, indicating significant wideband content. Note that there is a significant loss of tracking that can be attributed to the large frequency deviation of this signal. The corresponding IF estimate of the AF-RLS algorithm using an rate change factor of $R = 4$ and $\omega_d = 1.309/\pi$ is described in Fig. (3) (b), where the frequency transformations have enabled the better tracking of the IF of the large deviation CPM signal. Fig. (3) (c) describes the dramatic effect that these frequency transformations have on the average SEP for a rate change factor of $R = 2$.

Consider a two-component CPM signal environment in Fig. (4), where the components are both 1-REC-CPM signals with modulation indices $h_1 = h_2 = 0.715$, T_b = 1 ms and f_s = 10 kHz, *normalized carrier separation* (NCS) parameter (carrier separation normalized by the average Carson bandwidth of the components) of 1.46 and a relative power ratio (MPR) of 0 dB. With this parameter setting, there is a significant amount of spectral overlap. Fig. (4) (a) depicts the composite CPFSK signal over 15 symbol periods. Fig. (4) (b) describes the IF estimates derived from the AS-LMS algorithm. Fig. (4) (c) depicts the IF estimates derived from the AF-RLS algorithm. It is observed that the AF-RLS is better in tracking and separating out the IF components in the composite CPFSK signal than the AS-LMS algorithm partly due to the relatively increased sensitivity of the LMS algorithm to the conditioning of the input correlation matrix which further deteriorates as the spectral overlap between the components increases [12] and partly due to the absence of gradient related noise problems that plague the LMS.

Another useful observation is the fact that the symbols are from a zero-mean constellation and the mean of the IF estimate:

$$\hat{\omega}_c = \frac{1}{P} \sum_{n=0}^{P-1} \hat{\omega}_i[n].$$

can serve as an estimate of the carrier frequency of the signal. This is useful in carrier frequency recovery applications and in scenarios where a simple Doppler shift present in the received signal will manifest itself as a non zero mean in the IF estimates of these algorithms that is eventually subtracted from the IF estimate during the detection process.

VI. Conclusions

In this paper, we have presented an efficient CPM demodulation approach that combines adaptive linear predictive IF tracking implemented in the form of the adaptive step size and adaptive forgetting factor LMS and RLS algorithms along with frequency transformations derived from multirate and heterodyne operations. The frequency transformations convert the wideband CPM signal into a narrowband version making it more amenable to IF tracking based demodulation. The adaptive step size and memory aspects allow for efficient tracking of the IF in a non stationary or a dynamic SNR environment making them more suitable for the CPM demodulation problem. Simulation results have shown that there is a significant reduction in the demodulation error in comparison to the fixed step-size/forgetting factor LMS/RLS algorithms. These algorithms are also robust to the presence of Doppler shifts in the received signals.

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