Prior ICA Based Blind Multiuser Detection in DS-CDMA Systems

Malay Gupta, Balu Santhanam
Department of Electrical and Computer Engineering
University of New Mexico
Albuquerque, New Mexico 87131
Email: malay@ece.unm.edu

***Abstract*- ICA based blind source separation methods suffer from inherent scaling and permutation ambiguities which would cause problems in a DS-CDMA multiuser detection and MAI mitigation scenario. Recently ICA methods incorporating prior information about the desired user's code were proposed as an add-on to the subspace MMSE detector. In this paper, we propose an Q-R decomposition based technique to initialize the ICA algorithm in order to remove the permutation ambiguity as well as to avoid the explicit subspace computations. It is shown via simulations that the proposed technique is more robust in the case of highly correlated unequal energy users.**

I. Introduction

Multiple access interference (MAI) constitutes a significant bottleneck in achieving the envisaged capacity of a *direct sequence-code division multiple access* (DS-CDMA) system. Inadequacy of the conventional detector to deal with MAI has motivated the development of optimum multiuser detector [1] and its suboptimal counterparts [2], [3], [4]. These detectors either require complete knowledge of the MAI [2], training data [3] or involve long decoding delays [4]. To overcome these limitations, a class of spectrally efficient blind detectors was proposed. Most of the blind detection techniques in wireless communication literature [5], [6], [7], [8], [9] are however, based on subspace computations that are computationally demanding and may not work well in highly loaded CDMA systems. It is also to be noted that most of the blind detection techniques listed above utilize only the *second order statistics* (SOS) of the received data.

Independent component analysis (ICA) is a blind source separation (BSS) technique [10] that involves the computation of *higher order statistics* (HOS). ICA based techniques assume non-Gaussianity and independence of the sources. These assumptions are vital for ICA algorithms to work. Fortunately, these conditions are generally met in a typical communications system. It is the non-Gaussianity and source independence assumption that facilitates the use of HOS in ICA based techniques over SOS based techniques. However, permutation and scaling ambiguity [11] associated with the ICA algorithms require special attention in a communications system setting.

Efforts towards eliminating the indeterminacy problems have recently been reported in [12], where this indeterminacy is eliminated on the basis of prior knowledge about the source kurtosis. In our recent work [13], we proposed a *code constrained ICA* (CC-ICA) algorithm based on subspace concepts for multiuser detection problem. Prior information about the desired user's signature code was utilized to constrain the progress of the ICA algorithm in order to remove the inherent permutation ambiguity. Recently a multiuser detector based on the ICA approach was introduced in [14], however due to permutation ambiguity problem the ICA part has been incorporated as an add on to the subspace based quasi-blind *minimum mean square error* (MMSE) detector. In this paper, we present an algorithm based on the orthogonal-triangular (Q-R) decomposition of the observations to avoid explicit subspace computations following the recently proposed prior ICA (Pr-ICA) approach in [15]. The resulting algorithm is very useful in a multiuser CDMA environment where prior information about the desired user's code is generally available with the receiver.

II. System Model

A CDMA channel is characterized by the fact that there is no separation between the users either in the frequency domain or in the time domain. The composite received signal in continuous time domain can be represented as

$$y(t) = \sum_{k=1}^{K} \sum_{i=-\infty}^{+\infty} A_k b_k(i) s_k(t - iT_s - \tau_k) + \sigma n(t), \qquad (1)$$

where T_s is the symbol time interval, $s_k(t)$ is the deterministic signature waveform assigned to the k^{th}user in the channel, A_k is the amplitude of the k^{th}user's signal, b_k(i) is the i^{th}data symbol transmitted by the k^{th}user, $n(t)$ is additive white Gaussian noise (AWGN) with unit power spectral density, σ^2 is the noise power spectral density, τ_k is the channel delay for the k^{th} user. In the above system model it is assumed that the data symbols are independent, identically distributed (i.i.d.) random variables. The signature waveform assumes the following form

$$s_k(t) = \sum_{n=0}^{N-1} c_k(n) p_k(t - nT_c), \qquad (2)$$

where N is the number of chips per symbol, $T_c = \frac{T_s}{N}$ is the chip interval, $c_k(n)$ is the n^{th}chip in the spreading code of the k^{th}user, $p_k(t)$ is the received chip waveform of the k^{th}user, filtered by the transmitter, receiver and the channel. Considering a symbol synchronous system, i.e. $\tau_1 = \tau_2 = ... = \tau_K = 0$, chip matched filtering and sampling of the received signal at the chip rate N/T_s, we obtain a length N vector $\mathbf{y}$. The matrix formulation of the composite signal in AWGN channel for a given signaling interval i is given as

$$\mathbf{y}(i) = \mathbf{SAb}(i) + \mathbf{n}(i), \tag{3}$$

where $\mathbf{S} = [\mathbf{s_1}, \mathbf{s_2}, ..., \mathbf{s_K}]$ is a $N \times K$ matrix of correlated user signature codes. $\mathbf{A} = \text{diag}\,[A_1, A_2, ..., A_K]$ is a $K \times K$ diagonal matrix of user amplitudes and $\mathbf{b}(i) = [b_1(i), b_2(i), ..., b_K(i)]^T$ is a K dimensional user data at time $t = i$. In the remaining sections we will assume that user 1 is the desired user and the receiver has perfect knowledge of it's signature code and timing.

III. Indeterminacy in ICA Solutions

In a CDMA channel, where the receiver observes the superposition of the signals due to all the active users, the situation is very similar to the *linear generative* signal model widely used in ICA literature, which for a AWGN channel is given as

$$\mathbf{y} = \mathbf{Sb} + \mathbf{n}, \tag{4}$$

where by hypothesis all the source cumulants are diagonal. In particular, the two point correlation between the user symbols at the same time is given as:

$$K^0_{i,j} \equiv \langle b_i(t) b_j(t) \rangle = \delta_{i,j} K^0_i, \tag{5}$$

where $\delta_{i,j}$ is the Kronecker delta and $\langle z_1, z_2, ..., z_k \rangle$ denotes the cumulant of the k random variables $z_1, z_2, ..., z_k$. Without loss of generality one can always assume that all the sources have zero means

$$\langle b_k \rangle = 0, \quad k = 1, ..., K. \tag{6}$$

If this is not the case, one has to estimate the mean values of each source and subtract it from that source. The above signal model is very similar to (3) with the difference of $\mathbf{S}$ in place of $\mathbf{SA}$. In ICA literature, the unknown matrix $\mathbf{S}$ is generally termed as the *mixing matrix* and the components of the vector $\mathbf{b}$ are the unknown sources. Given T realizations of $\mathbf{y}$ it is desired to blindly estimate both the matrix $\mathbf{S}$ and the corresponding realizations of $\mathbf{b}$. In other words no other information apart from the observations $\mathbf{y}$ is assumed in ICA computations. An important distinction of ICA methods with other blind identification methods is that no particular structure is attributed to the matrix $\mathbf{S}$. ICA has traditionally been used in blind source separation problems such as the *cocktail party* problem, where the signal from a room full of speakers all talking at once is observed by a set of microphones. Separation of these unknown speakers from the observed data alone is precisely the task of ICA. In these applications there is an inherent ambiguity up to a scaling and permutation. Permutation ambiguity refers to the order of the speakers, i.e. which speaker's voice comes first. The scaling ambiguity refers to the case where the separated sources differ from the original sources by a scaling factor. If we denote the true values of $\mathbf{S}$ and $\mathbf{b}$ as $\mathbf{S}_0$ and $\mathbf{b_0}$, then it can easily be seen that $\mathbf{S}_0\mathbf{M}$ and $\mathbf{M^{-1}b_0}$ will also produce the same observations as in (4). $\mathbf{M}$ could be any $K \times K$ non-singular matrix. The structure of $\mathbf{M}$ is further specified by the waveform preserving relations [16] to be

$$\mathbf{M} = \mathbf{\Lambda^{-1}P}, \tag{7}$$

where $\mathbf{P}$ is a permutation matrix and $\mathbf{\Lambda}$ is a non-singular diagonal matrix. The scaling ambiguity is generally taken care of by constraining the column vector norm of the mixing matrix to be unity. However, the multiuser detection problem in communication applications does not allow for these ambiguities. Specifically, permutation ambiguity in a communications application translates into loss of control over the index of the user whose symbols are to be estimated.

IV. Prior ICA based Multiuser Detection

As noted in [15], the permutation ambiguity in ICA solution may be removed by initializing the ICA computations by some appropriately chosen basis vector. We propose the use of the prior information about the signature code of the desired user to obtain useful initialization for multiuser detection problem. For this purpose, we notice that any $N \times K$ matrix $\mathbf{S}$ can be decomposed as a product of a triangular matrix $\mathbf{L^{-1}}$ and an orthogonal (unitary) matrix $\mathbf{Q}$. This gives us the following relation

$$\mathbf{Q} = \mathbf{LS}. \tag{8}$$

It is evident from (8), that the prior information about $\mathbf{S}$ can be used to initialize $\mathbf{Q}$ for further computations. In a CDMA system we might have partial knowledge about the signature code matrix $\mathbf{S}$. For example in the CDMA downlink scenario where the goal is to detect a single user, the mobile handset may have prior information about a single column of the matrix $\mathbf{S}$. Whereas in the uplink scenario the base station may have prior information about all the active users in the channel or about certain users whose signature codes have been *locked*. In that case, multiple columns of $\mathbf{S}$ are known to the multiuser detector. In the above discussion, it was assumed that the deterministic information about the signature code about the desired user/users is known. In a more general setting when only available information is statistical in nature such as the information about the pdfs of the entries of $\mathbf{S}$, we have [15]

$$p_{q_{11}}(q_{11}) = \frac{p_{s_{11}}(q_{11}/l_{11})}{|l_{11}|}, \tag{9}$$

where $p_{q11}(q11)$ is the pdf of the entry at position $\{i, j\}_{i=1,j=1}$ of the matrix $\mathbf{Q}$. Note that the relation (9) is obtained when the matrix $\mathbf{L}$ in (8) is a lower triangular matrix. In this case, prior statistical information about the signature code matrix is transformed into the prior statistical information about the rotation matrix $\mathbf{Q}$. Various ICA algorithms incorporating the prior statistical information are proposed in [15].

It is to be noted that $\mathbf{L}$ in (8) is a whitening matrix which can be obtained from the SOS of the observations $\mathbf{y}$. In the present method $\mathbf{L}$ is obtained by performing the Q-R decomposition on the observations, that is $\mathbf{Y^T} = \mathbf{Q_1R_1}$, where $\mathbf{Y^T}$ is a $T \times K$ matrix, $\mathbf{Q_1}$ is a $T \times K$ matrix of orthonormal columns and $\mathbf{R_1}$ is a $K \times K$ upper triangular matrix obtained from the *economy size* Q-R decomposition in MATLAB. $\mathbf{L}$ is then given as $\mathbf{L} = \sqrt{T}\mathbf{R_1^{-T}}$. Prewhitening reduces the task of estimating the mixing matrix $\mathbf{S}$ to a simpler task of estimating $\mathbf{Q}$. This transformation reflects as a computational saving in ICA algorithms, since estimation of a $n \times n$ size $\mathbf{S}$ involves estimation of n^2 unknowns, where as estimation of $\mathbf{Q}$ involves estimation of $n(n-1)/2$ unknowns. It is also to be noted that $\mathbf{Q}$ merely rotates the constellation to obtain independent components from uncorrelated components obtained after the whitening step. A typical example of matrix $\mathbf{Q}$ in a simple 2×2 case could be a *Givens matrix* denoted by

$$\mathbf{Q} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix},$$

where the unknown is only a single parameter α.

Once the initial estimate of $\mathbf{Q}$ has been obtained from the relation (8), the estimate of the source data can be expressed as

$$\hat{\mathbf{b}} = \mathbf{Q}^T\tilde{\mathbf{y}}, \tag{10}$$

where $\tilde{\mathbf{y}}$ is the whitened version of $\mathbf{y}$. The task of the ICA algorithm is then to restore the independence in $\hat{\mathbf{b}}$ by further optimizing $\mathbf{Q}$ using the HOS. It is to be noted that in [14] the ICA basis vector $\mathbf{w}$ was initialized by

$$\mathbf{w} = \frac{1}{[\mathbf{s}_1^T\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^T\mathbf{s}_1]}\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^T\mathbf{s}_1, \tag{11}$$

which is the linear MMSE detector in terms of the signal subspace parameters. It is noted that this computation requires the eigen-decomposition of the sample data correlation matrix given by

$$\mathbf{R} = \frac{1}{T}(\mathbf{y}\mathbf{y}^T),$$

where T is the number of snapshots of $\mathbf{y}$ used in determination of the matrix $\mathbf{R}$. The philosophy of doing so was to exploit the independence and non-Gaussianity of the source signals by the use of the HOS which is not possible by the MMSE based detectors alone. Similar initialization could be done with the help of the subspace decorrelating detector given as

$$\mathbf{w} = \mu\mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K)^{-1}\mathbf{U}_s^T\mathbf{s}_1, \tag{12}$$

where μ is given as

$$\mu = \frac{1}{[\mathbf{s}_1^T\mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K)^{-1}\mathbf{U}_s^T\mathbf{s}_1]}.$$

The ICA basis vector can also be initialized by the RAKE receiver, for details see [14]. As stated earlier, in our method we make use of the relation (8) to initialize the ICA basis vector for further computations. It is worthwhile mentioning that depending on the amount of prior information we can detect more than one users simultaneously by the proposed method.

V. Algorithm Implementation

There have been many information theoretic approaches towards the computation of the ICA, for a comprehensive review see [17]. A quick summary of various ICA algorithms can also be found in [18]. The computation of ICA may be done either in the batch mode processing as well as in adaptive fashion. For the purposes of this paper, we maximize the generalized version of the cumulant based cost function [10], first proposed in [19]

$$J_G(\mathbf{w}) = [\mathbf{E}\{\mathbf{G}(\mathbf{w^T x})\} - \mathbf{E}\{\mathbf{G}(\nu)\}]^{\mathbf{2}}, \tag{13}$$

where $\mathbf{w}$ is a N-dimensional vector constrained so that $E\{(\mathbf{w}^T\mathbf{x})^2\} = 1$, ν is a Gaussian variable of zero mean and unit variance. A suitable choice for G in a sub-Gaussian source case is given by

$$\begin{aligned} G(u) &= \frac{1}{4}u^4 \\ g(u) &= u^3, \end{aligned} \tag{14}$$

where $g(.)$ is the derivative of $G(.)$. If we are interested in extracting a single independent component the approach is termed as the *sequential* or the *deflation* approach [20]. Extraction of multiple independent components at the same time is possible by the *symmetric* approach [18]. The algorithm for detection of a single desired user by the proposed method then becomes:

1) Collect T snapshots in a matrix $\mathbf{Y}$
2) Compute the lower triangular matrix $\mathbf{L}$ from the data using Q-R decomposition technique.
3) Compute one column of matrix $\mathbf{Q}$ from the relation (8).
4) Initialize $\mathbf{w}_0$ as the first column of $\mathbf{Q}$, that is $\mathbf{w}_0 = \mathbf{Q}(:,1)/||\mathbf{Q}(:,1)||$.
5) Update the ICA basis vector as follows: $\mathbf{w}(t+1) = \mathbf{w}(t) - \zeta[E\{\mathbf{y}g(\mathbf{w}^T\mathbf{y})\} - \beta\mathbf{w}]/[E\{g^{\cdot}(\mathbf{w}^T\mathbf{y})\} - \beta]$
6) Update the norm constraint $\mathbf{w}(t+1) = \frac{\mathbf{w}(t+1)}{||\mathbf{w}(t+1)||}$
7) If $|\mathbf{w}^T(t)\mathbf{w}(t+1)|$ is not close enough to 1, go back to step 6.

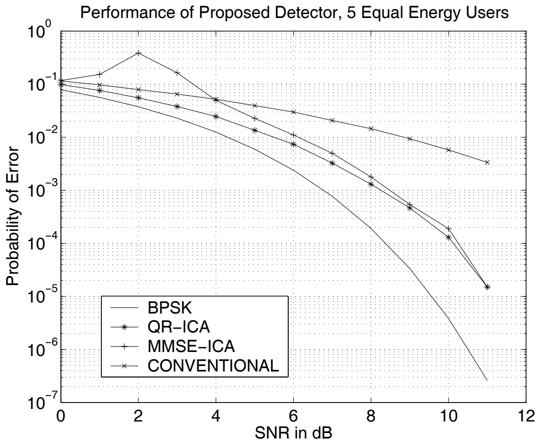
The constant β in step 6 is given as

$$\beta = E\{\mathbf{w}^T\mathbf{y}g(\mathbf{w}^T\mathbf{y})\},$$

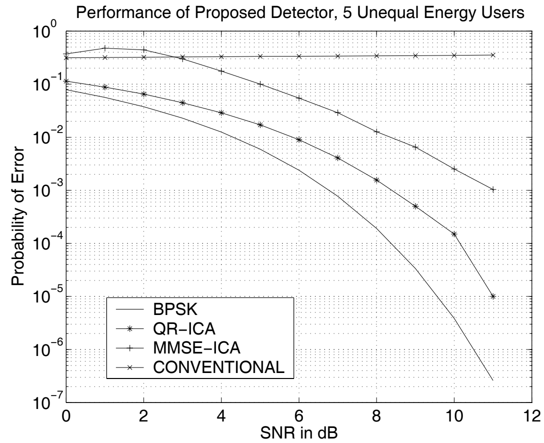
and ζ in step 5 is a step size parameter that may change with the iteration count. In the present algorithm, we avoid the need for explicit subspace computations as done in [14] which make the present algorithm attractive from computational point of view apart from eliminating the indeterminacy in ICA solution. The algorithm presented above is for the case when the deterministic prior information about the user signature code is available with the receiver, however it's generalization in a CDMA system might be possible with the help of [15].

VI. Simulation Results

In this section, we simulate the performance of the proposed algorithm in a multiuser synchronous CDMA system. For simplicity we consider the AWGN channel. Perfect knowledge of the desired user's signature code and timing is assumed.



(a)



(b)

Fig. 1. Performance of Prior ICA based detector in comparison with MMSE initialized detector with K=5 Users:(a) Equal Energy Scenario., (b) Unequal Energy Scenario. Correlation between users is as per Table-I

We consider the cases of perfect power control and no power control in the system. The number of data points taken in the simulations is 1000 and the simulation results are averaged over 200 Monte-Carlo experiments. The signature codes of the users are generated by first generating unit energy Hadamard codes of length $N = 16$ and then the desired correlation between them is induced as per the Table-I. The number of users considered in the simulation is $K = 5$ and $K = 10$.

TABLE I

Cross-Correlation of the Interfering Users Code with that of the Desired User.

User Number	Cross-Correlation with User No. 1
1	1.000
2	0.400
3	-0.032
4	0.097
5	0.226
6	-0.032
7	-0.290
8	0.097
9	0.226
10	0.200

For the first example, we take five equal energy users in the CDMA channel with correlation values as per table-I and compare the performance of the proposed detector with the ICA assisted MMSE detector of [14]. Both MMSE-ICA detector and the proposed QR-ICA detectors are quasi-blind methods in the sense that they do not have any information about the users except the desired one. In the simulations, we compute the initial ICA basis vector using the triangular-orthogonal (Q-R) decomposition of the received data, it was however noted that a similar performance can be obtained if we chose the ICA basis vector with the help of a generalized whitening matrix, computed using the signal subspace parameters. The learning constant ν was taken to be 0.35 in the simulations. In Fig. 1 (a) we simulate the performance of the system in a varying SNR environment with K=5 equal Energy users. Data modulation is *binary phase shift keying* (BPSK). It is seen that the ICA basis vector initialized by the proposed algorithm is able to achieve better BER performance than the previously proposed MMSE initialized ICA approach. For comparison purposes, the performance is also compared with the conventional matched filtering approach where the detector is simply taken to be the signature code of the desired user. The conventional mono-component detector performs poorly due to the correlated signature codes of the users in the channel. For Fig. 1 (b), we take 5 unequal energy users with the correlation values given in Table-I. The interfering users are 10 dB above the desired user's power. In this case also the performance of the proposed detector is better than the MMSE-ICA detector. Due to the power imbalance in the system, in this case the conventional detector has probability of error almost equal to 0.5 (near-far effect). Fig. 2 depicts the performance under the same environment but with increased number of users. The performance in this case is similar to that of Fig. 1. It is seen from the comparison of Fig. 1 (a) and Fig. 2 (a) that in case of equal energy users the performance of MMSE initialized ICA and QR initialized ICA are close in comparison with that of Fig. 1 (b) and Fig. 2 (b), this is because the power imbalance causes the conditioning of the received data correlation matrix to deteriorate. This deterioration adversely affects the accuracy of the subspace parameters and hence the performance of the MMSE-ICA detector deteriorates where as QR-ICA has no subspace computations involved and it's performance is not affected as much by the power imbalance.

VII. Conclusions

In this paper, we have presented a prior-ICA based multiuser detection approach towards solving the indeterminacy problem arising in ICA algorithms. The proposed detector incorporates prior information regarding the desired user's signature code,

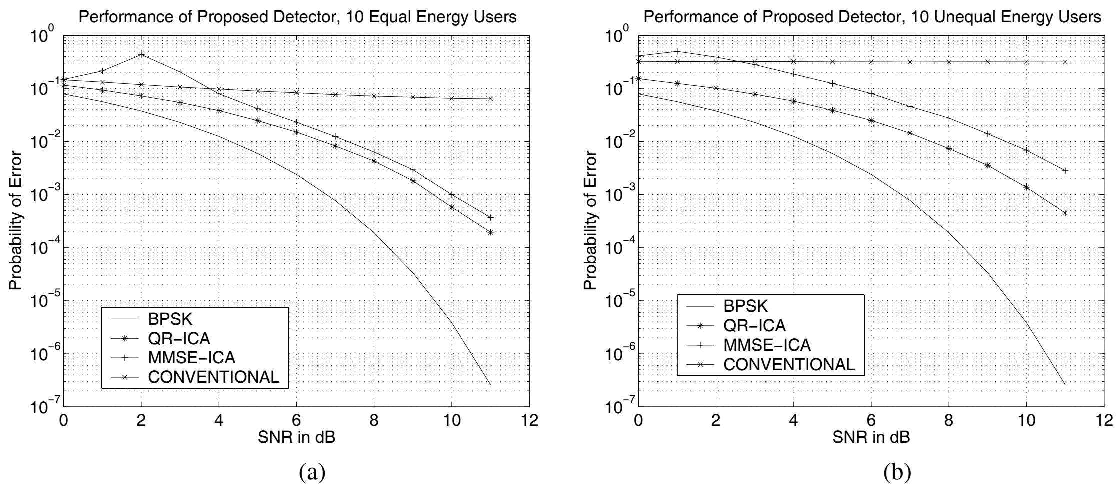


Fig. 2. Performance Comparison of Q-R initialized ICA detector with MMSE initialized detector with K=10 Users: (a) Equal Energy Scenario, (b) Unequal Energy Scenario. Correlation between users is as per Table-I

initializes the ICA basis vector in the ICA computations using a Q-R decomposition on the received data, and avoids the need for tedious subspace computations required by the subspace MMSE and CC-ICA approaches. Simulation results indicate that the performance of the proposed algorithm is significantly better than the subspace MMSE-ICA approach in the absence of power control in the system.

REFERENCES

[1] S. Verdú, "Minimum probability of error for asynchronous gaussian multiple-access channels," *IEEE Trans. on Inform. Theory*, vol. 32, no. 1, pp. 85 – 96, 1986.

[2] R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. on Inform. Theory*, vol. 35, no. 1, pp. 123 – 136, 1989.

[3] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. on Commun.*, vol. 42, pp. 3178 – 3188, Dec 1994.

[4] M. K. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications," *IEEE Trans. on Commun.*, vol. 38, no. 4, pp. 509 – 519, 1990.

[5] M. Honig, U. Madhow, and S. Verdu, "Blind adaptive multiuser detection," *IEEE Trans. on Inform. Theory*, vol. 41, pp. 944 – 960, Jul 1995.

[6] S. Roy, "Subspace blind adaptive detection for multiuser CDMA.," *IEEE Trans. on Commun.*, vol. 48, no. 1, pp. 169 – 175, 2000.

[7] X. D. Wang and H. V. Poor, "Blind multiuser detection: A subspace approach," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 677 – 690, Mar 1998.

[8] X. D. Wang and H. V. Poor, "Blind adaptive multiuser detection in multipath CDMA channels based on subspace tracking," *IEEE Trans. on Sig. Process.*, vol. 46, pp. 3030 – 3044, Nov 1998.

[9] J. H. Wen and C. K. Wen, "Adaptive recursive algorithm for complementary subspace-based blind multiuser detection," *IEEE Trans. on Circuits and Systems II-Analog and Digital Signal Processing*, vol. 48, pp. 1132 – 1136, Dec 2001.

[10] P. Comon, "Independent component analysis, a new concept ?," *Signal Processing*, vol. 36, pp. 287 – 314, Apr 1994.

[11] T. Lang, R. W. Liu, V. C. Soon, and Y. F. Huang, "Indeterminacy and identifiability of blind identifaction," *IEEE Trans. on circuits and systems*, vol. 38, no. 5, pp. 499 – 509, 1991.

[12] W. Lu and J. C. Rajapakse, "Eliminating indeterminacy in ICA," *Neurocomputing*, vol. 50, pp. 271 – 290, Jan 2003.

[13] M. Gupta and B. Santhanam, "ICA based blind adaptive MAI suppression in DS-CDMA systems," *Proc. of IEEE-DSP Workshop - Taos Ski Valley*, pp. 201 – 205, Aug 2004.

[14] T. Ristaniemi and J. Joutsensalo, "Advanced ICA-based receivers for block fading DS-CDMA channels," *Signal Processing*, vol. 82, pp. 417 – 431, Mar 2002.

[15] J. Igual, L. Vergara, A. Comacho, and R. Miralles, "Independent component analysis with prior information about the mixing matrix," *Neurocomputing*, vol. 50, pp. 419 – 438, Jan 2003.

[16] L. Tong, Y. Inouye, and R.-W. Liu, "Waveform-preserving blind estimation of multiple independent sources," *IEEE Trans. on Sig. Proc.*, vol. 41, pp. 2461–2470, Jul 1993.

[17] T. W. Lee, M. Girolami, A. J. Bell, and T. J. Sejnowski, "A unifying information-theoretic framework for independent component analysis," *Computers Mathematics Appl.*, vol. 39, no. 11, pp. 1 – 21, 2000.

[18] A. Hyvärinen and E. Oja, "Independent component analysis: Algorithms and applications," *Neural Nets*, vol. 13, pp. 411 – 430, May-June 2000.

[19] A. Hyvärinen, "Fast and robust fixed-point algorithms for independent component analysis," *IEEE Trans. on Neural Nets*, vol. 10, pp. 626 – 634, May 1999.

[20] N. Delfosse and P. Loubaton, "Adaptive blind separation of independent sources - a deflation approach," *Signal Processing*, vol. 45, pp. 59 – 83, Jul 1995.