ICA BASED BLIND ADAPTIVE MAI SUPPRESSION IN DS-CDMA SYSTEMS

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***Abstract*- The performance of existing multiuser detectors is dependent on available information about the structure of multiple access interference (MAI). The decorrelating detector requires complete knowledge of MAI. The minimum output energy (MOE) detector is a semiblind approach which exploits the information about the desired user only. The performance of MOE detector is suboptimal compared to that of the decorrelator. The proposed approach is a semiblind code constrained-independent component analysis (CC-ICA) based approach that exploits the prior information about the signature code of the desired user to constrain the ICA vector to lie in the orthogonal complement of the interfering users data. Simulation results indicate that the performance of the proposed approach approaches that of the decorrelator using much less prior information.**

1. INTRODUCTION

MAI constitutes a significant bottleneck in achieving the envisaged capacity of a *code division multiple access* (CDMA) system. Inadequacy of the conventional detector to deal with MAI has motivated the development of optimum multiuser detector [1] and its suboptimal counterparts [2], [3], [4]. The drawback of the above detectors are that they either require complete knowledge of the MAI [2] or training data [3] or the decision delay is too large [4]. To overcome these limitations, a class of spectrally efficient blind detectors based on constrained minimum output energy and subspace concepts are proposed in [5] [6] [7]. Subspace methods are gaining popularity due to their minimal side information requirement and increase in available computational capabilities. It can be shown that both the decorrelating detector and the MMSE/MOE detectors can be put into a blind approach framework using subspace concepts. Subspace methods usually identify the signal subspace by projecting the received data onto an orthogonal basis vectors spanning the signal subspace obtained from the data covariance matrix based on the widely used statistical technique *principal component analysis* (PCA). *Independent component analysis* (ICA), an extension of PCA, is a recent technique [8] that assumes source independence and tries to restore this attribute. The source independence assumption could very well be justified in most multiuser communication scenarios. Traditionally the ICA has been used in blind source separation problems i.e., the *cocktail party* problem for speech processing. The main goal here is to separate the signals. i.e. distinguish between the different speakers. In these applications, there is an inherent ambiguity up to a scaling and permutation. Permutation ambiguity refers to the order of the speakers i.e. which speaker's voice comes first. The scaling ambiguity refers to the case where the separated sources differ from the original sources by a scaling factor, this scaling ambiguity can be tolerated in places where most of the information conveyed is centered in the shape of the waveform rather than it's amplitude [9]. The structure of the CDMA channel where all the users transmit their data without any temporal or spectral separation immediately poses a similarity to the cocktail party problem and prompts the use of ICA in this scenario, however the nature of the problem in communication applications does not allow for the scaling and permutation ambiguity. Recently an adaptive receiver based on the ICA concept has been introduced in [10], however due to the ill-posed nature of the problem [11], the ICA part has been incorporated as an add on to the existing MMSE or rake receiver. Efforts towards eliminating the indeterminacy problems have recently been reported in [12], where this indeterminacy is eliminated on the basis of prior knowledge about the source kurtosis. In this paper, we assume the knowledge of the signature code of the desired user and present an algorithm to remove the indeterminacy in ICA solution by imposing a norm constraint and constraining the ICA solution to lie in the orthogonal complement of the interfering users data.

2. SYSTEM MODEL

A CDMA channel is characterized by the fact that there is no separation between the users either in the frequency do-

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main or in the time domain. The signal received at the receiver in continuous time domain can be represented as

$$y(t) = \sum_{k=1}^{K} \sum_{i=-\infty}^{+\infty} A_k b_k(i) s_k(t - iT_s - \tau_k) + \sigma n(t), \quad (1)$$

where we have used the following notation: T_s is the inverse of the data rate or the symbol time interval, $s_k(t)$ is the deterministic signature waveform assigned to the k^{th}user in the channel, A_k is the received amplitude of the k^{th}user's signal and A_k^2 is referred to as the energy of the k^{th}user, b_k(i) $\in$ [-1, +1] is the i^{th}data symbol transmitted by the k^{th}user, $n(t)$ is additive white Gaussian noise with unit power spectral density, σ^2 is the noise power spectral density, τ_k is the delay introduced for the k^{th} user. In the above system model it is assumed that the data symbols are independent, identically distributed (i.i.d.) random variables. The signature waveform has the form

$$s_k(t) = \sum_{n=0}^{N-1} c_k(n) p_k(t - nT_c), \quad (2)$$

where N is the number of chips per symbol, $T_c = \frac{T_s}{N}$ is the chip interval, $c_k(n)$ is the n^{th}chip in the spreading sequence of the k^{th}user, $p_k(t)$ is the chip waveform of the k^{th}user, received at the receiver end, filtered by the transmitter, receiver and the channel. Considering a symbol synchronous system, i.e. $\tau_1 = \tau_2 = ... = \tau_K = 0$, chip matched filtering and sampling the received signal at the chip rate N/T_s, we have a length N vector as the received data vector $\mathbf{y}$. The matrix formulation of the composite signal in AWGN channel for a given signaling interval i is given as

$$\mathbf{y}(i) = \mathbf{SAb}(i) + \mathbf{n}(i), \quad (3)$$

where : $\mathbf{S} = [\mathbf{s_1}, \mathbf{s_2}, ..., \mathbf{s_K}]$ is a KxK matrix of correlated user signature codes. $\mathbf{A} = \text{diag}\,[A_1, A_2, ..., A_K]$ is diagonal matrix of user amplitudes and $\mathbf{b}(i) = [b_1(i), b_2(i), ..., b_K(i)]^T$ is a K dimensional vector of user data at time $t = i$. By hypothesis all the source cumulants are diagonal, in particular the two point correlation between the user symbols at the same time is given as

$$K_{i,j}^0 \equiv \langle b_i(t) b_j(t) \rangle = \delta_{i,j} K_i^0 \quad (4)$$

where $\delta_{i,j}$ is the Kronecker delta and $\langle z_1, z_2, ..., z_k \rangle$ denotes the cumulant of the k random variables $z_1, z_2, ..., z_k$. Without loss of generality, one can always assume that all the sources have zero means

$$\langle b_k \rangle = 0, \quad k = 1, ..., K$$

If this is not the case, one has to estimated the mean values of each of the input and subtract it from that input. It is shown in [13] that an asynchronous CDMA system with K users can be thought of a single user system subject to ISI, where each bit of a user overlaps with $(K-1)$ to $(2K-1)$ interfering bits depending on the relative delay between the users. In the remaining sections, we will assume that user 1 is the desired user and the receiver has the perfect knowledge of the desired user's signature code and it's timing.

3. ICA BASED MULTIUSER DETECTOR

ICA of a random vector consists of searching for a linear transformation that minimizes the statistical dependence between its components and could be implemented by using both neural as well as statistical algorithms. One of early formulations of the ICA [8] is based on the concept of mutual information in which the expansion of mutual information is utilized as a function of cumulants of increasing order. Mutual information between a random vector $\mathbf{x}$ and $\mathbf{y}$, which are derived from a distributions $p_{\text{x}}(x)$ and $p_{\text{y}}(y)$ is given as

$$I(\mathbf{x};\mathbf{y}) = \sum_x \sum_y p_{\text{x,y}}(x,y) \;\log \frac{p_{\text{x,y}}(x,y)}{p_{\text{x}}(x) p_{\text{y}}(y)}, \quad (5)$$

It is however noted in the above equation that the mutual information is infact the *relative entropy* [14] between the joint distribution $p_{\text{x,y}}(x,y)$ and the product distribution $p_{\text{x}}(x)p_{\text{y}}(y)$. In another notation the relative entropy is defined as the *Kullback-Leibler* distance between the distributions $p_{\text{x}}(x)$ and $p_{\text{y}}(y)$ and is denoted as

$$D\left(p_{\text{x,y}}(x,y) || p_{\text{x}}(x) p_{\text{y}}(y)\right),$$

In our case, where the goal is to compute ICA on a given random vector, we rephrase the above definitions to obtain the mutual information as the relative entropy between the density of random vector $\mathbf{x}$ and the density assuming component-wise independence of $\mathbf{x}$. This gives us the following expression for the mutual information

$$I(\mathbf{x}) = D\left(p_{\text{x}}(\mathbf{x}) || \prod_{i=1}^{i=N} p_{\text{x}}(x_i)\right), \quad (6)$$

Most of the ICA algorithms try to minimize the above mutual information in order to compute the independent components. This usually consists of finding a KxN demixing matrix/linear transformation matrix $\mathbf{W}$, which when multiplied by the random vector yields independent components. $\mathbf{W}$ is found by minimizing/maximizing some appropriately defined cost function, typically based on a measure of mutual information. Most of the ICA algorithms have a preprocessing stage, where the data is whitened to yield uncorrelated components, this preprocessing step reduces the problem of finding $\mathbf{W}$ to finding an orthogonal (unitary) matrix $\mathbf{Q}$, that rotates the signal constellation to yield independent components from uncorrelated components. It is to be noted that the whitening matrix can be obtained from the second order statistics of the received signal, whereas the computation of orthogonal matrix, usually employs higher order statistics such as higher order (> 2) cumulants, this fact was first brought up by Comon [8] and Cardoso [15]. In ICA, a typical signal generating model is

$$\mathbf{y} = \tilde{\mathbf{S}}\tilde{\mathbf{b}}, \tag{7}$$

In the present case (refer to eq.3) we have $\tilde{\mathbf{S}} = \mathbf{SA}$ as the memoryless *mixing matrix* and the noise component $\mathbf{n}$ can be incorporated in the model of (7) as added source components. A multi-user detector can be formulated by extracting all the independent components at once, this is particularly useful in a CDMA uplink scenario where it is reasonable to assume that the base station has information about all the active users in the channel, however in a CDMA downlink scenario, where the emphasis is on the extraction of a single desired user, we are interested in obtaining one row of the matrix $\mathbf{W}$. To compute $\mathbf{W}$ or $\mathbf{Q}$ variety of cost functions or contrast functions have been proposed in the literature. There have been many information theoretic approaches towards the computation of the ICA, for a comprehensive review see [16], a quick summary of various ICA algorithms can also be found in [17]. Specifically, for the purposes of this paper, we implement the CC-ICA detector based on a modified version of Hyvärinen's fixed point algorithm proposed in [18]. The cost function takes form of a generalized version of the cumulant based cost function proposed by Comon in [8], which is also considered in [18].

$$J_G(\mathbf{w}) = [\mathbf{E}\{\mathbf{G}(\mathbf{w^T x})\} - \mathbf{E}\{\mathbf{G}(\nu)\}]^{\mathbf{2}}, \tag{8}$$

where $J_G(\mathbf{w})$ is the cost function we are interested in optimizing, $\mathbf{w}$ is a N-dimensional vector constrained so that $E\{(\mathbf{w}^T\mathbf{x})^2\} = 1$, ν is a Gaussian variable of zero mean and unit variance. G in this paper is taken as

$$\begin{aligned} G(u) &= \frac{1}{4}u^4 \\ g(u) &= u^3, \end{aligned} \tag{9}$$

where $g(.)$ is the derivative of $G(.)$. We follow the "sequential" approach using the nonlinearity $G(.)$ in (9) to compute the ICA. This sequential scheme is also called as the *deflation* approach. The same concept is extended to the multiuser detection case, where we need to separate out more than one sources simultaneously, namely the "symmetric" approach. Following the sequential approach, minimization of the cost function in (8), will extract the data from one user, but there is no control onto which user's data is being extracted. The above stated limitation constitutes a major hurdle for the application of ICA based approaches in communication applications. The removal of this *inherent ambiguity* is precisely the goal of this paper.

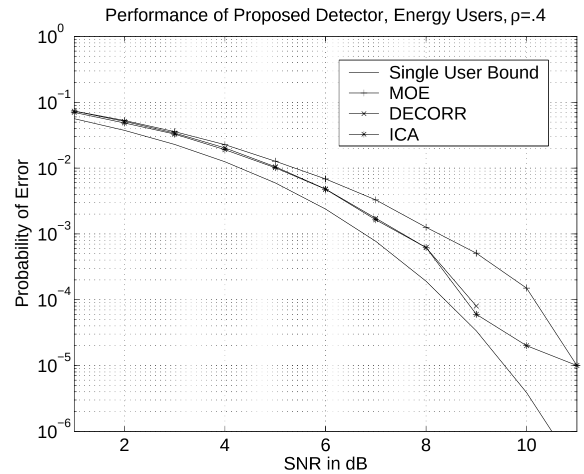

Fig. 1. Performance comparison of the proposed ICA based detector to that of MOE and decorrelating detector. Number of users in the channel is 2 and perfect power control is assumed. Correlation between the users is $\rho = 0.4$.

4. CODE-CONSTRAINED ICA RECEIVER

The minimization of the cost function listed in (8), for deflation approach yields a Nx1 vector $\mathbf{w}$, such that

$$x_i = \mathbf{w}^T\mathbf{y} = s_j \quad \text{where} \quad i \neq j, \tag{10}$$

It is to be noted in the above equation that there is no control over which user is extracted. We would like to extract the user of interest given the knowledge of the spreading code of the desired user. This is achieved by imposing a norm constraint and constraining the ICA basis vector to lie in the space orthogonal to the signal space spanned by the interfering users. Let us denote the value of the weight vector at time instant k as $\mathbf{w}_k$, the value of the weight vector at $(k+1)^{th}$ time instant is obtained as

$$\mathbf{w}_{k+1} = \Pi_{s_1}^{\perp}\mathbf{w}_k, \tag{11}$$

The above projection ensures that the ICA weight vector belong to null space of the interfering users. Clearly, an estimate of the interference subspace is required in practice. We assume that a one-shot estimate of the interference subspace is extracted from the sample correlation matrix based on samples of the received vector with the desired signal component removed, i.e.,

$$\tilde{\mathbf{y}} = \mathbf{y} - \mathbf{s}_1(\mathbf{s}_1^T\mathbf{y}) \tag{12}$$

$$\tilde{R} = \frac{1}{N_{in}}(\tilde{\mathbf{y}}\tilde{\mathbf{y}}^T)$$

where N_{in} is the number of snapshots used in determination of the interference correlation matrix $\tilde{R}$. This matrix is

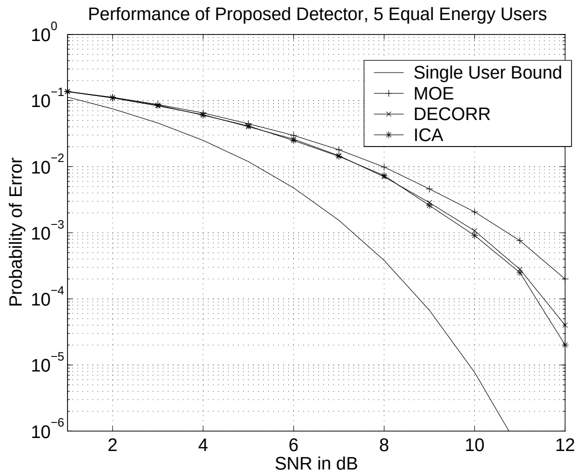

Fig. 2. Performance comparison of the proposed ICA based detector to that of MOE and decorrelating detector. Number of users in the channel is 5 and perfect power control is assumed. Correlation between users is as per Table-I

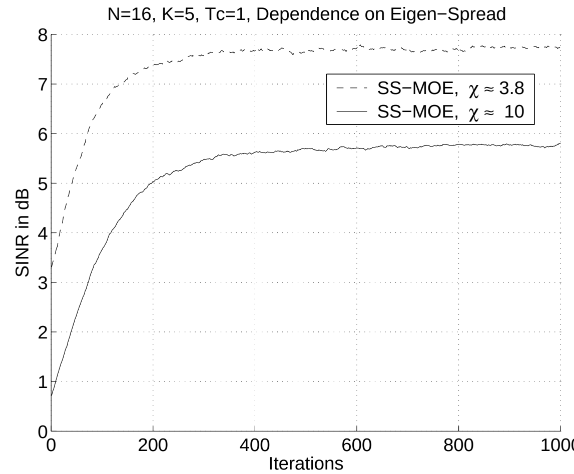

Fig. 3. SINR degradation of Sub-Space MOE detector due to the increasing condition number. Achievable SINR decreases as the correlation between the users increases thereby increasing the probability of error. Number of users in the channel is 5.

of rank $N-1$ and the most significant $K-1$ eigenvectors constitute the interference subspace Π_{s_1}. The prewhitening step is carried out to reduce the problem to the estimation of on orthogonal matrix, thereby reducing some of the computational overheads. We can further combine the dimensionality reduction step with the whitening step to exclude the problem of overfitting, or 'fitting to the noise' in the data. Note that the proposed method requires eigen-decomposition of the received data to estimate the signal subspace and the interfering subspace, and for some this computation is very demanding, as it is. In this regard, we need to point the attention towards [19], where it is envisaged that some day neural processing will make the eigen-computation a trivial task, that would not only compute the eigen-structure, but also compute the tasks such as finding the projection following the estimation of signal subspace in a single step.

5. SIMULATION RESULTS

In this section, we simulate the performance of the proposed detector in a multiple user scenario in a synchronous additive white Gaussian (AWGN) CDMA channel. We consider the cases when the power control is present and when the power control is not present. The simulation results are averaged over 100 monte-carlo experiments. The number of data points taken in simulations is 1000. The signature codes of the users are generated by first generating unit energy Hadamard codes of length 16 and then the desired correlation among them is induced as per Table-I.

TABLE I

Cross-Correlation of the Interfering Users Code with that of the Desired User.

User Number	Cross-Correlation with User No. 1
1	1.000
2	0.300
3	-0.320
4	0.280
5	0.700

As the first example we take two equal energy user CDMA channel with correlation of $\rho = 0.4$ and compare the performance of the proposed detector with the MOE detector of [5] and decorrelating detector of [2]. It is to be noted that MOE detector and the proposed CC-ICA detectors are quasi-blind methods in the sense that they do not have any information about the users except the desired one. The performance of the MOE detector is inferior to that of the decorrelating detector due to the fact that it has lesser prior information available, however the proposed detector's performance is close to the decorrelating detector although using the same information as that of the MOE detector, refer to Fig. 1. As a second example, in Fig.2 we take 5 equal energy with the correlation values given in Table-I. In this case also the performance of the proposed detector is better than the MOE detector and close to that of decorrelating detector. Although we note that there is a performance degradation as compared to two user case due to the highly correlated nature of the user signature codes. This degrada-

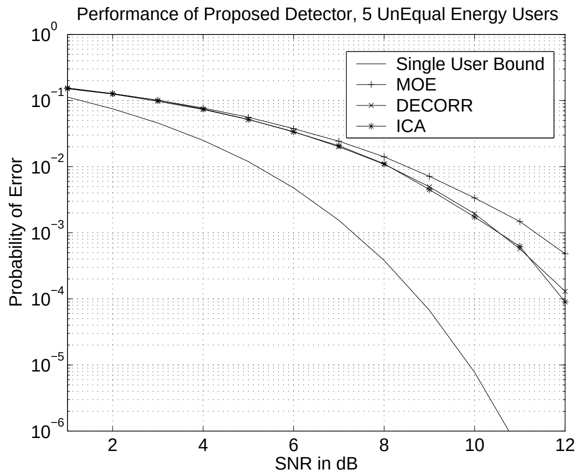

Fig. 4. Performance comparison of the proposed ICA based detector to that of MOE and decorrelating detector in absence of power control. Number of users in the channel is 5 and correlation between them is given in Table-I

tion in the performance could be explained in terms of the condition number χ of the interfering user subspace. As the condition number of the data correlation matrix increases the achievable signal to interference noise ratio (SINR) decreases. Since the probability of error is dependent on the SINR, the probability of error increases as the SINR decreases, see Fig.3. SINR is defined in a CDMA system as

$$SINR = \frac{(\mathbf{w^T s_1})^2}{\sum_{i\neq 1} A_i^2(\mathbf{w^T s_i}) + \sigma^2 \mathbf{w^T w}} \tag{13}$$

where σ^2 the noise variance. As a third example, in Fig. 4 we take the case when the interfering users are 10dB above the desired user's signal power, we note that we obtain the similar performance gain over the MOE detector.

6. CONCLUSIONS

In this paper, we have presented an algorithm based on the code-constrained ICA application in multi-user CDMA system to remove the inherent indeterminacy problem in ICA computations. The indeterminacy in ICA computation is removed by constraining the CC-ICA detector to lie in the orthogonal complement of the interfering users data space. Simulation results indicate that the performance of the proposed detector approaches that of the decorrelating detector without requiring the complete knowledge of the structure of MAI. The performance of the proposed detector is better than that of MOE detector using the same amount of prior information.

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