COMPONENT ENUMERATION OF MULTICOMPONENT AM–FM SIGNALS VIA GENERALIZED ENERGY OPERATORS

Balu Santhanam

Dept. of E.E.C.E., University of New Mexico
Albuquerque, NM: 87131-1356
Email: bsanthan@eece.unm.edu

ABSTRACT

Enumerating the number of signal components present in a superposition of signals is a problem that finds application in radar systems, multiuser communication problems, beamforming and array processing applications. The related problem of enumerating the number of components present in a superposition of non-stationary signals is a more challenging problem because of the nature of the components that may be well separated in frequency at one instant and overlap at a later instant. In this paper, we apply the Teager-Kaiser energy operator and higher-order generalizations of the operator to the problem of non-stationary signal component enumeration. The singularity of these instantaneous signal operators is used as a cue to track the number of components present in a multicomponent AM–FM signal.

1. INTRODUCTION

The problem of enumerating the number of components present in a superposition of signals is one that finds applications in radar systems, where one might be interested in determining the number of targets [2], in multiple user communications, where one is interested in the number of users [6], in array processing and beamforming applications, where one is interested in the number of narrowband sources impinging on a array of antennas [4]. The problem of component enumeration becomes even more challenging when the components are non-stationary.

As noted in [1], the situation is complicated by the fact that decomposition of a multicomponent signal into its constituent components is a local phenomena, where the components could be well separated in the time-frequency plane at one instant of time and overlap at a later instant of time. For the components to be well defined, the spread of the *instantaneous frequency* (IF) of the components should be narrow in relation to the instantaneous bandwidth of the IF of the composite signal [1]. Classical methods for component enumeration of stationary signals include the evaluation of the rank of a Toeplitz/Hankel matrix of signal samples [3], which requires a SVD or computation of the eigenvalues of a observability measure [6]. Detecting abrupt changes in the number of components, however, requires algorithms that possess good time resolution characteristics.

The Teager-Kaiser energy operator [9], higher-order energy operators [8], and related generalizations [10] are ideal candidates because they possess the desired simplicity, efficiency, and excellent time resolution. The energy operator specifically has been used to detect the onset of an abrupt event such as the presence of a transient in a AM-FM background [5]. In this paper, we will investigate the application of the Teager-Kaiser energy operator and its generalizations to the problem of component enumeration of nonstationary signals. Specifically we will exploit the singularity of these instantaneous operators and the GDE of the composite signal [11] for component enumeration of multicomponent AM–FM signals.

2. ENERGY DEMODULATION PRIMER

The Teager–Kaiser energy operator is a nonlinear, differential operator that computes the energy of a signal $x(t)$ via:

$$\Psi_c(x) = [\dot{x}(t)]^2 - x(t)\ddot{x}(t),$$

where the dot denotes the time derivative. The discrete–time energy operator applied to the signal $x[n]$ is defined via:

$$\Psi_d(x) = x^2[n] - x[n+1]x[n-1].$$

The *energy separation algorithm* (ESA) developed in [9] uses this operator to separate amplitude modulations from frequency modulations to accomplish monocomponent AM–FM signal demodulation. Discrete versions of the ESA (DESA's) [9] and applications of the ESA to the problems of AM–FM speech analysis–synthesis, AM–FM vocoding, speech formant frequency and bandwidth tracking have been investigated in [12].

Higher order generalizations of the energy operator, i.e., *higher-order energy operators* (HOEO) for the continuous–time and the discrete–time case are defined via [8]:

$$\begin{aligned}\Upsilon_k(x) &= \dot{x}(t)x^{(k-1)}(t) - x(t)x^{(k)}(t)\\ \Upsilon_k(x) &= x[n]x[n+k-2] - x[n-1]x[n+k-1].\end{aligned}$$

These operators for sinusoidal input signals measure the higher-order energies of a classical harmonic oscillator normalized to half unit mass [7]. The *energy demodulation of Mixtures* (EDM) algorithm developed in [8] uses these HOEO's to accomplish separation and demodulation of two

This was supported by a start-up grant at the University of New Mexico, Albuquerque

component AM–FM signals. For the sake of brevity, we will adopt the signal model and performance measures described in [8]. The underlying assumption is that the component IF/IA signals vary slowly with respect to their carriers.

3. COMPONENT ENUMERATION

3.1. Monocomponent AM–FM Signals

The *generating difference equation* (GDE) for a monocomponent sinusoidal signal that is invariant to both the amplitude and frequency is given by [8, 11]:

$$D[n] = \Psi(x[n]) - \Psi(x[n-1]) = 0.$$

In the case of narrowband AM–FM signals, where the information signals are slowly time–varying this relation holds approximately. The test signal $D[n]$ can therefore be used to detect the presence of a single component by thresholding the samples of $D[n]$ that are close enough to zero within a threshold η_o that is dependent of the SNR of the signal environment, i.e.,

$$T[n] = \begin{cases} 1 & |D[n]| \leq \eta_o \\ 0 & \text{otherwise.} \end{cases}$$

Specifically in the case where the signal $x[n]$ is noisy the test signal $D[n]$ can be smoothed using simple binomial smoothing to provide robustness. A large proportion of samples of the decision variable $T[n]$ being one indicates that $D[n] \approx 0$. The monocomponent detection problem can then be posed as a binary hypothesis testing problem of the form:

$$\begin{aligned} \mathbf{H}_1 &: \hat{p} \geq p_o(\eta_o) \\ \mathbf{H}_0 &: \hat{p} < p_o(\eta_o), \end{aligned}$$

where $\hat{p}$ is the proportion of decision variable values that is 1 and p_o is a threshold that depends on the SNR of the signal environment. Modelling each of the N samples of the decision variable, $T[n]$ as independent trials of a binomial random variable and treating a zero decision variable sample as a success, for a large N the variable

$$Z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{N}}}$$

is a standard Gaussian random variable via the central limit theorem. The hypothesis detection problem can then posed in terms of a Neyman-Pearson test of the form:

$$Z \underset{\mathbf{H}_o}{\overset{\mathbf{H}_1}{\gtrless}} Q^{-1}(\alpha).$$

where α is the probability of a false-alarm. Fig. (1) describes a component enumeration example where the composite signal described in Fig. (1)(a) has two components present in its first half and just one component present during the second half with a SNR of 27 dB. The additional benefit in energy operator based component enumeration is that the spikes in the energy operator output can be used to detect the presence or the onset of an event [5]. The presence of a energy discontinuity in $D[n]$ indicates the presence of an event after 500 samples, which in this example is a change in the number of components. Fig. (1)(b) describes the test signal $D[n]$ after 5–time binomial smoothing of the energy signal $\Psi(x[n])$. Fig. (1)(c) describes the decision variable $T[n]$ using a threshold of $\eta_o = 0.06$. The proportion of ones in the decision variable for this example over the second half of the signal was 98.6 %, while in the first half this proportion was 5.0 %.

3.2. Multicomponent Case

For signals that contain more than one component, we will use the fact that for stationary sinusoids a Hankel matrix of signal values is of rank $2M$ when there are M components in the signal or when the component IF and IA signals are slowly time-varying [3]. For test signals, we will use the generalized energy operators obtained via the Toeplitz determinant approach suggested in [10]. The instantaneous operators, $D_r[n]$ of interest are defined via the determinant of a Toeplitz matrix of instantaneous signal samples [10]. Specifically the GDE of a M component sinusoidal signal, that is invariant to the both the frequencies and amplitudes, in terms of these instantaneous operators is given by:

$$D_{M+1}(x[n]) = 0.$$

These Toeplitz determinant operators are then computed for various orders and then the value of the determinant is then thresholded and the proportion of the zero determinant samples is computed. The number of components in the signal can then be determined from the model order for which a significant proportion of determinant samples is close to zero within a threshold p_o that depends on the SNR. Figure 2 describes a two-component example where two components are present during both halves but the component IF's overlap over the second half. Fig. (2)(c) describes the performance of the determinant proportion method for $\eta_o = 0.1, p_o = 0.8$ for a SNR of 30 dB, MPR of 6 dB.

The singularity of these instantaneous operators will however, depend on the spectral proximity of the components and the relative strengths of the components. The *normalized carrier separation* (NCS) parameter described in [8] is defined as the separation between the component carrier frequencies normalized by the average Carson bandwidth of the components. Physically this parameter measures the spectral proximity of the components. For NCS, parameters larger than 1, the components are more or less spectrally disjoint, although they are not completely separated from each other. For NCS parameters lesser than 1, the components overlap spectrally and component interaction increases. For NCS parameters less than 0.1, we are in the co-channel signal range and the components are no longer distinct. As noted in [8], the energy equations in the EDM algorithm become ill-conditioned as the NCS parameter decreases below 1 and finally become singular as the NCS decreases further. This is a consequence of the fact that the two component signal starts to resemble a monocomponent signal as the NCS parameter decreases.

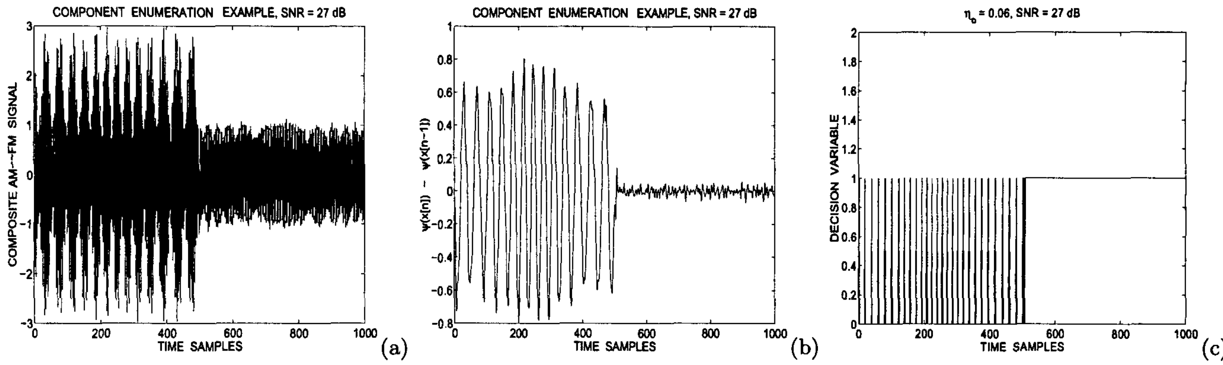

Figure 1: Component enumeration example: (a) composite AM–FM signal, (b) test signal $D[n]$ after 5-time binomial smoothing, (c) decision variable for monocomponent detection over the two halves of the signal. The first half of the signal contains two components while the second contains just one component.

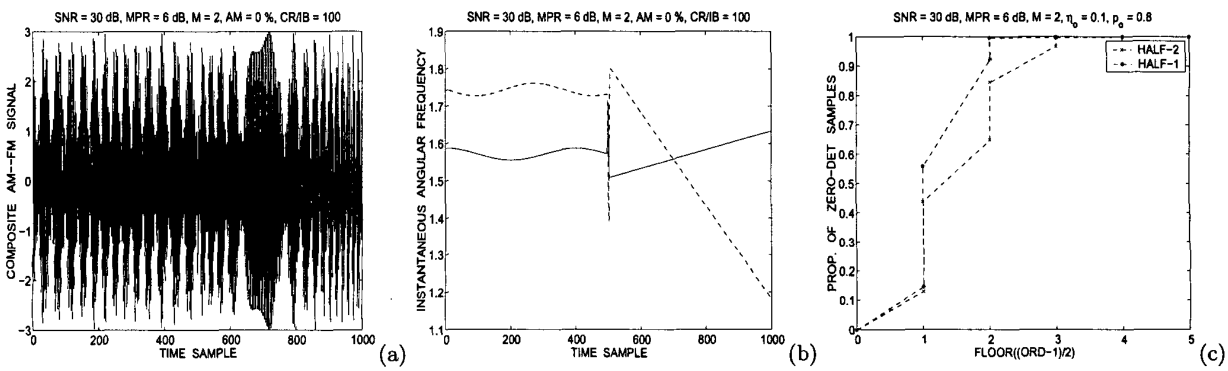

Figure 2: Two–component signal example where both halves of the signal contain 2 components. In the first half the component IF's are well–separated, while in the second half they cross–over.

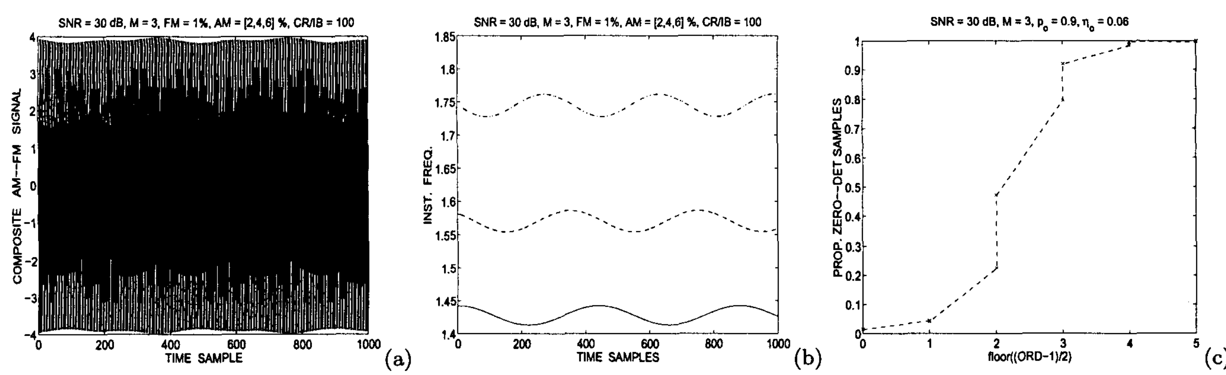

Figure 3: Three–component example: (a) composite AM–FM signal, (b) component IF's, (c) proportion of zero–determinant samples for various model orders using the Toeplitz-determinant proportion technique with a SNR of 30 dB, $\eta_o = 0.06$ and $p_o = 0.9$.

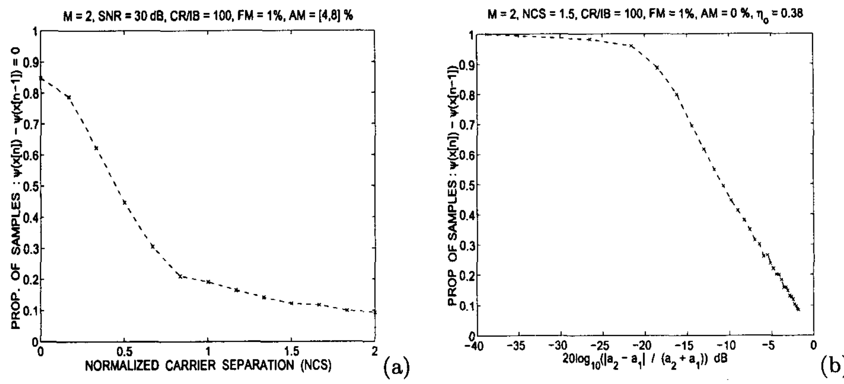

Figure 4: Effect of NCS and relative component power: (a) proportion of samples where $D[n] = 0$ for different NCS parameters with $\eta_o = 0.05$, (b) proportion of samples where $D[n] = 0$ for different relative component amplitudes.

In a similar fashion, the relative power or amplitude ratio between the components plays a role in singularity. Specifically the interaction between the components of a two-component AM–FM signal is a maximum when the components are of equal magnitude [1]. For larger relative power ratios, one of the components is stronger than the other and the components are therefore isolated in terms of their power. The GDE of a mono-component AM–FM signal can therefore be used both as a measure of singularity and as a mono-component detection tool. Figure (4) describes the proportion of singular samples of a two-component sinusoidally modulated AM–FM signal in terms of the NCS and relative amplitude ratio parameters.

4. SUMMARY AND CONCLUSIONS

A component enumeration algorithm for multicomponent AM–FM signals that exploits the GDE of the composite signal and singularities of higher-order generalizations of the Teager-Kaiser energy operator was presented. The component enumeration problem was shown to be equivalent to a Neyman-Pearson binary hypothesis testing problem with respect to the proportion of singular samples of these higher-order instantaneous operators. Simple binomial smoothing of these operators provides robustness in the presence of noise.

The advantage of this GDE-singularity-threshold approach to component enumeration is that it avoids the need to compute the rank or the eigenvalues of a matrix. Furthermore this approach employs generalizations of the energy operator that are simple, efficient, and possess excellent time resolution characteristics needed for the tracking of abrupt changes in a nonstationary signal.

REFERENCES

[1] L. Cohen and C. Lee, "Instantaneous Frequency, Its standard deviation and Multicomponent Signals," *Proc. of SPIE*, Vol. 975, pp. 186-208, 1988.

[2] P. Gulden, M. Vossiek, E. Storck and P. Heide, "Application of State Space Frequency Estimation Techniques to Radar Systems," *Proc. of ICASSP-01*, Vol. 5.

[3] B. D. Rao and K. S. Arun, "Model Based Processing of Signals: A State Space Approach," *Proc. of IEEE*, Vol. 80, No. 2, pp. 283-309, 1992.

[4] M. Wax and T. Kailath, "Detection of Signals by Information Theoretic Criteria," *IEEE Trans. Acoust., Speech, Sig. Process*, Vol. 33, No. 2, pp. 387-392.

[5] R. B. Dunn, T. F. Quatieri, and J. F. Kaiser, "Detection of Transient Signals Using the Energy Operator," *Proc. of ICASSP-93*, Vol. 3, pp. 145-148.

[6] B. A. Hedstrom and R. L. Kirlin, "Detection, Enumeration, and Estimation of Multiple Co-Channel Signals," *Proc. 6th SP Wkshp. on Statistical Sig. and Array Process.*, pp. 70-73, Oct. 1992.

[7] P. Maragos and A. Potamianos, "Higher order Differential Energy Operators," *IEEE Sig. Process. Lett.*, Vol. 2, No. 8, pp. 152-154, 1995.

[8] B. Santhanam and P. Maragos, "Energy Demodulation of Two–Component AM–FM Signal Mixtures," *IEEE Sig. Process. Lett.*, vol. 3, pp. 294-298, Nov. 1996.

[9] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "On Amplitude and Frequency Demodulation Using Energy Operators," *IEEE Trans. Sig. Process.*, vol. 41, pp. 1532-1550, April 1993.

[10] R. Kumaresan, A. G. Sadasiv, and C. S. Ramalingam, and J. F. Kaiser, "Instantaneous Nonlinear Operators For Tracking Multicomponent Signal Parameters," *Proc. 6th SP Wkshp. on Statistical. Signal and Array Process.*, pp. 404 - 407, Oct. 1992.

[11] A. Zayezdny and I. Druckmann, "A New Method of Signal Description and its Applications to Signal Processing", *Signal Process.*, Vol. 22, pp. 153 - 178", 1991.

[12] A. Potamianos and P. Maragos, "Speech Analysis and Synthesis using an AM-FM Modulation Model," *Speech Communication*, Vol. 28, No.3, pp. 195-209, July 1999.