Energy Demodulation of Two–Component AM–FM Signal Mixtures

Balasubramaniam Santhanam, *Student Member, IEEE,* and Petros Maragos, *Fellow, IEEE*

Abstract— **In this letter, an algorithm is presented for the separation and energy-based demodulation of two–component mixtures of AM–FM signals. The proposed algorithm is based on the generating differential or difference equation of the mixture signal and nonlinear differential energy operators.**

I. Introduction

AM–FM signals of the form $x(t) = a(t)\cos[\int_0^t \omega(\tau)d\tau]$ are very useful in analog communication systems and have been used recently in [1], [2], [3] to model speech resonances. To demodulate $x(t)$ into its amplitude envelope $|a(t)|$ and instantaneous frequency signal $\omega(t)$ the *energy separation algorithm* (ESA) was recently proposed in [1], [3]:

$$\sqrt{\frac{\Psi(\dot{x})}{\Psi(x)}} \approx \omega(t) \quad , \quad \frac{\Psi(x)}{\sqrt{\Psi(\dot{x})}} \approx |a(t)|,$$

where

$$\Psi(x) \quad \stackrel{\triangle}{=} \quad (\dot{x})^2 - x\ddot{x}$$

is the Teager–Kaiser energy operator [4], [5] and dots denote time derivatives. The ESA is efficient, has very low computational complexity and excellent time resolution. However, if x(t) is a multicomponent AM–FM signal, then bandpass filtering is needed to isolate each component before applying the ESA. The bandpass filtering causes problems when the components have overlapping spectra. In this paper, we present the *energy demodulation of mixtures* (EDM) algorithm for the demodulation of two-component AM–FM signals of the form

$$x(t) = a_1(t)\cos\left[\int_0^t \omega_1(\tau)d\tau\right] + a_2(t)\cos\left[\int_0^t \omega_2(\tau)d\tau\right]$$

using differential energy operators and the generating differential or difference equations pertaining to the composite signal. The envisioned applications are in the areas of co–channel signal separation and demodulation and speech formant separation and demodulation. First, we exploit the structural properties of a mixture of two sinusoidal signals by treating the mixture signal as a solution to a generating differential or difference equation (GDE) [6]. The coefficients of the GDE are then expressed in terms of generalizations of the energy operator Ψ to higher orders [7], to achieve both component separation and demodulation of the components into instantaneous frequency and amplitude signals.

In the last section, we compare the EDM algorithm with the *instantaneous Toeplitz determinant* (ITD) algorithm in [8], [9] and with adaptive linear prediction implemented via the LMS as described in [10].

Manuscript received February 22, 1996. This work was supported by the National Science Foundation under Grant MIP-93963091. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. V. J. Mathews.
The authors are with the School of Electrical & Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA. (email: santhan@eedsp.gatech.edu).
Publisher Item Identifier S 1070-9908(96)07927-8.

II. Continuous–Time Algorithm

The two–component AM–FM signal is modeled instantaneously by the two–component sinusoidal signal

$$x(t) = a_1\cos(\omega_1 t + \theta_1) + a_2\cos(\omega_2 t + \theta_2).$$

This mixture signal satisfies the following fourth-order GDE:

$$x^{(4)} + c_1\ddot{x} + c_2 x = 0,$$

where $x^{(n)} = d^n x/dt^n$ and

$$c_1 = (\omega_1^2 + \omega_2^2) \quad , \quad c_2 = \omega_1^2\omega_2^2.$$

The k^{th}-order differential energy operator is defined as [7]

$$\Upsilon_k(x) \quad \stackrel{\triangle}{=} \quad \dot{x}x^{(k-1)} - xx^{(k)}.$$

As a special case for $k = 2$, we obtain the Teager-Kaiser energy operator $\Upsilon_2 \equiv \Psi$. Second–order operators have dimensions of energy (per unit mass), third–order operators have dimensions of energy velocity, while fourth–order operators have dimensions of energy acceleration.

Using the GDE and its derivative and solving the resultant 2×2 linear system of equations yields the following expressions for the coefficients:

$$c_1 = -\frac{\Upsilon_5(x)}{\Upsilon_3(x)} \quad , \quad c_2 = \frac{\Upsilon_3(\ddot{x})}{\Upsilon_3(x)}.$$

Then, the EDM estimates for the frequencies ω_1 and ω_2 are computed as

$$\omega_{1,2} = \sqrt{\frac{c_1 \pm \sqrt{c_1^2 - 4c_2}}{2}}.$$

These frequency estimates are then used in conjunction with 2nd-order energy operators to develop the EDM estimates for the amplitude as follows:

$$\begin{aligned} \Psi(x^{(3)}) - \omega_1^2\omega_2^2\Psi(\dot{x}) &= (a_1^2\omega_1^6 - a_2^2\omega_2^6)(\omega_1^2 - \omega_2^2) \\ \Psi(\ddot{x}) - \omega_1^2\omega_2^2\Psi(x) &= (a_1^2\omega_1^4 - a_2^2\omega_2^4)(\omega_1^2 - \omega_2^2) \end{aligned}$$

$$\begin{aligned} a_{1,2}^2 = \; & \frac{\omega_{2,1}^4(\Psi(x^{(3)}) - \omega_1^2\omega_2^2\Psi(\dot{x}))}{\omega_1^4\omega_2^4(\omega_1^2 - \omega_2^2)^2} \\ & - \frac{\omega_{2,1}^6(\Psi(\ddot{x}) - \omega_1^2\omega_2^2\Psi(x))}{\omega_1^4\omega_2^4(\omega_1^2 - \omega_2^2)^2}. \end{aligned}$$

The GDE coefficients and hence the frequencies and amplitudes are time-invariant quantities for sinusoidal signals, but become slowly time-varying (lowpass) quantities for AM–FM signals.

III. **Discrete–Time Algorithm**

Discrete–time two–component AM–FM signals are modeled instantaneously as a two–cosine signal:

$$x[n] = a_1 \cos(\Omega_1 n + \theta_1) + a_2 \cos(\Omega_2 n + \theta_2).$$

The mixture satisfies the following fourth-order generating difference equation

$$c_1(x_{n-1} + x_{n-3}) + c_2 x_{n-2} = -(x_n + x_{n-4}),$$

where we use the compact notation $x_n = x[n]$ and

$$c_1 = -2(\cos\Omega_1 + \cos\Omega_2) \quad , \quad c_2 = 4\cos\Omega_1\cos\Omega_2 + 2.$$

By evaluating the GDE at time instants n and $n+1$ and solving the 2×2 linear system of equations, we obtain

$$c_1 = \frac{\Upsilon_3(x_{n-3}) - \Upsilon_3(x_{n-1})}{\Psi(x_{n-1}) - \Psi(x_{n-2})}$$

$$c_2 = \frac{\Psi(x_n) - \Psi(x_{n-3})}{\Psi(x_{n-1}) - \Psi(x_{n-2})} + \frac{\Upsilon_4(x_{n-2}) - \Upsilon_4(x_{n-3})}{\Psi(x_{n-1}) - \Psi(x_{n-2})},$$

where Υ_k is the k^{th}-order discrete-time energy operator defined in [7] as

$$\Upsilon_k(x_n) \quad \triangleq \quad x_n x_{n+k-2} - x_{n-1}x_{n+k-1}.$$

and Ψ is the discrete-time Teager-Kaiser energy operator [4], [5]:

$$\Psi(x_n) = (x_n)^2 - x_{n+1}x_{n-1}.$$

Note that, for $k = 2$ the discrete Υ_k becomes identical to the discrete Ψ.

The discrete-time EDM frequency estimation algorithm is then given by

$$\Omega_{1,2} = \cos^{-1}\left(\frac{-c_1}{4} \pm \frac{\sqrt{c_1^2 - 4c_2 + 8}}{4}\right).$$

The corresponding EDM amplitude estimation algorithm involves the use of symmetric differences and 2nd-order energy operators:

$$\begin{aligned} a_{1,2}^2 \quad = \quad & \frac{S_{2,1}^4(\Psi({\Delta_s}^3 x) - S_1^2 S_2^2 \Psi(\Delta_s x))}{S_1^4 S_2^4 (S_2^2 - S_1^2)^2} \\ & - \frac{S_{2,1}^6(\Psi({\Delta_s}^2 x) - S_1^2 S_2^2 \Psi(x))}{S_1^4 S_2^4 (S_2^2 - S_1^2)^2}, \end{aligned}$$

where $S_{1,2} = \sin(\Omega_{1,2})$, and

$$\Delta_s x = \frac{x_{n+1} - x_{n-1}}{2} \quad , \quad {\Delta_s}^m x = \Delta_s({\Delta_s}^{m-1} x).$$

Similar results for discrete frequency estimation have been obtained in [11] for two–component sinusoidal signals in the context of AR signal modeling. The novelties of our procedure in the discrete case are: (1) its application to demodulating two–component AM–FM signals; (2) the use of energy operators which give a physically meaningful interpretation to the solution; and (3) amplitude estimation via energy equations instead of least squares.

Finally, note that estimates of the two carrier frequencies and mean amplitudes can be obtained as the time averages of the estimated instantaneous frequency and amplitude signals over the duration of the original signal.

IV. **Performance of the EDM**

As an example, consider a discrete-time mixture signal of two sinusoidally modulated and spectrally close FM signals:

$$\begin{aligned} x[n] &= \sum_{i=1}^{2} a_i \cos\left(\int_0^n \Omega_i[m]dm\right) \\ \Omega_i[n] &= \Omega_{ci} + \Omega_{mi}\cos(\Omega_{fi} n + \theta_i) \quad , \quad i = 1\ ,\ 2. \end{aligned}$$

with parameters

$$\Omega_{ci} = \tfrac{\pi}{4}, \tfrac{\pi}{4} + \tfrac{\pi}{50}, \;\; \theta_i = 0, \pi, \;\; \Omega_{mi} = \Omega_{fi} = \tfrac{\Omega_{ci}}{100}.$$

Despite the significant amount of spectral overlap of the two above mixture components, as shown in Fig. 1(a), the EDM can recover important parts of the original frequency signals, as shown in Fig. 1(b), although with some error whose magnitude increases with the amount of spectral overlap. Post–smoothing after the EDM yields the *smoothed EDM* (SEDM) algorithm, which can suppress a significant amount of demodulation error as shown in Fig. 1(c). Post–smoothing involves moving-average filtering to remove beating at the carrier difference frequency, while 9-point median filtering is employed to remove spikes in the GDE coefficients.

Next we define some performance-related parameters of the mixture signal. The *normalized carrier separation* and *carrier to information bandwidth ratio* parameters are

$$\text{SEP} = \frac{|\Omega_{c2} - \Omega_{c1}|}{\sum_i(\Omega_{fi} + \Omega_{ai} + \Omega_{mi})} \quad , \quad \text{CR/IB} = \frac{\Omega_c}{\max(\Omega_f, \Omega_a)}.$$

The SEP parameter measures the the spectral separation between the components. The denominator of this parameter is the Carson bandwidth of the AM–FM signal, which is a modest under-estimate of the essential bandwidth of the signal. The *signal to interference ratio* of the mixture and the *carrier to frequency deviation ratio(s)* are

$$\text{SIR} = 20\log\left(\frac{\sigma_{x1}}{\sigma_{x2}}\right) \quad \text{and} \quad \text{CR/FD} = \frac{\Omega_c}{\Omega_m},$$

where σ_{xi} are the standard deviations of the components of the mixture. The SIR parameter measures the power of the first component relative to the second component, while the CR/FD parameter measures the strength of frequency modulation.

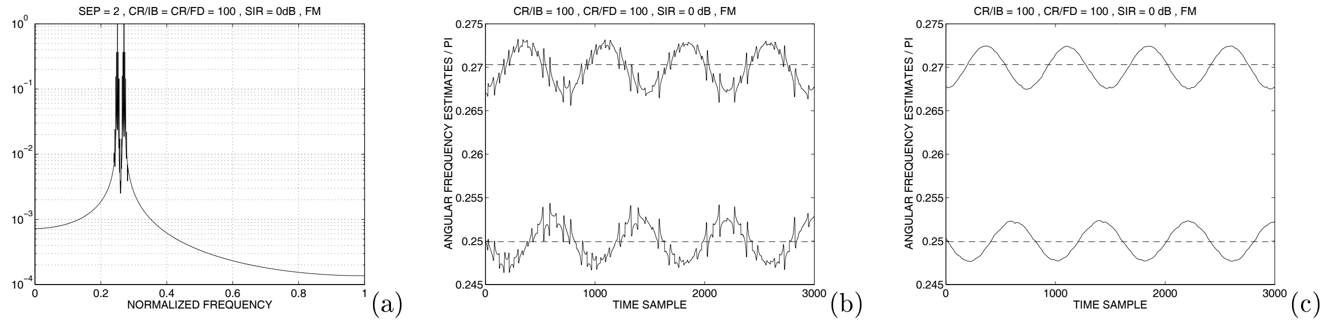

Fig. 1. (a) Fourier spectral magnitude of mixture signal. (b) Angular frequency estimates of the EDM as fractions of π. (c) Frequency estimates of the SEDM, i.e. the estimates in (b) after post–smoothing.

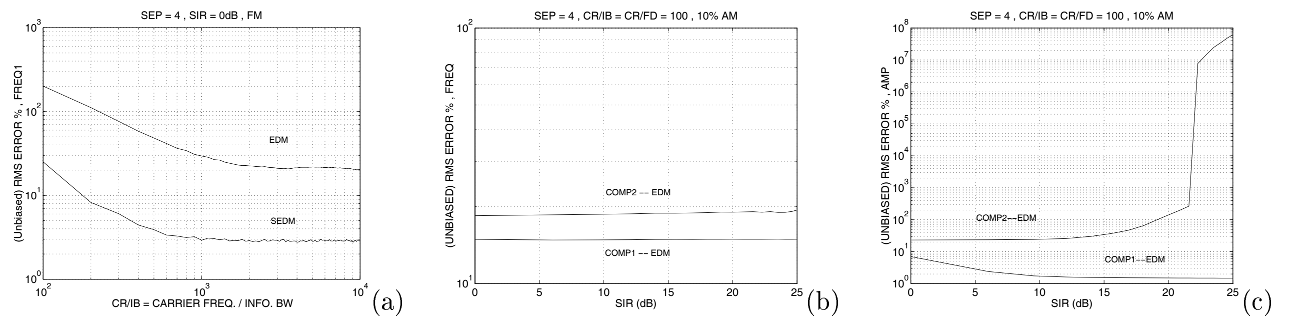

Fig. 2. (a) Effect of CR/IB on EDM frequency demodulation obtained by averaging over FM modulation indices $\in \{0.1 - 1\}\%$. (b,c) Effect of SIR on EDM frequency and amplitude demodulation.

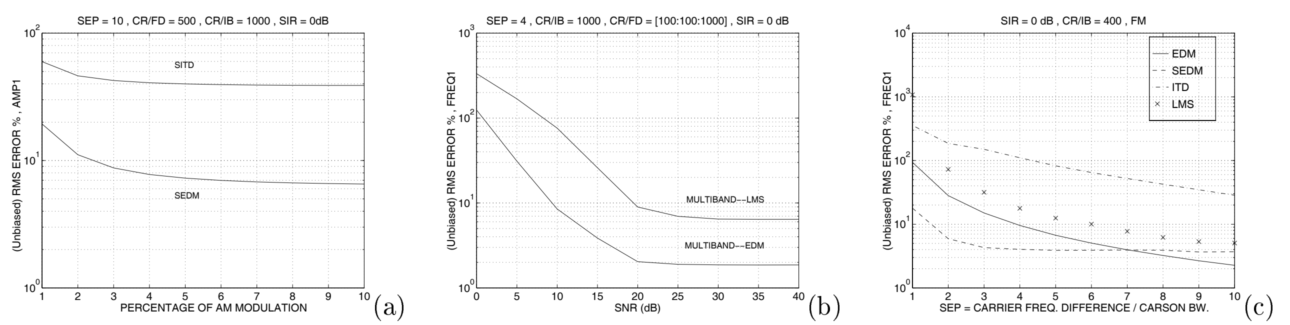

Fig. 3. (a) Comparison of amplitude demodulation between the SEDM and the SITD for different AM modulation percentages. (b) Effect of SNR on the performance of the multiband–EDM (MEDM) and the multiband–LMS (MLMS) (averaged over various CR/FD). (c) Effect of spectral separation on the performance of the EDM, the ITD and the LMS (averaged over CR/FD).

When (the SEP parameter decreases and) the spectral overlap between the components increases beyond a limit, the energy equations of the EDM become ill-conditioned and finally singular as described in the case of the unmodulated sine mixture by

$$\Upsilon_3(x) = 0 \quad , \quad \Psi(x_{n-1}) - \Psi(x_{n-2}) = 0,$$

resulting in an increase in the demodulation error. As the CR/IB parameter increases, the FM mixture approaches a stationary sinusoidal mixture, resulting in a decrease in the demodulation error as described in Fig. 2(a). The CR/FD parameter measures the strength of signal modulations. As the CR/FD parameter increases, the strengths of the modulations in the components decrease. As the SIR increases, the strength of the first component in the mixture increases and that of the second component decreases. For large SIR parameters, the stronger component dominates the mixture, resulting in a decrease in the amplitude demodulation error of the stronger component, while that of the weaker one rapidly increases as described in Fig. 2(c). The EDM frequency estimator on the other hand is derived from the GDE of the composite signal invariant to amplitudes [6] and consequently frequency demodulation is independent of SIR as described by Fig. 2(b). In situations where the signal of interest is corrupted with additive white Gaussian noise (AWGN), the EDM is used in conjunction with the multiband filtering scheme proposed in [12]. Namely, in what we call the *multiband–EDM* (MEDM), the mixture signal is first filtered through a bank of FIR filters that sample the frequency domain densely. An energy detector [12] determines the channel that is more active (largest mean short-time energy operator output). The output of this channel is then used for demodulation via the EDM, while the outputs of all the other less active channels are discarded.

V. Comparisons

Although the EDM, the modified ITD[1] [8] and the LMS [10] have a common goal of demodulating two–component AM–FM signals, they differ in terms of their nature, their complexity, their time resolution, and noise suppression capabilities. The EDM, the ITD, and (4th order) adaptive linear prediction via the LMS are all based on estimating the coefficients of the GDE of the composite signal. The EDM algorithm uses higher-order energy operators to instantaneously estimate the coefficients, the ITD uses instantaneous operators obtained as determinants of 4×4 Toeplitz matrices to perform this task, while the LMS uses signal correlations to estimate the coefficients in a least squares and adaptive sense.

The ITD and the LMS as implemented in [8], [10] require the computation of 4 or more GDE coefficients. The LMS also requires extensive optimization of several parameters like the step size and filter order. The EDM algorithm exploits the inherent symmetry of the problem requiring the computation of only two GDE coefficients and does not require extensive optimization of parameters.

The time resolution of the ITD is governed by the assumption that the GDE coefficients are stationary over four time–steps. The time resolution and the length of the transients in the LMS estimates are governed by the order of the predictor and the step size parameter. The EDM algorithm is nonadaptive, does not exhibit transients in the estimates and requires the GDE coefficients stationary over two time–steps, thereby enabling better tracking of the instantaneous frequencies and amplitudes of two–component AM–FM signals.

The amplitude estimation algorithm of the ITD and the LMS involves the integration of the instantaneous frequency estimates and the least squares solution to a system of linear equations in the amplitudes and phase offsets to smooth out the noise accumulated during integration process. These factors coupled with non-ideal linear filtering of the FM signal in the ITD[2] contribute to a larger amplitude estimation error in the ITD as compared to the EDM as evident in Fig. 3(a) for different AM modulation percentages. Similarly for the case of EDM versus LMS.

The MEDM is used for demodulation of noisy signals, where the filters are optimized for minimum harmonic distortion. For small SNR parameters, filters with small bandwidths are needed, while for large SNR parameters, filters with larger bandwidths are more appropriate. Noise suppression in the multiband–EDM is achieved by picking the most energetic of the channels and discarding all the other less active channels. The comparison of MEDM with the MLMS) (LMS in conjunction with multiband filtering) for different SNR parameters is depicted in Fig. 3(b).

As the SEP parameter decreases, the EDM and the ITD develop singularities which manifest as beating in the estimates, while the LMS coefficients develop convergence problems. For smaller separations, the beating in the EDM and the ITD intensifies and swamps the information signals, while the LMS coefficients do not converge even for large step sizes. The comparison of the algorithms for different SEP parameters is shown in Fig. 3(c).

[1] The original algorithm in [8] had mistakes of the kind $\sqrt{\cos^2 \omega} = \cos\omega$. Our modification was to solve for the $\cos(\Omega_1)$ and $\cos(\Omega_2)$ instead of solving for $\cos(\Omega_1 \pm \Omega_2)$.

[2] Energy equations applied to the output of a linear filter approximately hold only if the frequency response is narrow and flat in its passband [12].

VI. Conclusion

Applied to a two-component AM–FM signal mixture, the EDM algorithm yields efficient estimates for the instantaneous amplitude and frequency signals of each component. For a mixture of two cosines the EDM algorithm yields exact quantities. The EDM algorithm is computationally efficient because it does does not require any optimization of parameters and exploits the natural symmetry of the signal mixture to estimate a minimal number of coefficients of the generating differential/difference equation via the use of differential energy operators. These operators have a low complexity and excellent time resolution. Further they have the advantages over previous linear predictive approaches in that they do not require signal correlations and give a physically meaningful interpretation to the solution that involves manipulation of low-pass energy signals.

References

[1] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "On Separating Amplitude from Frequency Modulations Using Energy Operators," in *Proceedings of ICASSP–92*, vol. 2, (San Fransisco, CA), pp. 1 – 4, March 1992.

[2] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "On Amplitude and Frequency Demodulation Using Energy Operators," *IEEE Trans. on Sig. Proc.*, vol. 41, pp. 1532 – 1550, April 1993.

[3] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "Energy Separations in Signal Modulations with Application to Speech Analysis," *IEEE Trans. on Sig. Proc.*, vol. 41, pp. 3024 – 3051, Oct 1993.

[4] J. F. Kaiser, "On a simple algorithm to calculate the energy of a signal," in *Proceedings of ICASSP–90*, (Albuqerque, New Mexico), April 1990.

[5] J. F. Kaiser, "On Teager's Energy Algorithm and its Generalization to Continuous signals," in *Proceedings IEEE DSP Workshop*, (New Paltz, NY), Sep 1990.

[6] A. Zayezdny and I. Druckmann, "A new method of signal description and its applications to signal processing," *Signal Processing*, vol. 22, pp. 153 – 178, 1991.

[7] P. Maragos and A. Potamianos, "Higher Order Differential Energy Operators," *IEEE Sig. Proc. Letters*, vol. 2, pp. 152 – 154, Aug 1995.

[8] R. Kumaresan, A. G. Sadasiv, C. S. Ramalingam, and J. F. Kaiser, "Instantaneous Nonlinear Operators For Tracking Multicomponent Signal Parameters," in *Proc. of the 6^{th} SP Workshop on Statistical Signal and Array Processing*, (Victoria, BC, Canada), pp. 404 – 407, Oct 1992.

[9] R. Kumaresan and A. G. Sadasiv, "An Invariance Property of the Determinant of a Matrix Whose Elements are a Sum of Sinusoids and Its Application," *Archiv fur Electronik and Ubertragungstechnik (Int'l J. Electronics and Communication)*, vol. 47, no. 2, pp. 119 – 122, 1993.

[10] L. J. Griffiths, "Rapid Measurement of Digital Instantaneous Frequency," *IEEE Trans. on ASSP*, vol. 23, pp. 207 – 222, April 1975.

[11] T.-H. Li and B. Kedem, "Iterative Filtering for Multiple Frequency Estimation," *IEEE Transactions on Signal Processing*, vol. 42, pp. 1120 – 1132, May 1994.

[12] A. C. Bovik, P. Maragos, and T. F. Quatieri, "AM–FM Energy Detection and Separation in Noise Using Multiband Energy Operator," *IEEE Trans. on Sig. Proc.*, vol. 41, pp. 3245 – 3265, Dec 1993.