# POWER SYSTEMS STABILIZATION WITH FEEDBACK LINEARIZATION

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#### Abstract

Input-state feedback linearization is discussed in this paper as a method for power system control. This method linearizes for a range of operating points and is more appropriate than the conventional methods since the operating point can change frequently in power systems. A single generator, infinite bus model is used to illustrate the design.

#### 1. INTRODUCTION

The control theory of linear systems is a very well defined area and has been used in many years with good results. In the real world nearly all systems are nonlinear, and are therefore usually linearized before being controlled.

A feedback-linearization approach can be used to cancel out the nonlinearities of a nonlinear system by feeding them back into the system, and gives a linear closed-loop system, which can be controlled with linear control theory.

To the best of our knowledge, in power systems only few papers have been published on this method [1-4], and they have only dealt with the single-input case. In this paper the objective is to consider the multi-input case, and to find out if input-state feedback linearization can be used on a single generator, infinite bus model where the inputs are the mechanical torque, and the generator field voltage.

# 2. FEEDBACK LINEARIZATION OF POWER GENERATOR CONNECTED TO AN INFINITE BUS

When studying power system stabilizers the system is often simplified to a single-generator, infinite bus model as shown in Fig. 2.1.

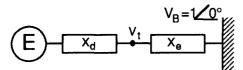


Figure 2.1: Single-generator, infinite bus model

There exist different state-space models of the single-generator, infinite bus model. Their dimensions can vary, depending on how accurate a model is required, and what states are of interest [5]. The model that will be used here has dimension n=3, and the state vector has the following components:  $\delta$ , the synchronous generator phase angle,  $\omega$ , the rotor angular velocity, and  $e'_q$ , the voltage behind the transient reactance of the generator. The system is described by the following nonlinear state equations [6]:

$$\dot{x}_1 = \omega_{B} \cdot (x_2 - 1) 
\dot{x}_2 = -A \cdot B \cdot x_3 \cdot \sin(x_1) - A \cdot C \cdot \cos(x_1) - A \cdot u_1 
\dot{x}_3 = -(D + D \cdot E) \cdot x_3 + D \cdot F \cdot \cos(x_1) + D \cdot u_2$$
(2.1)

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where 
$$\underline{x} = \begin{bmatrix} \delta \ \omega \ e_q^{\top} \end{bmatrix}^T$$
,  $\underline{u} = \begin{bmatrix} P_m & E_{fd} \end{bmatrix}^T$ , and 
$$A = \frac{\omega_o}{2 \cdot H} \quad B = \frac{V_\infty}{(x_d^{\top} + x_e)} \quad C = \frac{(x_d^{\top} + x_e) \cdot V_\infty^2}{2 \cdot (x_d^{\top} + x_e)(x_q + x_e)}$$
 
$$D = \frac{1}{T_{do}^{\top}} \quad E = \frac{(x_d - x_d^{\top})}{(x_d^{\top} + x_e)} \quad F = \frac{(x_d - x_d^{\top})}{(x_d^{\top} + x_e)} \cdot V_\infty$$

(For the definition of the parameters see Appendix A)

After finding out if the system is feedback linearizable, a state transformation can be found [7]. This gives a linear system that has the following state vector:

$$\underline{z} = \begin{bmatrix} x_1 - x_{1o} & \omega_B \cdot (x_2 - 1) & x_3 - x_{3o} \end{bmatrix}^T = \begin{bmatrix} \Delta \delta & \omega_B \cdot \Delta \omega & \Delta e_q \end{bmatrix}^T$$

and input transformation:

$$u_{1} = \frac{v_{1}}{\omega_{B} \cdot A} + B \cdot x_{3} \cdot \sin(x_{1}) + C \cdot \sin(2x_{1})$$

$$u_{2} = \frac{v_{2}}{D} + (1 + E) \cdot x_{3} - F \cdot \cos(x_{1})$$
(2.2)

The resulting linear system is then:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$
 (2.3)

## 4. CONTROL DESIGN AND SIMULATIONS

In this section the responses of the nonlinear, and the linearized systems to a disturbance are simulated. The disturbance is generated by a drop in the infinite bus voltage  $V_{\infty}$  from 1.0 p.u. to 0.0 p.u. at time 1.0 s. The values of the system's parameters used in these simulations are given in the Appendix and are from a generator in the Icelandic power system.

In order to evaluate the performance of feedback linearizing controllers, we choose a linear state-feedback controller, where the feedback matrix is:

$$\mathbb{K} = \begin{bmatrix} 30170 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 3.1 shows the response of the system in (2.1) with the state feedback. In figure 3.2 the system is feedback linearized, and the oscillations are suppressed, the disturbance is smaller and the settling time is much faster, than that of figure 3.1.

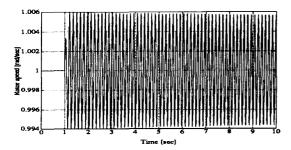


Figure 3.1: Response of nonlinear system to disturbance at 1 sec.

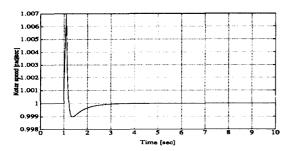


Figure 3.2: Response of linearized system to disturbance at 1 sec.

## 4. Backstepping

The system in (2.1) is not complete; because  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are not accessible inputs. To obtain a more realistic model the linear dynamics of governor, turbine, voltage regulator, and exciter were added. These systems have the following state space equations:

Governor and turbine:

$$\dot{\xi}_{1-3} = \mathbf{A}_1 \cdot \dot{\xi}_{1-3} + \dot{\mathbf{b}}_1 \cdot \mathbf{r}_1 
\mathbf{u}_1 = [0 \ 0 \ 1] \cdot \dot{\xi}_{1-3}$$
(4.1)

Voltage regulator and exciter:

$$\dot{\xi}_{4-5} = \mathbf{A}_2 \cdot \underline{\xi}_{4-5} + \underline{b}_2 \cdot \mathbf{r}_2$$

$$\mathbf{u}_2 = [0 \ 1] \cdot \underline{\xi}_{4-5}$$
(4.2)

The governor and turbine system is non-minimum phase, but the voltage regulator and exciter system is minimum phase. The two inputs from (2.2) are known, but we need to find the physical accessible inputs  $r_1$  and  $r_2$ . To do that, we tried to use backstepping [8]. Because the system in (4.1) is non-minimum phase there is no simple solution, but the system in (4.2) has a simple backstepping solution. On the other hand, the solution for  $r_2$  does not give the correct response.

# 5. CONCLUSIONS

The use of input-state feedback linearization on a single generator, infinite bus model has been presented in this paper. The model used was linearizable, and as simulations show, the linearized system permits the use of linear control with good results. It also permits the continued use of robust linear controller or any other controller from linear

control theory. We have also investigated the use of backstepping with limited success.

#### **APPENDIX**

ω: rotor angular velocity

 $\delta$ : phase angle

 $e_q^{\prime}$ : transient induced voltage  $\omega_B$ : infinite-bus angular velocity

V<sub>∞</sub> : infinite-bus voltage
P<sub>m</sub> : mechanical power
E<sub>fd</sub> : field voltage
H : inertia constant

T<sub>do</sub>: field time constant

 $\begin{array}{llll} x_d & : & d\text{-}axis \ synchronous \ reactance} \\ x_d' & : & d\text{-}axis \ transient \ reactance} \\ x_\alpha & : & q\text{-}axis \ synchronous \ reactance} \end{array}$ 

x<sub>e</sub>: external reactance

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