Lumped-Element Region

At any frequency, a transmission line can be shorted to a length below which the line operates in as a lumped-element circuit.

The boundary is defined by all combination of ω and l for which the magnitude of the *propagation coefficient* $l\gamma(\omega)$ remains **small**, i.e., **less than** Δ.

Δ is typically set to **0.25** (1/4)

$$|l\gamma(\omega)| < \Delta$$

l is the length of the transmission line

$\gamma(\omega)$ is the propagation coefficient (neper/m)

For typical digital transmission applications, the propagation coefficient **increases monotonically**.

Therefore, the inequality need be checked only at the maximum length and maximum anticipated frequency.

The boundary of the lumped element region can be approximated

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)}$$

Start with propagation coefficient

Lumped-Element Region

Assume R, L and C are constants that do not vary with frequency.

Substituting into boundary condition, solve for l

$$|l\gamma(\omega)| < \Delta \longrightarrow l_{\mathrm{LE}} = \left|\frac{0.25}{\sqrt{(j\omega L + R)(j\omega C)}}\right|$$

Since the boundary is 'fuzzy', we can substitute precise calculations with asymptotic approximations.

Here, the two boundaries are defined for l (meters) depending on the relative value of jωL and R. Both are constrained by $\Delta = 0.25$.

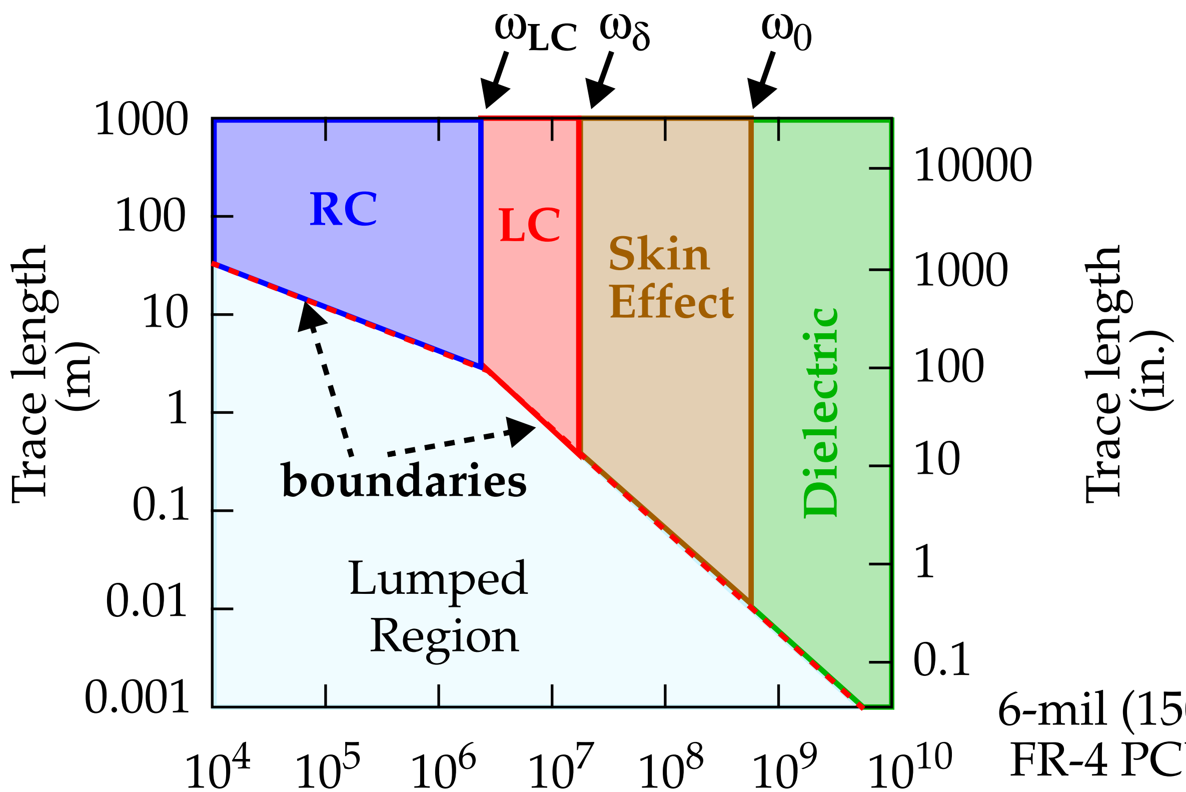
$$l\gamma = l\sqrt{(j\omega L + R)(j\omega C)}$$

$$l_{LE} \approx \frac{\Delta}{\sqrt{\omega R_{DC} C}} \quad \text{for } (\omega < R_{DC}/L) \quad \text{(RC product dominates)}$$

$$l_{LE} \approx \frac{\Delta}{\omega\sqrt{LC}} \quad \text{for } (\omega > (R_{DC}/L)) \quad \text{(LC product dominates)}$$

These constraints ensure the transmission line delay **remains much smaller** than the signal's rise and fall times.

Lumped-Element Region



6-mil (150 µm), 50-Ω, FR-4 PCB stripline

A discontinuity in the boundary is evident in the figure, creating a two-segment boundary.

The ω outside the sqrt() in the second constraint indicates that *l* decreases more rapidly beginning with LC mode.

Lumped-Element Region

Because the delay of the line is short, the source and load exert an almost ***instantaneous*** influence on the system behavior.

The *tight coupling* between the source and load impedances indicate that lumped-element operation rarely requires *termination*.

- Except in cases involving very low-impedance drivers coupled to large reactive loads.

Bear in mind that ***two*** fully independent modes of propagation still exist (out and back).

The short line causes a portion of the signal's transition to propagate to the load, interact and reflect back to the source, affecting the input impedance.

In contrast, on long lines, the long transit time *disconnects* the source and load in the temporal sense.

- Here, information about the load reflects back to the source too late to affect the progress of an individual rising or falling edge.

Lumped-Element Region: Input Impedance

Our objective is to examine the **input impedance** of a lumped-element structure under various conditions of loading.

The following portions of a Taylor-series expansion may be used to approximate H and H^{-1} in the lumped-element region

$$\frac{H^{-1}+H}{2} \approx 1 + \frac{(l\gamma)^2}{2}$$

$$\frac{H^{-1}-H}{2} \approx (l\gamma) + \frac{(l\gamma)^3}{6}$$

Applying these to our general equation for **input impedance**.

$$Z_{\text{in, loaded}} = Z_C\left(\frac{\left(\frac{H^{-1}+H}{2}\right)+\frac{Z_C}{Z_L}\left(\frac{H^{-1}-H}{2}\right)}{\left(\frac{H^{-1}-H}{2}\right)+\frac{Z_C}{Z_L}\left(\frac{H^{-1}+H}{2}\right)}\right)$$

Lumped-Element Region: Input Impedance

Neglecting all but the constant and linear terms yields

$$Z_{\text{in, loaded}} = Z_C \left\{ \frac{1 + \frac{Z_C}{Z_L}(l\gamma)}{(l\gamma) + \frac{Z_C}{Z_L}} \right\}$$

Under conditions that the line is **lightly loaded** ($Z_L >> Z_C$), the right hand terms in the numerator and denominator vanish, leaving

$$Z_{\text{in, open-circuited}} \approx Z_C \left\{ \frac{1}{l\gamma} \right\}$$

Plugging in reveals that the input impedance of a short, unloaded line looks entire capacitive

$$Z_{\text{in, open-circuited}} \approx \sqrt{\frac{(j\omega L + R)}{j\omega C}} \left\{ \frac{1}{l\sqrt{(j\omega L + R) j\omega C}} \right\} = \frac{1}{l * j\omega C}$$

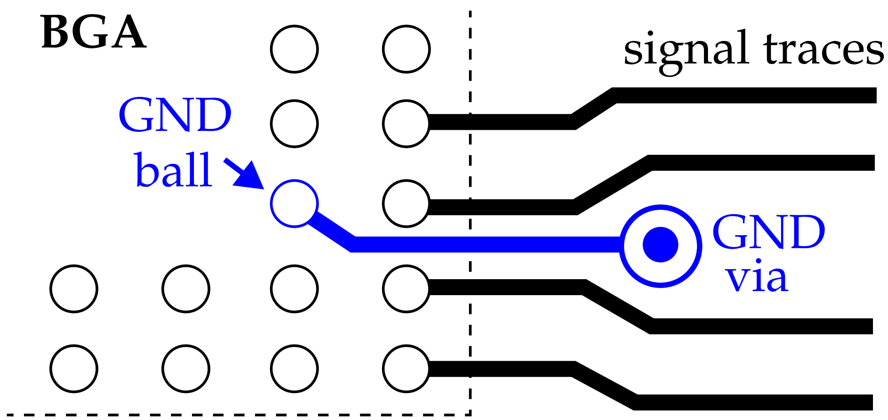
Lumped-Element Region: Input Impedance

And the total capacitance is the total distributed capacitance of the line, i.e., l*C.

Remember, this works only when the line delay is short compared to the signal rise and fall time (1/6 and 1/3 at most) AND

The line is lightly loaded at its endpoint.

Consider the case when the line is short-circuited to ground at the far end.



Short 'jumper' connection is routed from the GND pin to a GND via.

This is commonly done (but not a good idea) because of congestion around the BGA pins

What is the effective input impedance of this trace leading to GND, from the perspective of the chip?

Lumped-Element Region: Input Impedance

Since the line is shorted, the impedance of the load is much lower than the line impedance, i.e., $Z_L << Z_C$.

This term inflates the right-hand terms, causing them to dominate

$$Z_{\text{in, loaded}} = Z_C \left\{ \frac{1 + \frac{Z_C}{Z_L}(l\gamma)}{(l\gamma) + \frac{Z_C}{Z_L}} \right\}$$

This yields a simple expression for input impedance.

$$Z_{\text{in, short-circuited}} \approx Z_C \{ l\gamma \}$$

Plugging in shows the input impedance is either **inductive** or **resistive**, depending on the ratio of $j\omega L$ to R.

$$Z_{\text{in, short-circuited}} \approx l^*(j\omega L + R)$$

Lumped-Element Region: Input Impedance

In digital apps, the *inductance* of the trace is usually much more significant.

The amount of inductance is the total distributed inductance ($l*L$) of the transmission line, defined by the trace and its return path.

This equation is useful for computing **ground-bounce** when a current $i(t)$ passes through the trace to GND.

In a third case, when the transmission line is properly terminated, ($Z_L = Z_C$), the numerator and denominator are equal yielding Z_C.

One final point is that for lines operated at frequencies below the onset of the LC region, the input impedance **is not constant**.

Rather it is a strong frequency-varying quantity with phase at 45 degrees.

Accurately matching the impedance Z_C in this region **is not trivial**, so its fortunate that most short lines do not need termination.

Lumped-Element Region: Circuit Gain

To compute the gain, substitute

$$\frac{H^{-1} + H}{2} \approx 1 + \frac{(l\gamma)^2}{2}$$

$$\frac{H^{-1} - H}{2} \approx (l\gamma) + \frac{(l\gamma)^3}{6}$$

into

$$G_{\mathrm{FWD}} = \frac{v_3}{v_1} = \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)\left(1 + \frac{Z_S}{Z_L}\right) + \left(\frac{H^{-1} - H}{2}\right)\left(\frac{Z_S}{Z_C} + \frac{Z_C}{Z_L}\right)\right]}$$

This yields

$$G = \left[\left(1 + \frac{Z_S}{Z_L}\right) + l\gamma\left(\frac{Z_S}{Z_C} + \frac{Z_C}{Z_L}\right) + \frac{(l\gamma)^2}{2}\left(1 + \frac{Z_S}{Z_L}\right) + \frac{(l\gamma)^3}{6}\left(\frac{Z_S}{Z_C} + \frac{Z_C}{Z_L}\right)\right]^{-1}$$

Lumped-Element Region: Circuit Gain

What are the conditions needed to achieve gain flatness?

As the term $l*\gamma$ approaches zero, all terms associated with its various powers vanish, and the propagation function approaches

$$G = \frac{Z_L}{(Z_S + Z_L)}$$

This is exactly what you would expect if the source and load were directly connected (no line).

We assumed that the magnitude of the coefficient $l*\gamma$ in the lumped element region remains less than $\Delta = 1/4$.

This allows you to ignore the right-most two elements in the gain Eq.

The second term can be ignored too if

$$\left| l\gamma \frac{Z_S}{Z_C} \right| << 1 \quad \text{and} \quad \left| l\gamma \frac{Z_C}{Z_L} \right| << 1 \quad \text{and} \quad |l\gamma| < 0.25$$

Lumped-Element Region: Circuit Gain

Inserting definitions for γ and Z_C

$$|Z_S| << \left|\frac{1}{l * j\omega C}\right|$$

$$|l * (j\omega L + R)| << |Z_L|$$

Therefore, for the line to not exert any deleterious influence over signal quality, these conditions must ALSO be met, above and beyond

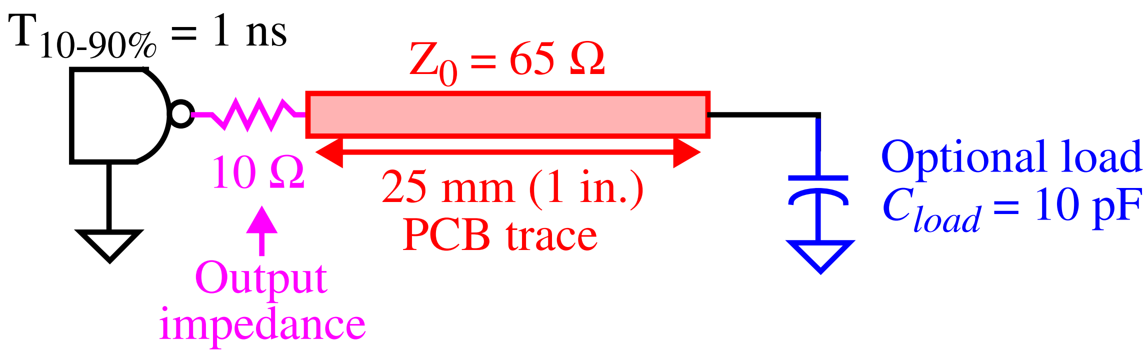
$$|l\gamma| < 0.25$$

In words:

- The **source impedance** of the driver MUST be *much smaller* than the impedance represented by the **total shunt capacitance** of the line.
- The **total series impedance** of the line MUST remain *much smaller* than the impedance of the **load**.

Lumped-Element Region

Example



Effective dielectric constant: 3.8

High frequency propagation velocity:

$$v_0 = \frac{c}{\sqrt{3.8}} = 1.54 \times 10^8 \text{ m/s}$$

DC resistance: 3 Ω/m

Operating frequency (corresponds to the center of the spectral lobe associated with each rising and falling edge)

$$\omega = \frac{2\pi * (0.35)}{1 \text{ ns}} = 2.2 \times 10^9 \text{ rad/s}$$

Lumped Element Region: Example

Compute transmission line parameters, L and C

$$L = \frac{Z_0}{v_0} = 422\ \text{nH/m} \qquad C = \frac{1}{Z_0 v_0} = 100\ \text{pF/m}$$

Since the *inductive effects* of the line far outweighs the *resistance*, you can approximate the magnitude of the propagation coefficient using

$$|l\gamma| = l\omega\sqrt{LC}$$

$$|l\gamma| = 0.025 * 2.2\times10^{9}\sqrt{(422\ \text{nH/m})(100\ \text{pF/m})} = 0.357$$

This value is just outside the 'official' boundary of the lumped-element region at 0.25.

Check conditions

$$|Z_S| << \left|\frac{1}{l * j\omega C}\right|$$

$$|10\ \Omega| << \left|\frac{1}{0.025 * 2.2\times10^{9} * 100\ \text{pF/m}}\right| = 182\ \Omega \qquad \text{Yes}$$

Lumped Element Region: Example

And

$$|l*(j\omega L + R)| << |Z_L|$$

For the case of no load capacitance (infinite Z_L), this condition clearly holds.

Therefore, for rise/fall times no faster than 1 ns, this microstrip without a load induces practically ***no*** distortion in the transmitted wfm.

What about the 10 pF case?

$$|Z_L| = \frac{1}{(2.2\times10^9)(10\times10^{-12})} = 45.5\ \Omega \longleftarrow Z_L = \frac{1}{j\omega C}$$

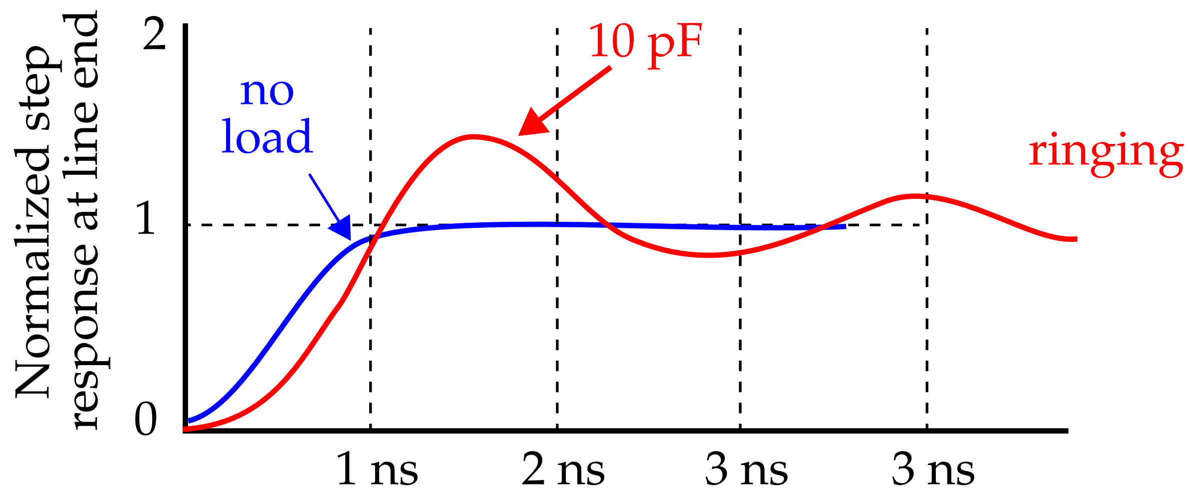
$$|l(j\omega L)| = 0.025*2.2\times10^9*422\ \text{nH/m} = 23.2\ \Omega$$

Here, the magnitude of Z_L exceeds the series impedance of the line by only a small amount (2:1)

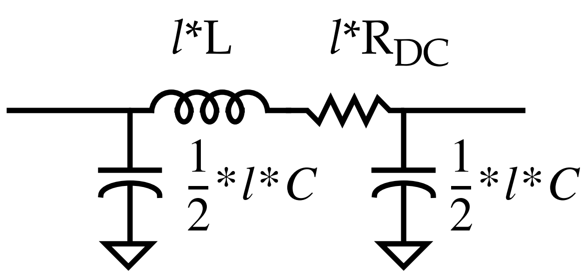
This implies the transmission line will have a noticable effect.

Lumped Element Region: Example

Plotting



The *pi-model* can be used to approximate the behavior of a short transmission line



Lumped Element Region: Example

From our example, the values of the variables for the **unloaded** version are

$$l*L = 0.025*422 \text{ nH/m} = 10.6 \text{ nH}$$

$$\frac{1}{2}(l*C) = 0.5*0.025*100 \text{ pF/m} = 1.25 \text{ pF}$$

When driven by a low-impedance source, the capacitor on the left plays only a small role.

The main effect is the R-L-C series-resonant circuit formed by the output resistance of the driver, the series inductance and capacitor on the right.

The resonant frequency under **no loading**

$$\omega_{res} = \frac{1}{\sqrt{(l*L)\left(\frac{1}{2}l*C\right)}} = \frac{1}{\sqrt{10.6\times10^{-9}*1.25\times10^{-12}}} = 8.7\times10^{9} \text{ rad/s}$$

This is well above the *spectral center of gravity* of the rising and falling edges (2.2 X 10^9 rad/s), so resonance does not occur.

Lumped Element Region: Example

With a 10 pF load, the situation changes.

The new load capacitance *adds* to the capacitance on the right in the *pi* model, reducing the resonance frequency

$$\omega_{res} = \frac{1}{\sqrt{(l*L)\left(\frac{1}{2}l*C + C_{load}\right)}} = \frac{1}{\sqrt{10.6\times10^{-9} * 11.25\times10^{-12}}} = 2.9\times10^{9}\ \text{rad/s}$$

This is close to the bandwidth of the driver at 2.2 X 10^9 rad/s.

The resonance is obvious in the plot.

The period of the resonance is

$$\frac{2\pi}{\omega_{res}} = \frac{2\pi}{2.9\times10^{9}\ \text{rad/s}} = 2.2\ \text{ns}$$

This corroborates the ringing observable in the plot.