RC Region (dispersive transmission line)

RC mode includes all combinations of ω and *l* for which the line behaves in a **distributed manner**.

Also, the frequency remains *well below* the point at which the magnitude of ωL approaches the DC resistance of the line, R_{DC}.

The RC region extends from DC up to frequency ω_{LC} (the LC mode cutoff).

At this point, the reactive component of the propagation coefficient, ωL, becomes equal to the magnitude of the resistive component, R_{DC}.

$$\omega_{LC} = \frac{R_{DC}}{L}$$

The length, *l*, of the transmission line where you need to start worrying about RC mode (vs. lumped-element mode) is obtained from

$$l_{LE} \approx \frac{\Delta}{\sqrt{\omega R_{DC} C}} \quad \text{for } (\omega < R_{DC}/L)$$

Boundary between lumped-element (LE) mode and RC mode

$$\Delta = 0.25$$

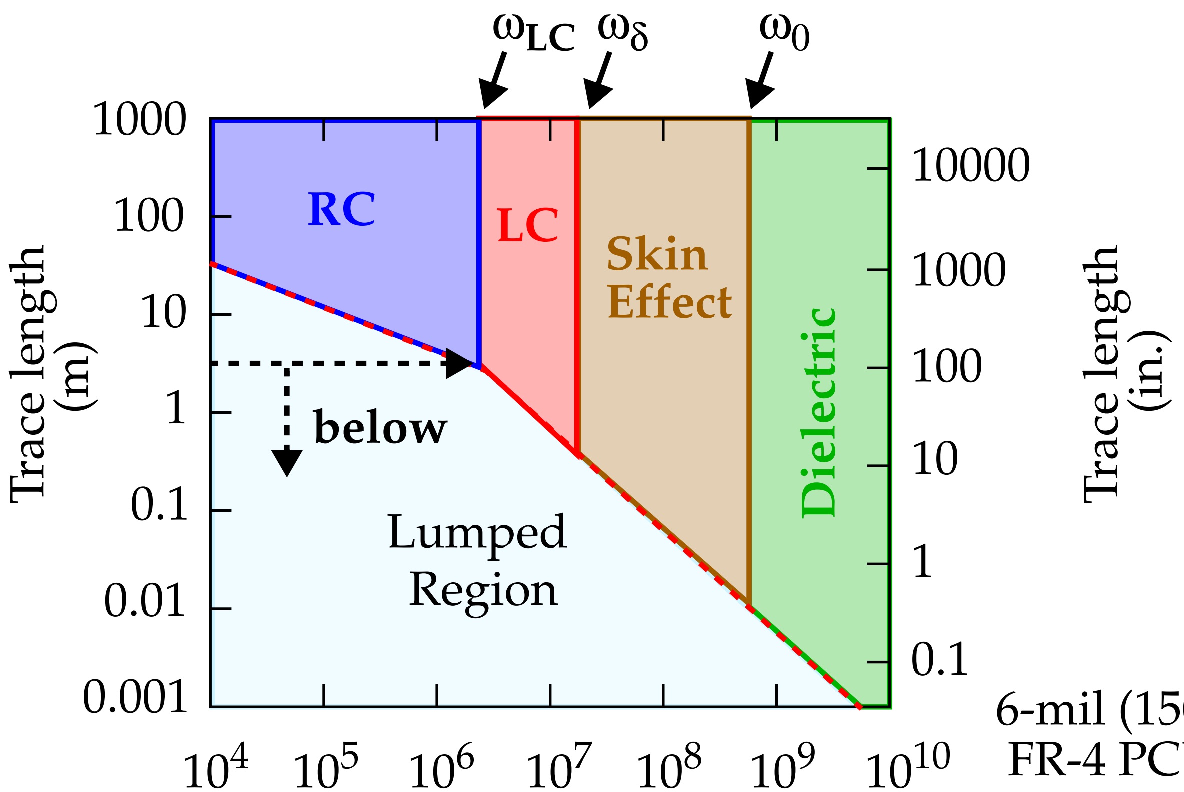
RC Region

Substituting ω_{LC} into this equation yields

$$l_{RC} = \frac{\Delta}{\sqrt{\omega R_{DC} C}} = \frac{\Delta}{\sqrt{\frac{R_{DC}}{L} R_{DC} C}} = \frac{\Delta}{R_{DC}}\sqrt{\frac{L}{C}}$$

$\Delta = 0.25$
L in H/m
C in F/m

Below this length, distributed RC behavior does NOT occur.



6-mil (150 µm), 50-Ω, FR-4 PCB stripline

RC Region

Therefore, we are in RC mode when the total DC resistance of the line, $l*R_{DC}$, **grows** to a value comparable to the high frequency impedance sqrt(L/C).

$$l_{RC}R_{RC} = \Delta\sqrt{\frac{L}{C}}$$

Note that from the figure in slide 2, we can go directly from lumped-element mode to LC mode, e.g., at 1 meter.

For the PCB trace, its resistance at one meter is only 6.3 Ω, which is much smaller than the line impedance of 50 Ω.

For this reason, PCB designers never need to worry about RC mode at the board level.

Telephone lines (24-gauge) will begin exhibiting RC mode around 100 m.

Interesting RC mode **does** occur on-chip, over much smaller wires.

This is due to the larger resistance of the wires, e.g., polysilicon.

RC Region: Input Impedance

The **input impedance** varies strongly with the length of the line and the type of load connected.

$$Z_{\text{in, loaded}} = Z_C\left(\frac{\left(\frac{H^{-1}+H}{2}\right)+\frac{Z_C}{Z_L}\left(\frac{H^{-1}-H}{2}\right)}{\left(\frac{H^{-1}-H}{2}\right)+\frac{Z_C}{Z_L}\left(\frac{H^{-1}+H}{2}\right)}\right)$$

Recall that line length is incorporated in H

$$H(\omega, l) = e^{-l\gamma(\omega)}$$

This complicates the design of reactive source and load networks needed to establish some target equalization goal in the propagation function.

The problem can be solved by providing end-termination such that $Z_L = Z_C$.

This *eliminates* reflections and makes the input impedance equal to Z_C, **independent of line length**.

RC Region: Input Impedance

A second solution is for the transmission line to be very long such that H takes on a value significantly less than 1.

Here, the inverse-gain, H^{-1}, vastly exceeds H, and allows H to be ignored in the input impedance expression, making $Z_{in,\ loaded} = Z_C$.

We gave the **characteristic impedance** earlier (ignoring G) as

$$Z_C = \sqrt{\frac{j\omega L + R}{j\omega C}}$$

But in RC mode, we assumed ωL to be small compared to R

$$Z_C = \sqrt{\frac{R}{j\omega C}} = \left(\frac{1-j}{\sqrt{2}}\right)\sqrt{\frac{R}{\omega C}} \qquad \text{since} \qquad \sqrt{\frac{1}{j}} = \sqrt{-j} = \frac{(1-j)}{\sqrt{2}}$$

This expression is a complex function of frequency with a phase angle of -45 degrees and a magnitude slope of -10 dB/decade

$$|Z_C| = \sqrt{\frac{R}{\omega C}} \rightarrow 20\log\left(\sqrt{\frac{1}{10}}\right)\text{dB} = -10\text{dB}$$

$$\angle Z_C = \tan^{-1}\left(\frac{(-1/\sqrt{2})\sqrt{R/(\omega C)}}{(1/\sqrt{2})\sqrt{R/(\omega C)}}\right) = -45°$$

RC Region: Input Impedance

As an example, the two wires (AWG 24) running from the central telephone switching office to your phone represent an RC transmission line.

The wires are twisted, yielding the following L, C and R values:

R = 0.165 Ω/m

L = 400 nH/m

C= 40 pF/m

$\omega = 1600 \text{ Hz} \times 2\pi = 10053 \text{ rad/s}$

$$Z_C = \sqrt{\frac{R + j\omega L}{j\omega C}} = \sqrt{\frac{0.165 + j0.00402}{j4.02\times10^{-7}}}$$

$$|Z_C| = \sqrt{\frac{\sqrt{0.0272 + 1.6\times10^{-5}}}{4.02\times10^{-7}}} = 640.6\ \Omega \qquad \angle Z_C = \sqrt{\frac{\tan^{-1}\left(\frac{0.00402}{0.165}\right)}{90°}} = -44.3°$$

$$|Z_C| = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{0.165}{j4.02\times10^{-7}}} = 640.7\ \Omega \qquad \angle Z_C = \sqrt{\frac{0°}{90°}} = -45°$$

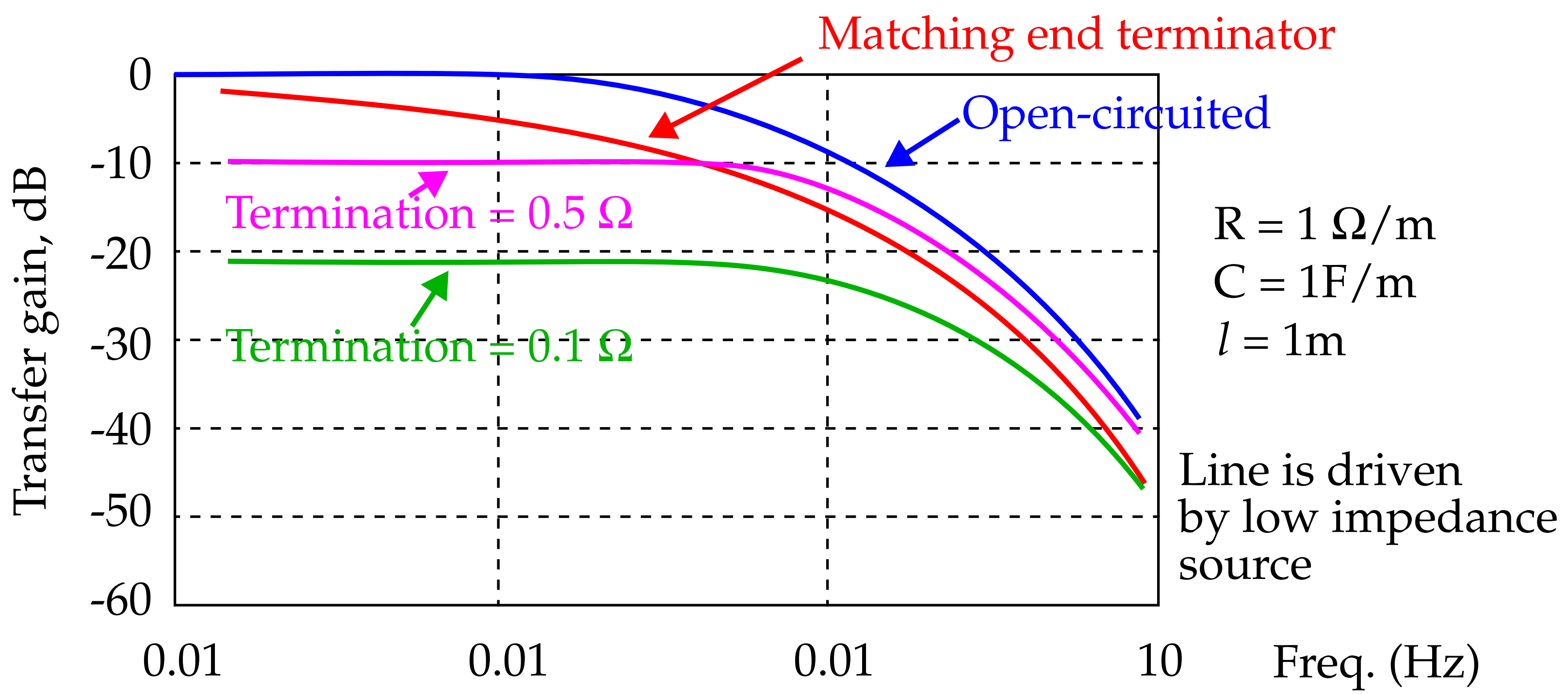
Telephone lines have a characteristic impedance of 600 Ω in the voice band, but at high frequencies, it reduces to 100 Ω.

Note that **characteristic impedance varies markedly with frequency**.

RC Region Propagation Function

For the best results, the termination **must** match Z_C over the entire frequency range spanned from ω_{LE} and ω_{LC}.

Consider the propagation function of a unit-sized RC transmission line



The response is shown for several cases: open-circuit and three loading conditions.

RC Region Propagation Function

Open-circuit response shows the *least overall loss* at high frequencies, and is the most common configuration.

The *matched-end terminator* curve is the response when the transmission line is configured with a matched end-termination impedance $Z_C(\omega)$.

The matched-end configuration degrades the line's response in two ways.

- It reduces the available signal at the end of the line.
- It introduces a *tilt* to the propagation function.

  The *tilt* introduces significant amounts of intersymbol interference, which can cause **bit errors**.

Binary signals tolerate *tilt* of no more than approximately 3 dB (at most 6 dB).

Therefore, although match-end termination makes the input impedance independent of line length, it causes **severe** degradation in the transfer response.

RC Region Propagation Function

The remaining curves are from *resistive load* configurations, equal to 1/2 and 1/10, respectively of the aggregate series resistance of the transmission line.

Although the signal attenuation is higher than the open-circuit configuration, the overall attenuation curve is **flattened**.

The *flattening* occurs up through higher frequencies than either of the previous cases, making it possible to send binary data at higher bandwidths.

These resistive termination schemes show a classic **gain-bandwidth** tradeoff.

You can *improve* the *bandwidth* at the expense of *reducing signal amplitude*.

The upper limit of the achievable bandwidth is defined by the onset of the LC mode of operation, i.e., when $j\omega L$ exceeds R.

In LC mode, a resistance of Z_0 is best for termination (to be discussed).

Using Z_0 eliminates reflections in LC mode while simultaneously providing a relatively flat propagation function in RC mode.

RC Region Propagation Coefficient

You can derive the propagation coefficient starting with

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)}$$

$$H(\omega, l) = e^{-l\gamma(\omega)}$$

In this region, ignore $j\omega L$ and high freq. dependencies of R and C

And simplifying for RC mode.

$$\gamma = \sqrt{R(j\omega C)}$$

$$H(\omega, l) = e^{-l\sqrt{R(j\omega C)}}$$

Now substitute for *H* in

$$G = \frac{v_3}{v_1} = \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)\left(1 + \frac{Z_S}{Z_L}\right) + \left(\frac{H^{-1} - H}{2}\right)\left(\frac{Z_S}{Z_C} + \frac{Z_C}{Z_L}\right)\right]}$$

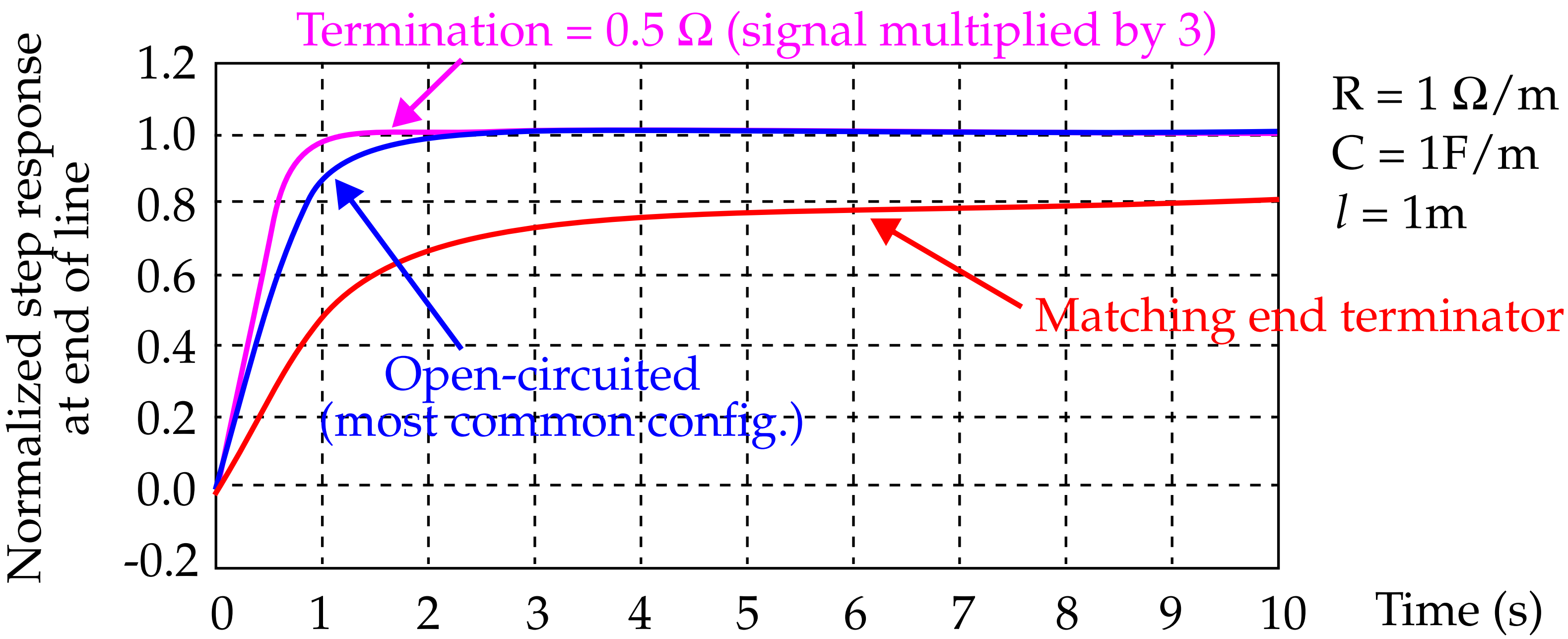
And analyze under various assignments/assumptions for Z_S, Z_C and Z_L,

e.g., $Z_S = 0$ and Z_L = infinity yields

$$G(\omega) = \frac{2}{H^{-1} + H}$$

RC Region Step Response

The normalized step response of a unit-sized RC transmission line



The degraded risetime is obvious for the case of the matching end-termination impedance equal to $Z_C(\omega)$.

The resistive termination shows **superior** risetime, at the cost of *reduced signal amplitude* --> in this case the DC gain is 1/3 given $Z_L = (l{*}R)/2$.

RC Region Step Response

In the RC region, risetime scales with the square of length.

Therefore, doubling length quadruples risetime.

Also, the speed of operation achievable scales inversely with the square of transmission line length.

In this case, you must *wait longer* (slower operational speed) for the signal to reach the same level of magnitude for longer length lines.

W.C. Elmore described a way to estimate the delay of RC circuit that is used (in variations) to validate on-chip timing.

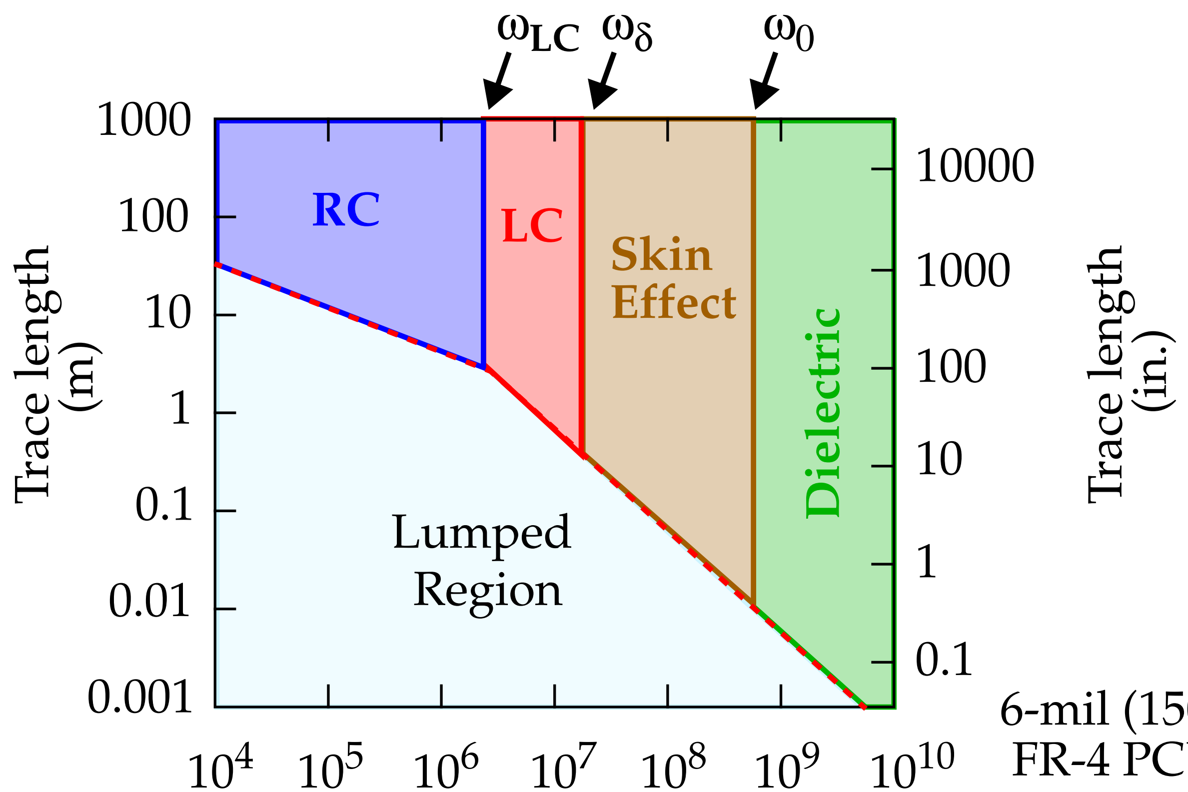
His technique works only with *well-damped* circuits composed of any number of series resistance and shunt capacitances.

It does not work with circuits involving *inductance*, *resonance*, *overshoot* or any form of poorly damped or non-monotonic behavior.

It can be used to quickly compute a reasonable **upper bound** on the delay of complicated tree and bus structures used on-chip.

LC Region

The LC region is characterized by the growth of inductive reactance to the point where it exceeds the magnitude of the DC resistance.



6-mil (150 μm), 50-Ω, FR-4 PCB stripline

At ω_{LC} point, ωL equals R

$$\omega_{LC} = \frac{R_{DC}}{L}$$

LC Region

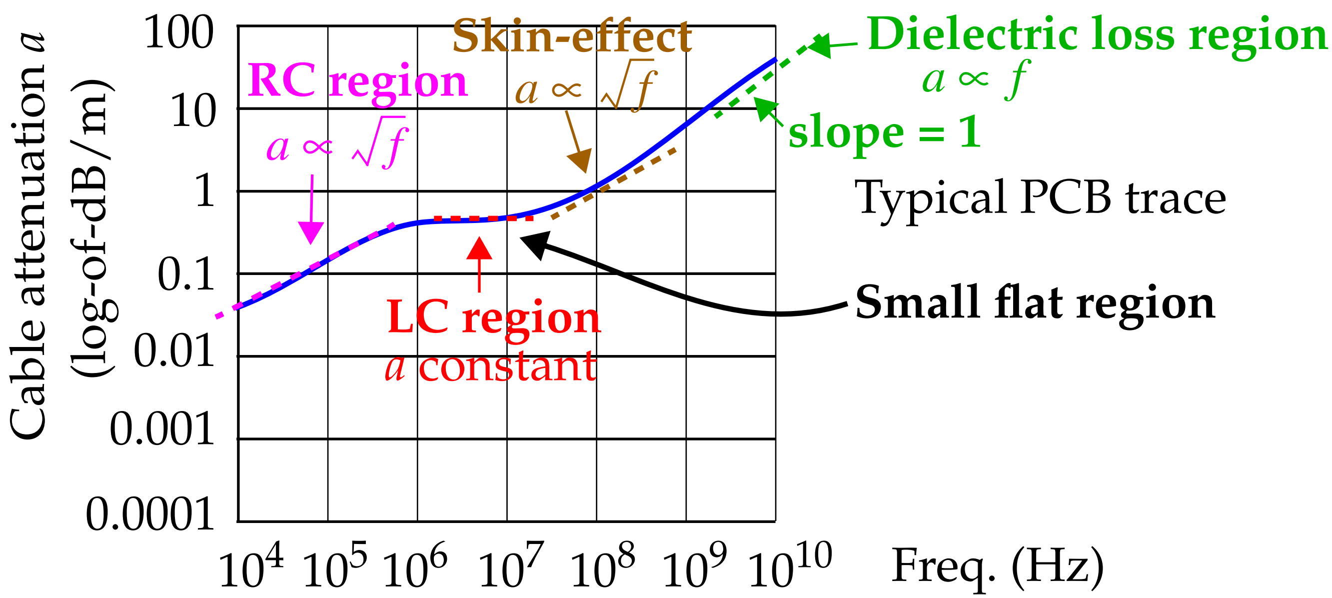
Recalling our earlier analysis, you are in the lumped-element region if

$$l_{LE} \approx \frac{\Delta}{\omega\sqrt{LC}} \qquad \text{for } (\omega > (R_{DC}/L))$$

An interesting feature of LC mode is that the attenuation does vary much with frequency.

In most digital applications, the LC region is fairly narrow (and can be non-existent)

Since dB is already a logarithmic unit, this is a double log



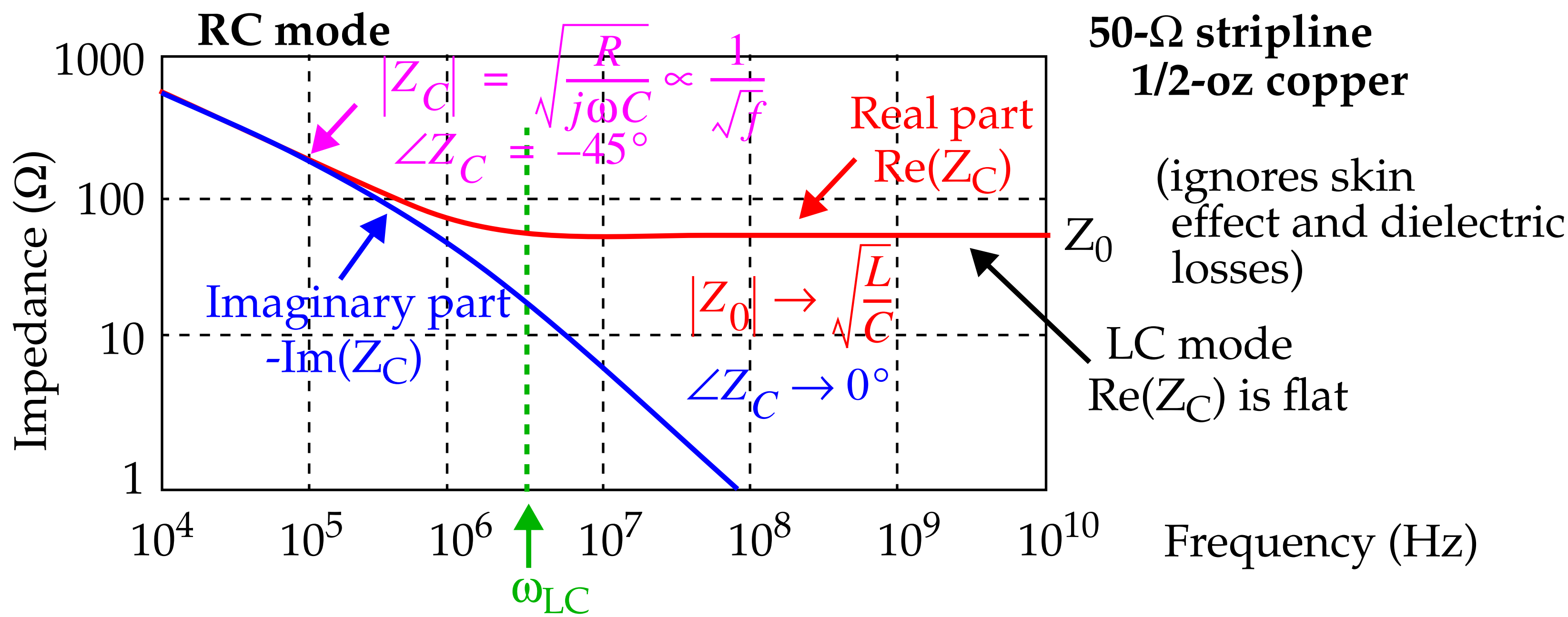
LC Region Characteristic Impedance

We have already derived characteristic impedance in the LC region from

$$Z_C(\omega) = \sqrt{\frac{j\omega L + R}{j\omega C}} \longrightarrow Z_0 = \sqrt{\frac{L}{C}}$$

The difference in Z_C and Z_0 at 3 times ω_{LC} is on order of 5%, at 10x its 0.5%.

However, near ω_{LC}, Z_C is significantly different.



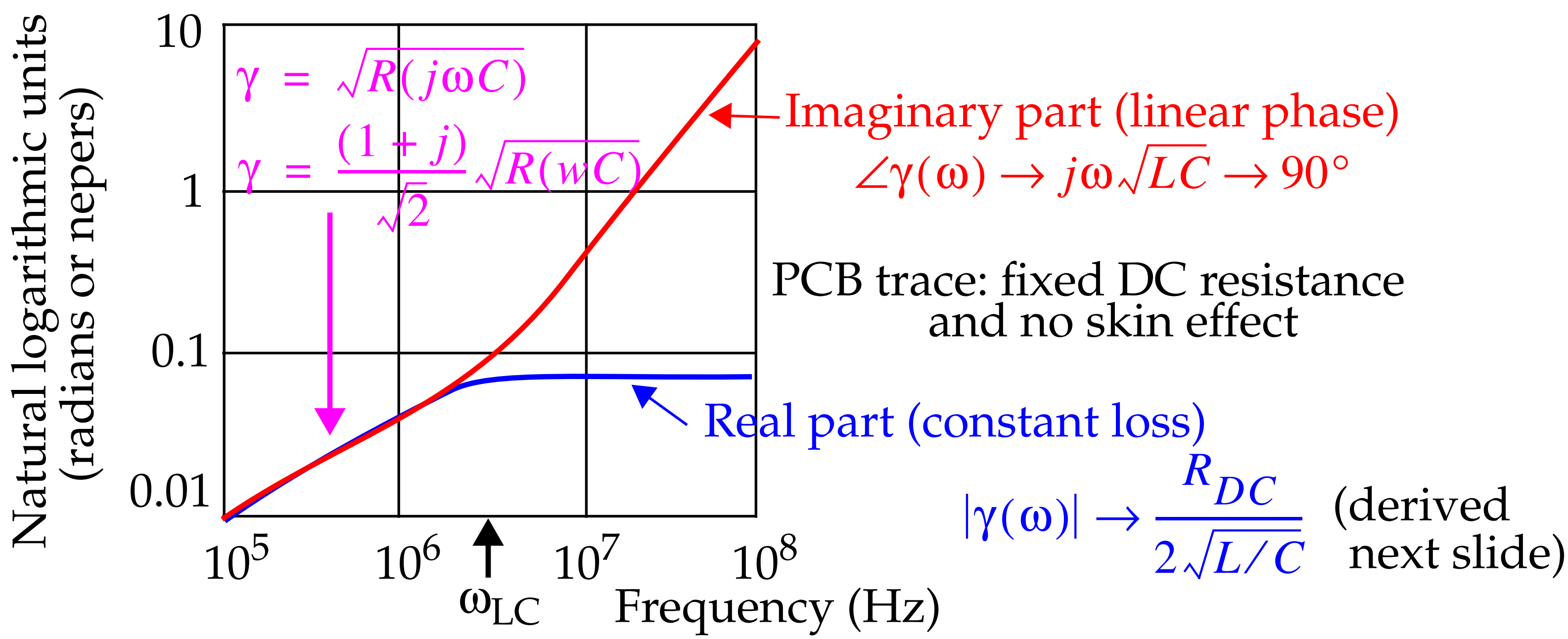
The impedance at $\omega = 0$ is **infinity** and decreases with higher frequencies.

LC Region Propagation Coefficient

At frequencies below ω_{LC} (approximately 3 MHz), both the real and imaginary components decrease at a rate proportional to the *inverse square root of frequency*.

Above ω_{LC}, the imaginary part goes to zero and the overall impedance flattens out to 50 Ω.

The **Propagation Coefficient** for this region graphically



LC Region Propagation Coefficient

Below ω_{LC} in the RC region, both the real part of the propagation coefficient (log of attenuation) and the imaginary part (phase in radians) rise together in proportion to the *square root of frequency*.

Above ω_{LC} in the LC region, attenuation and phase become de-coupled.

Here, the imaginary part grows linearly while the real part stays fixed.

Starting with the propagation coefficient, factor out a jω term.

$$\gamma(\omega) = \sqrt{(j\omega L + R)(j\omega C)}$$

$$\gamma(\omega) = j\omega\sqrt{LC}\sqrt{1 + \frac{R_{DC}}{j\omega L}}$$

The square root on the left can be approximated (valid for $\omega >> \omega_{LC}$)

$$\gamma(\omega) = j\omega\sqrt{LC}\left(1 + \frac{1}{2}\frac{R_{DC}}{j\omega L}\right) = j\omega\sqrt{LC} + \frac{R_{DC}}{2\sqrt{L/C}}$$

Then substitute Z_0 for sqrt(L/C).

LC Region Propagation Coefficient

This expression shows a linear-phase ramp

$$Im(\gamma) \rightarrow \omega\sqrt{LC}$$

and a steady-state value

$$Re(\gamma) \rightarrow \frac{R_{DC}}{2Z_0}$$

The linear phase indicates that the one-way propagation function *H* of an LC-mode transmission line acts like a large **time-delay element**.

$$t_p \triangleq \frac{1}{v_0} = \sqrt{LC} = \frac{\sqrt{\varepsilon_{re}}}{c} \text{ s/m}$$

The **delay** varies in proportion to the length of the transmission line, where doubling the length doubles the delay.

The transfer loss in nepers per meter or *resistive loss coefficient* (does NOT account for skin-effect)

$$\alpha_{r,\,DC} \triangleq Re[\gamma(\omega)] = \frac{1}{2}\frac{R_{DC}}{Z_0} \text{ neper/m}$$

LC Region Propagation Coefficient

In dB

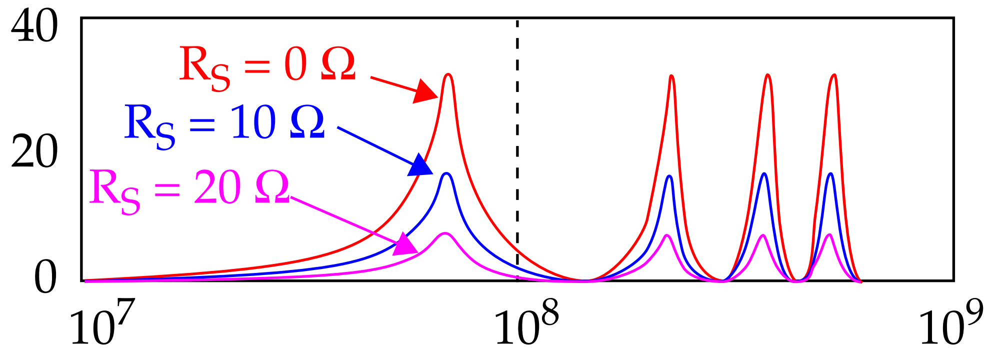
$$\alpha_{r,\,DC} \triangleq Re[\gamma(\omega)] = 4.34\frac{R_{DC}}{Z_0}\ \text{dB/m}$$

The magnitude of H is given by the real part of the propagation function

$$|H(\omega, l)| = e^{-l\frac{1}{2}\frac{R_{DC}}{Z_0}}$$

Doubling the length, doubles the loss.

The property that signals in the LC region have substantial phase delay and low attenuation indicates they may act as **high-Q** resonant circuits.



PCB trace (no skin effect, etc)

Open circuited at far end

LC Region Terminations

All LC transmission lines exhibit similar resonant peaks when driven by a source impedance less than Z_0.

Controlling Z_L and Z_S can resolve this problem.

There are **three classical ways** of *stabilizing* an LC transmission line, i.e., eliminating the resonance.

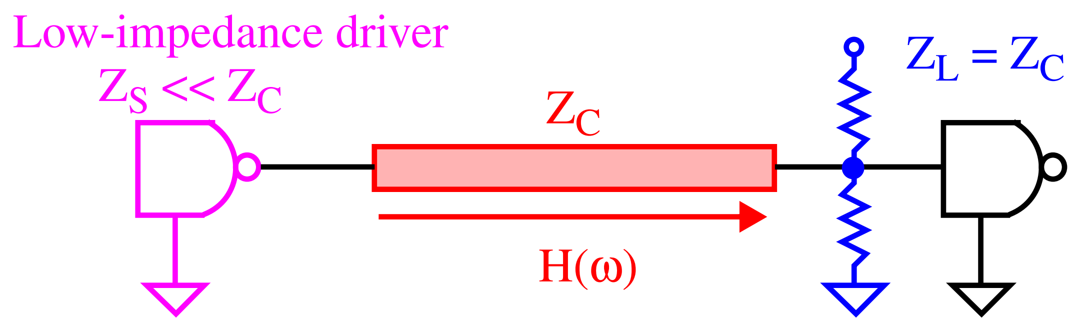
Each of these uses a **resistive termination** to provide a circuit gain that is *proportional* (and desirable) to the propagation function $H(\omega)$.

This strategy works well for PCB traces, which are relatively short in length, producing a propagation function H that is nearly flat with linear phase.

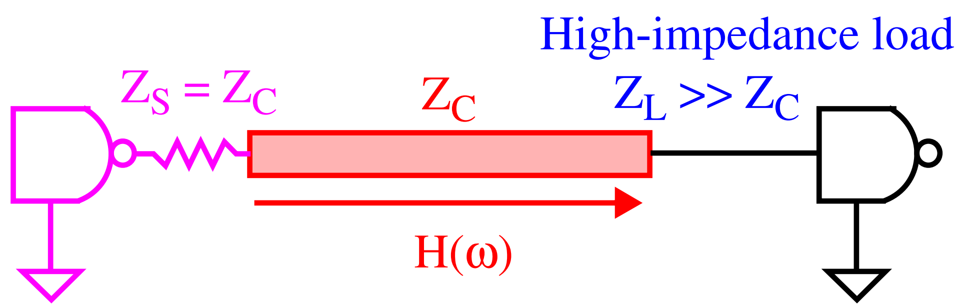
For PCB traces, the line acts like nothing more than a *time-delay* element with a small amount of attenuation.

LC Region Terminations

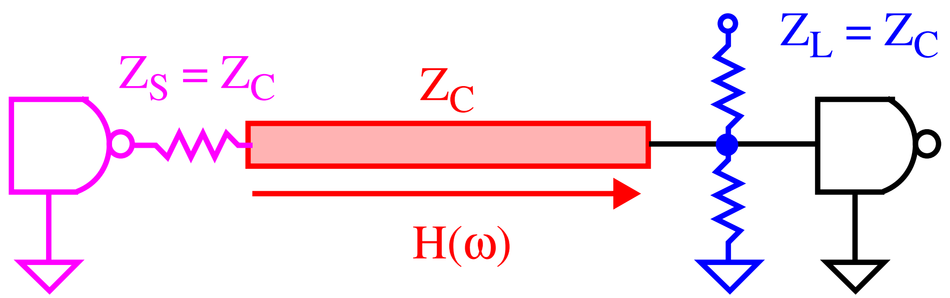
End Termination



Source Termination



Both-ends Termination



LC Region Terminations

For **end termination**, assuming that Z_L is close to Z_C and Z_S is much less than Z_C.

$$G = \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)\left(1 + \frac{Z_S}{Z_L}\right) + \left(\frac{H^{-1} - H}{2}\right)\left(\frac{Z_S}{Z_C} + \frac{Z_C}{Z_L}\right)\right]}$$

Substituting 1 for the Z_C/Z_L terms and 0 for Z_S/Z_C yields

$$G \approx \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)(1 + 0) + \left(\frac{H^{-1} - H}{2}\right)(0 + 1)\right]} = H$$

For **source termination**, assuming Z_S is close to Z_C and Z_L is much larger than Z_C yields

$$G \approx \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)\left(1 + \frac{1}{\infty}\right) + \left(\frac{H^{-1} - H}{2}\right)\left(1 + \frac{1}{\infty}\right)\right]} = H$$

LC Region Terminations

For **both-ends termination**, assuming $Z_S = Z_C = Z_L$ yields

$$G = \frac{1}{\left[\left(\frac{H^{-1} + H}{2}\right)(1+1) + \left(\frac{H^{-1} - H}{2}\right)(1+1)\right]} = \frac{H}{2}$$

Note that unlike RC mode, attenuation in LC mode does **not** vary with frequency, therefore the speed of operation is not directly *limited* by length.

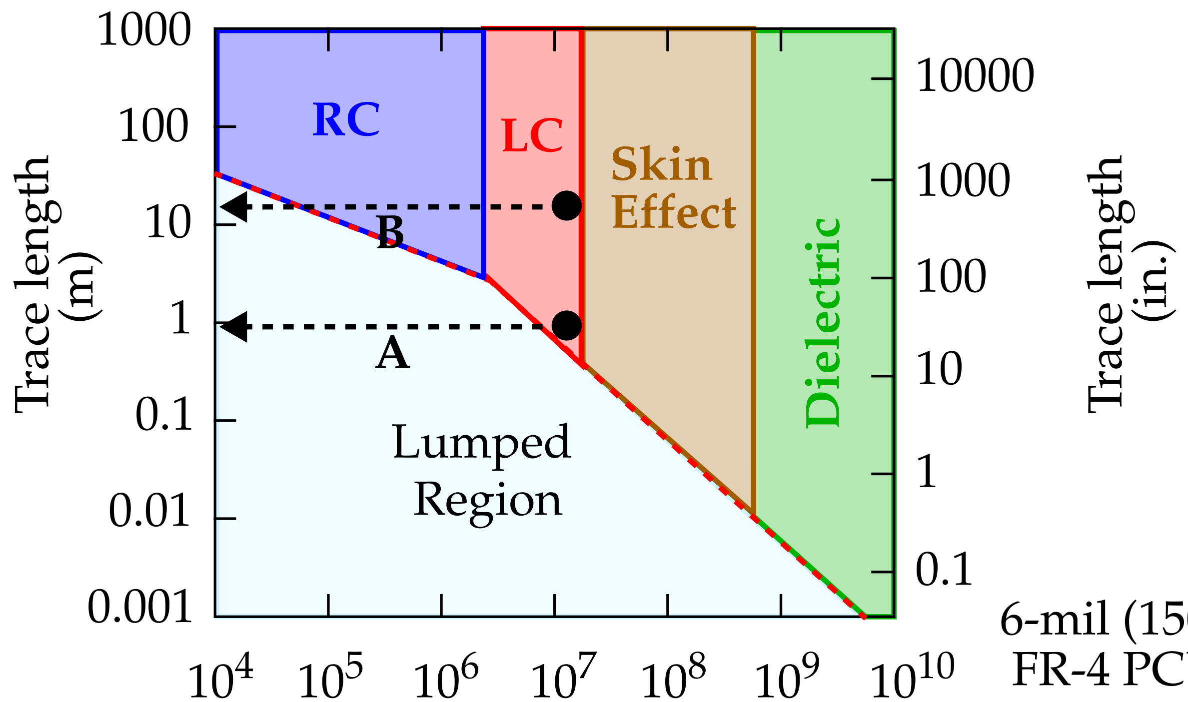
Since LC, skin-effect and dielectric-loss-limited regions all share the same asymptotic high-frequency value of Z_0, the same termination schemes work.

For the LC region, the propagation function H is flat while it is **not flat** in the skin-effect and dielectric-loss-limited regions.

For these regions, H acts as a low-pass filter, attenuating and dispersing the edges of signals.

Mixed-Mode Operation

System **A** is typical of PCB traces and consists of a transmission line with length less than l_{RC} and operates at frequencies over 0 - 20 MHz.



6-mil (150 µm), 50-Ω, FR-4 PCB stripline

It spans only the LC and lumped-element regions so terminating at Z_0 should work well, assuming you don't have a reactive load.

System **B** operates in three modes, where the response in RC mode is a strong function of frequency, requiring a *frequency-varying* termination network.