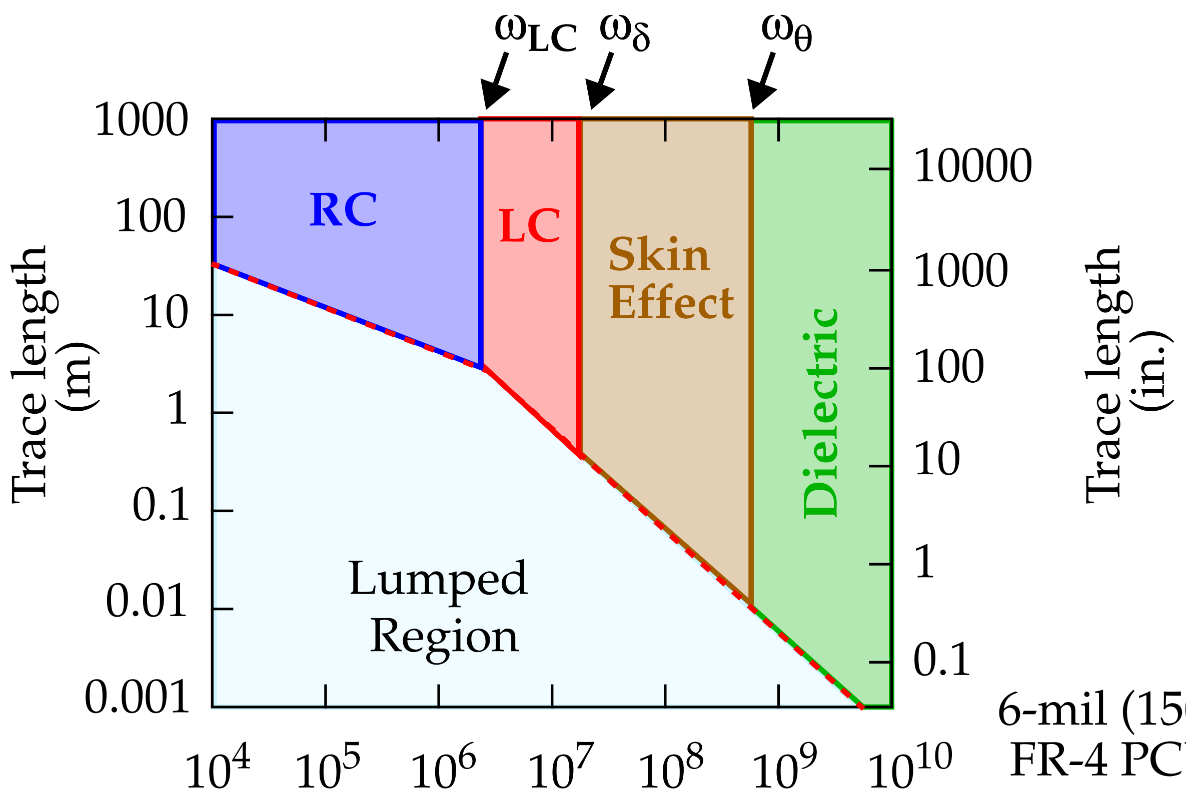
Skin-Effect Region

In the skin-effect region, the *internal inductance* of the conductors becomes significant compared to the DC resistance.



6-mil (150 µm), 50-Ω, FR-4 PCB stripline

ω_δ delineates the start of the region where the real part of the skin-effect resistance, R_{AC}, equals the DC resistance, R_{DC}.

Skin-Effect Region

Recalling from previous discussions

$$R_{DC} = \frac{k_a \rho}{a} \qquad Re[R_{AC}] = \frac{k_p k_r \sqrt{\omega\mu}}{p\sqrt{2\sigma}}$$

ω_δ is given by

$$\omega_\delta = \omega_0 \left(\frac{R_{DC}}{R_0}\right)^2$$

Here, ω_0 is a frequency well into the skin-effect region and R_0 is the value of R_{AC} at that frequency.

In skin-effect mode, the **characteristic impedance** remains fairly flat but the **line attenuation** in dB varies in proportion to the *square root of frequency.*

Characteristic impedance is given by

$$Z_C = \sqrt{\frac{j\omega L_0 + R(\omega)}{j\omega C}}$$

Skin-Effect Characteristic Impedance

The term L_0 refers to the *external inductance* of the line, since *internal inductance* is accounted for in $R(\omega)$.

External inductance is the value of series inductance assuming that current rides on the surface without penetrating the wire.

As you proceed to higher frequencies above ω_δ, the contribution of $R(\omega)$ becomes negligable, leaving

$$Z_0 \stackrel{\Delta}{=} \lim_{\omega \to \infty} \sqrt{\frac{j\omega L_0 + R(\omega)}{j\omega C}} = \sqrt{\frac{L_0}{C}}$$

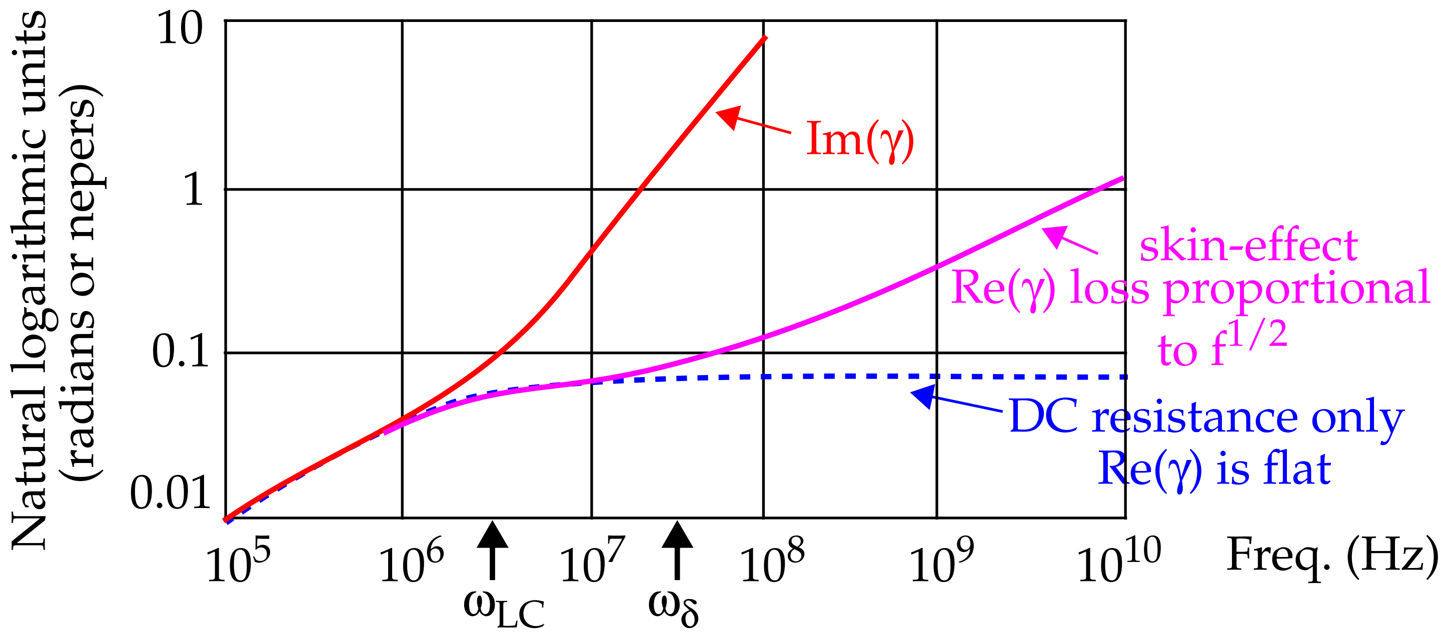
This indicates that although $R(\omega)$ grows in proportion to the square root of frequency, $j\omega L_0$ grows more quickly.

Therefore, once past the cross-over ω_{LC} (remember, this is where the inductance impedance equals the DC resistance), $R(\omega)$ diminishes in importance.

Skin-Effect Propagation Coefficient

Because the skin-effect onset is so close to the LC-mode onset, the flat region for the real part is very small.

PCB trace



As indicated, after ω_δ, attenuation (in dB) increases proportional to the square root of frequency.

The *decoupling* of phase and attenuation enable the construction of a line with an enormous phase delay and yet very low attenuation.

Skin-Effect Propagation Coefficient

Propagation function assuming operation at a frequency well in excess of ω_δ so that $R_{AC} >> R_{DC}$

$$\gamma(\omega) = \sqrt{(j\omega L_0 + R_{AC})(j\omega C)}$$

Factoring out common $j\omega L_0$ and $j\omega C$ terms

$$\gamma(\omega) = \sqrt{(j\omega L_0)(j\omega C)}\sqrt{1 + \frac{R_{AC}}{j\omega L}}$$

Assuming $\omega >> \omega_{LC}$ so that $|j\omega L_0| >> |R_{AC}|$

$$\gamma(\omega) = \sqrt{(j\omega L_0)(j\omega C)}\left(1 + \frac{1}{2}\frac{R_{AC}}{j\omega L_0}\right)$$

Distributing

$$\gamma(\omega) = \sqrt{(j\omega L_0)(j\omega C)} + \sqrt{(j\omega L_0)(j\omega C)}\frac{1}{2}\frac{R_{AC}}{j\omega L_0}$$

Skin-Effect Propagation Coefficient

Combining and substituting for characteristic impedance

$$\gamma(\omega) = \sqrt{(j\omega L_0)(j\omega C)} + \frac{1}{2}\frac{R_{AC}}{Z_0}$$

Factoring out $j\omega$ on the left and substituting expression for R_{AC} described earlier

$$\gamma(\omega) = j\omega\sqrt{L_0 C} + \frac{(1+j)}{2}\frac{R_0}{Z_0}\sqrt{\frac{\omega}{\omega_0}}$$

Like the LC region, the first term indicates linear phase and represent the bulk transport delay.

$$t_p \triangleq \frac{1}{v_0} = \sqrt{L_0 C}\ \text{s/m}$$

The second term is a **low-pass filter** whose attenuation in dB grows proportional to the *sqrt(f)*.

Skin-Effect Propagation Coefficient

Similar to the LC region, the **skin-effect loss coefficient** can be defined

$$\alpha_r \stackrel{\Delta}{=} Re[\gamma(\omega)] = \frac{1}{2}\frac{R_0}{Z_0}\sqrt{\frac{\omega}{\omega_0}} \text{ neper/m} = 4.34\frac{R_0}{Z_0}\sqrt{\frac{\omega}{\omega_0}} \text{ dB/m}$$

The low-pass filtering action will *slur* the rising edge of the step response, adding slew.

$$|H(\omega, l)| = e^{-l\frac{1}{2}\frac{R_0}{Z_0}\sqrt{\frac{\omega}{\omega_0}}}$$

Here again, doubling length, doubles the signal loss.

However, here signal loss is also **frequency dependent**, doubling the frequency multiplies the loss by the *sqrt(2)*.

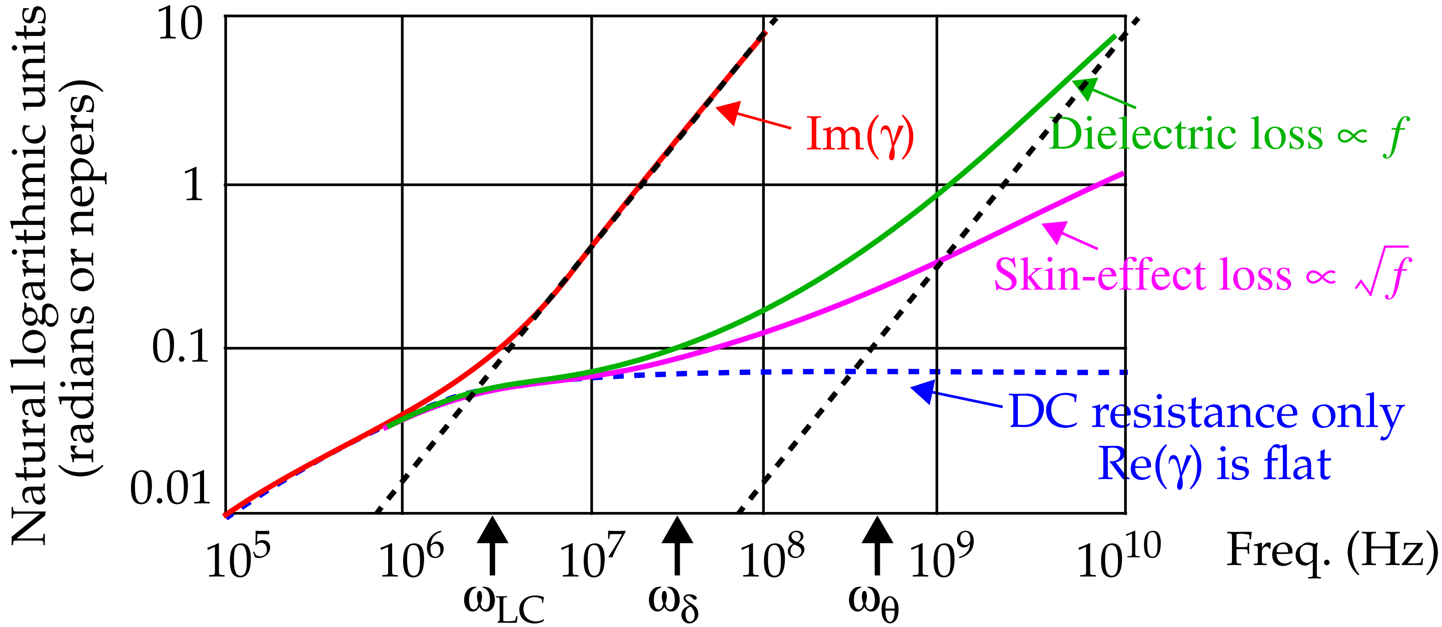
The termination approaches discussed with reference to the LC region work here as well (and in the dielectric-loss-limited region).

This is true because all three regions share the same asymptotic high-frequency value of characteristic impedance Z_0.

Dielectric-Loss-Limited Region

Dielectric-loss increases the slope of the Re(γ)

PCB trace



It is common in PCB problems to see skin-effect losses AND dielectric losses.

Waveguide-dispersion region begins when the frequency of the signal approaches the dimensions of your conductor.

For a stripline, the critical dimension is the spacing between the planes.

Strange modes appear, severe ringing occurs under perfect termination.

Example

Specs

- Length l = 0.6 m (23.6 in.) (backplane application)
- Conductor parameters: w = 152 μm (6 mil), t = 17.4 μm (1/2 oz. Cu), perimeter $p = 2(w + t)$ = 339 μm (13.35 mil)
- Conductivity of signal conductor $\sigma = 5.98 \times 10^7$ S/m
- Specification frequency for AC parameters: $\omega_0 = 2\pi \times 10^9$
- Characteristic impedance at ω_0: $Z_0 = 100\ \Omega$
- Effective dielectric constant $\varepsilon_R = 4.3$
- Effective loss tangent for FR-4 dielectric: $\tan\theta_0 = 0.025$
- Proximity factor $k_p = 3.2$

Computed values

- Propagation velocity above RC region

$$v_0 = \frac{c}{\sqrt{\varepsilon_R}} = 1.4457\times10^8 \text{ m/s} \qquad t_p = 175.7 \text{ ps/in.}$$

Example

- Differential inductance per meter

$$L = \frac{Z_0}{v_0} = 691 \text{ nH/m}$$

- Differential capacitance per meter

$$C = \frac{1}{Z_0 v_0} = 69.1 \text{ pF/m}$$

- DC resistance

$$R_{DC} = \frac{2}{(\sigma \mathrm{w} t)} = 12.64 \ \Omega\text{/m}$$

- AC resistance

$$R_0 = \frac{k_p}{p}\sqrt{\frac{\omega_0 \mu}{2\sigma}} = 76.74 \ \Omega\text{/m}$$

Example

Lumped-element region

$$\text{critical length} = \left(\frac{0.25}{R_{DC}}\right)\sqrt{\frac{L}{C}} = 1.97\ \text{m}$$

The trace length of $l = 0.6$ m falls short of the critical length, so we move from lumped-element directly to LC, skipping the RC region.

Other region transitions

Note: ω values multiplied by $1/(2\pi)$

$$\omega_{LC} = \left(\frac{\Delta}{l}\right)\sqrt{LC} = 9.58\ \text{MHz}$$ LC Region

$$\omega_{\delta} = \omega_0\left(\frac{R_{DC}}{R_0}\right)^2 = 27.1\ \text{MHz}$$ Skin-effect

$$\omega_{\theta} = \frac{1}{\omega_0}\left[\frac{(v_0 R_0)}{Z_0 \tan\theta_0}\right]^2 = 498\ \text{MHz}$$ Dielectric (haven't covered)