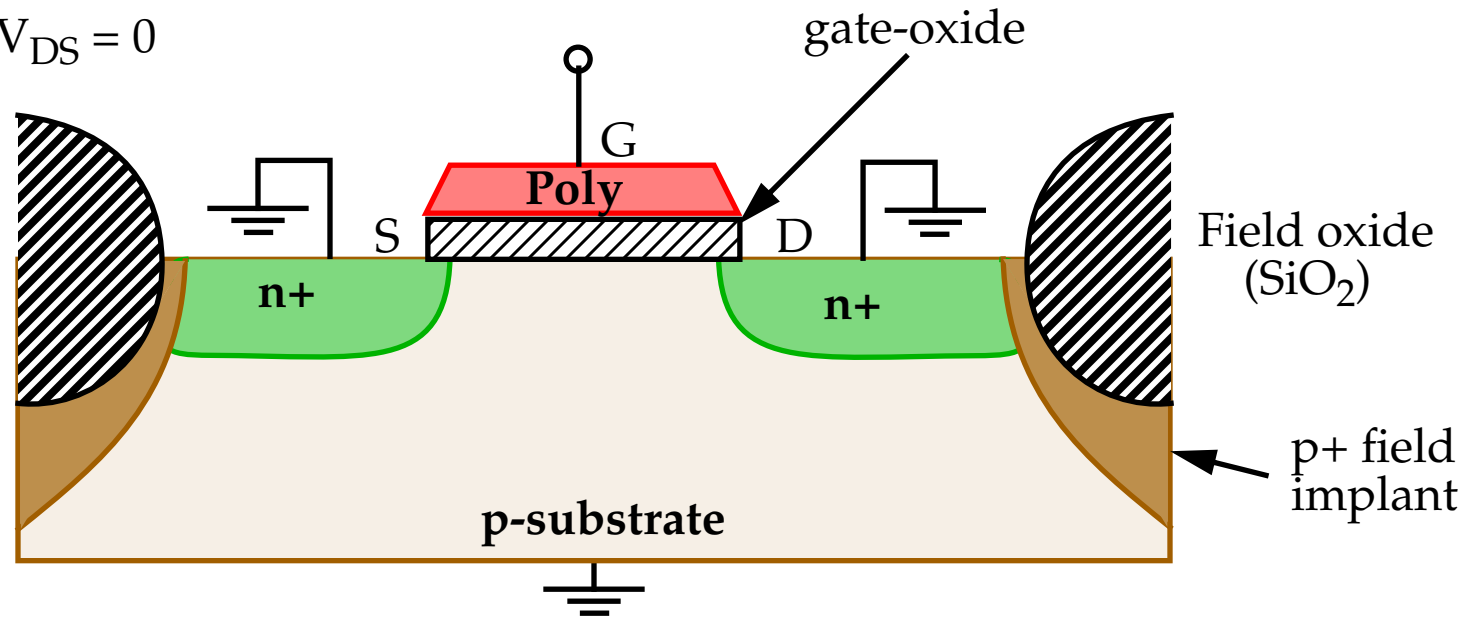


**Static Behavior**

An nMOS transistor cross-section:

$$V_{GS} = 0, V_{DS} = 0$$



Under zero bias, two back-to-back *pn*-junctions create a very high resistive path between source and drain.

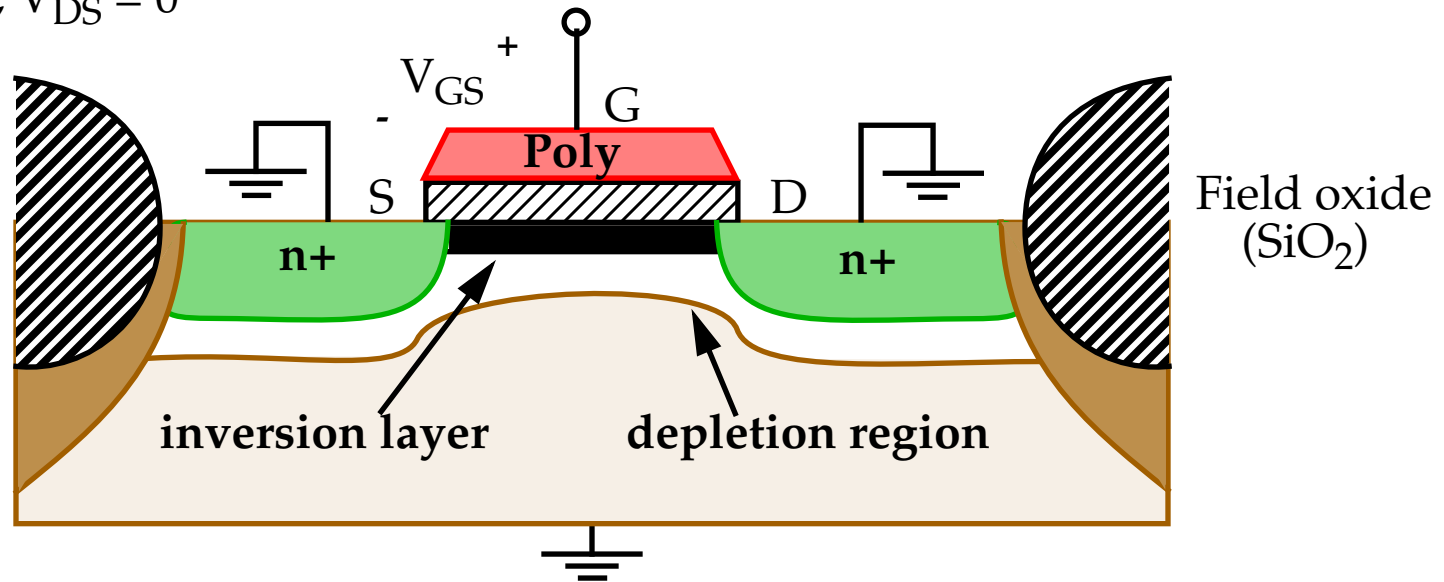
Applying a **positive bias** ( $V_{GS}$ ) to the gate (w.r.t. the source), creates a depletion region under the gate (repels mobile holes).

The depletion region is similar to the one occurring in a *pn*-junction.

**Static Behavior**

Inversion:

$$V_{GS} > 0, V_{DS} = 0$$



Depletion region expressions are similar to the diode expressions:

$$W_d = \sqrt{\frac{2\epsilon_{si}\phi}{qN_A}} \quad \text{Width}$$

$$Q_d = \sqrt{2N_A\epsilon_{si}\phi} \quad \text{Space charge}$$

$\phi$  = potential at the oxide-silicon boundary



## Static Behavior

Inversion:

At a critical value of  $V_{GS}$ , the substrate "**inverts**" to *n-type* material.

This is called strong inversion and occurs at a voltage that is twice the *Fermi Potential*:

$$\phi_F = -0.3V \text{ for typical p-type silicon substrates.}$$

Further increases in  $V_{GS}$  do **not** increase the depletion layer width.

The charge is offset with additional inversion-layer electrons (sourced from the heavily doped *n+* source region).

The *conductivity* of the n-channel is modulated by  $V_{GS}$ .

Under strong inversion, the *charge* in the depletion region is fixed and equals:

$$Q_{B0} = \sqrt{2qN_A\epsilon_{si}|-2\phi_F|}$$

## Static Behavior

Inversion:

A substrate bias voltage,  $V_{SB}$ , **increases** the surface potential needed to create strong inversion to:

$$|-2\phi_F + V_{SB}|$$

$V_{SB}$  is normally positive for n-channel devices.

This changes the charge in the depletion region:

$$Q_B = \sqrt{2qN_A\epsilon_{si}|-2\phi_F + V_{SB}|}$$

The value of  $V_{GS}$  where strong inversion occurs is *threshold voltage*,  $V_T$ .

$V_T$  depends on several components, many are material constants:

- difference in work function between gate and substrate material.
- oxide thickness
- Fermi voltage
- charge associated with impurities trapped at oxide-channel interface
- concentration of implanted ions

## Static Behavior

$V_T$  also depends substrate voltage,  $V_{SB}$ .

Rather than depend on the complete analytical form (which often is not a good predictor of  $V_T$ ), an empirical parameter is used,  $V_{T0}$ .

$V_{T0}$  is the threshold voltage with  $V_{SB} = 0$ .

$$V_T = V_{T0} + \gamma(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|}) \quad V_T \text{ is positive for nMOS} \\ \text{and negative for pMOS}$$

$$\gamma = \frac{\sqrt{2q\epsilon_{si}N_A}}{C_{ox}} \quad (\gamma \text{ is the } \textit{body effect coefficient})$$

$\gamma$  expresses the impact of changes in  $V_{SB}$ .

A negative bias on the well or substrate causes  $V_T$  to **increase**.

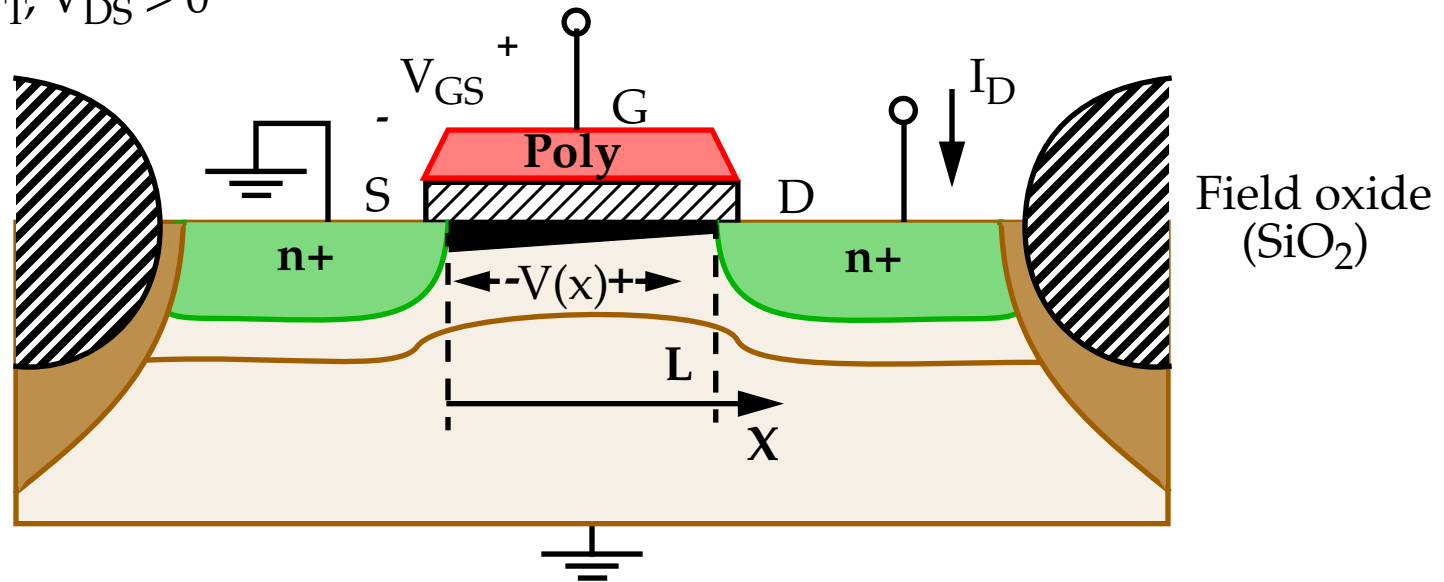
$$\text{Given: } V_{T0} = 0.75V \quad \gamma = 0.54 \quad 2\phi_F = -0.64V \quad V_{SB} = 5V$$

$$V_T = 0.75 + 0.54(\sqrt{|-2(-0.6) + 5V|} - \sqrt{|-2(-0.6)|}) = 1.6V!$$

**Static Behavior**

Current-Voltage Relations:

$$V_{GS} > V_T, V_{DS} > 0$$



- **Linear Region**

At a point  $x$  along the channel, the voltage is  $V(x)$ .

The gate-to-channel voltage at that point equals  $V_{GS} - V(x)$ .

**Static Behavior****Linear region**

Assume that this voltage exceeds the threshold everywhere along the channel.

The *induced channel* charge per unit area at point  $x$  is:

$$Q_i(x) = -C_{ox}[V_{GS} - V(x) - V_T]$$

The gate capacitance per unit area,  $C_{ox}$ , is expressed as:

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \quad \epsilon_{ox} = 3.97 \times \epsilon_0 = 3.5 \times 10^{-13} \text{ F/cm}$$

$t_{ox}$  is *gate oxide thickness*.

It is 10nm or smaller in contemporary processes.

For  $t_{ox} = 5\text{nm}$ ,  $C_{ox}$  is 7 fF/ $\mu\text{m}^2$ .

## Static Behavior

### Linear region

Current is given as the product of the *drift velocity* of the carriers and the available *charge*:

$$I_D = -v_n(x)Q_i(x)W$$

$v_n$  = drift velocity  
 $W$  = width of channel

The **electron velocity**,  $v$ , is related to the electric field through a parameter called the mobility ( $\mu$ ):

$$v_n = -\mu_n E(x) = \mu_n \frac{dV}{dx}$$

Combining the equations:

$$I_D dx = \mu_n C_{ox} W (V_{GS} - V - V_T) dV$$





## Static Behavior

### Linear region

Integrating this equation over the length of the channel yields the *current-voltage* relationship of a nMOS transistor.

$$I_D = k_n' \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \quad (1)$$

$$k_n = k_n' \frac{W}{L} \quad (\text{gain factor})$$

$$k_n' = \mu_n C_{ox} = \frac{\mu_n \epsilon_{ox}}{t_{ox}} \quad (\text{process transconductance parameter})$$

For typical n-channel devices with:  $t_{ox} = 20\text{nm}$

$$k_n' = 80\mu\text{A}/\text{V}^2$$

For small values of  $V_{DS}$ , the quadratic factor can be ignored and we observe a linear relationship between  $V_{DS}$  and  $I_D$ .

NOTE:  $W$  and  $L$  are *effective* width and length, not the drawn values.

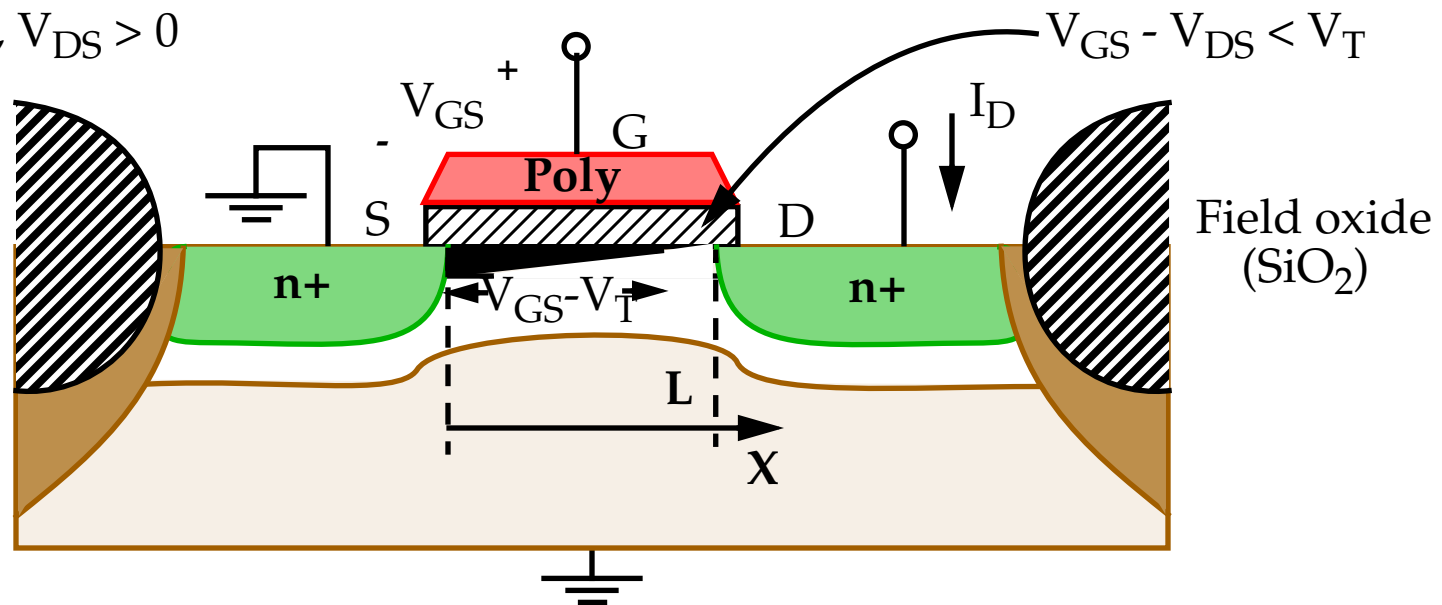
**Static Behavior****Saturation**

When  $V_{DS}$  is further increased, the channel voltage all along the channel may **cease** to be larger than the threshold, e.g.,

$$V_{GS} - V(x) < V_T$$

At that point, the induced charge is zero and the channel disappears or is *pinched off*.

$$V_{GS} > 0, V_{DS} > 0$$



## Static Behavior

Current-Voltage Relation, Saturation.

The voltage difference over the *induced channel* remains fixed at  $V_{GS} - V_T$  and the current remains constant (or **saturates**).

Replacing  $V_{DS}$  with  $V_{GS} - V_T$  in equation (1) (since this equation was derived over the channel) yields:

$$I_D = \frac{k_n'}{2} \frac{W}{L} (V_{GS} - V_T)^2 \quad (2)$$

This equation is not entirely correct, since the channel length changes as a function of  $V_{DS}$ .

Current **increases** as channel length ( $L$ ) **decreases**, according to equation (2).

A more accurate expression for current in saturation is:

$$I_D = \frac{k_n'}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$\lambda$ : empirical parameter called *channel-length modulation*

**Static Behavior**

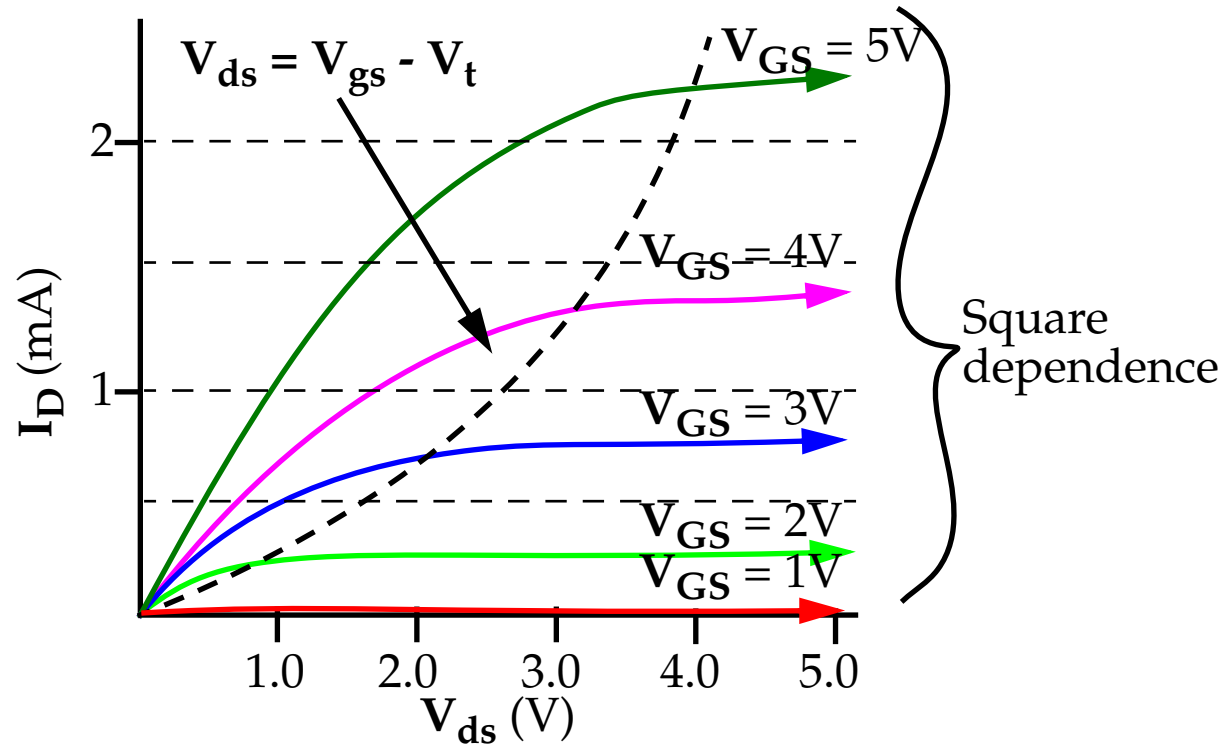
Current-Voltage curves

I-V nMOS transistor curves for a device with dimensions:

$$W = 100\mu m$$

$$L = 20\mu m$$

in a 1.2um process.



## Static Behavior

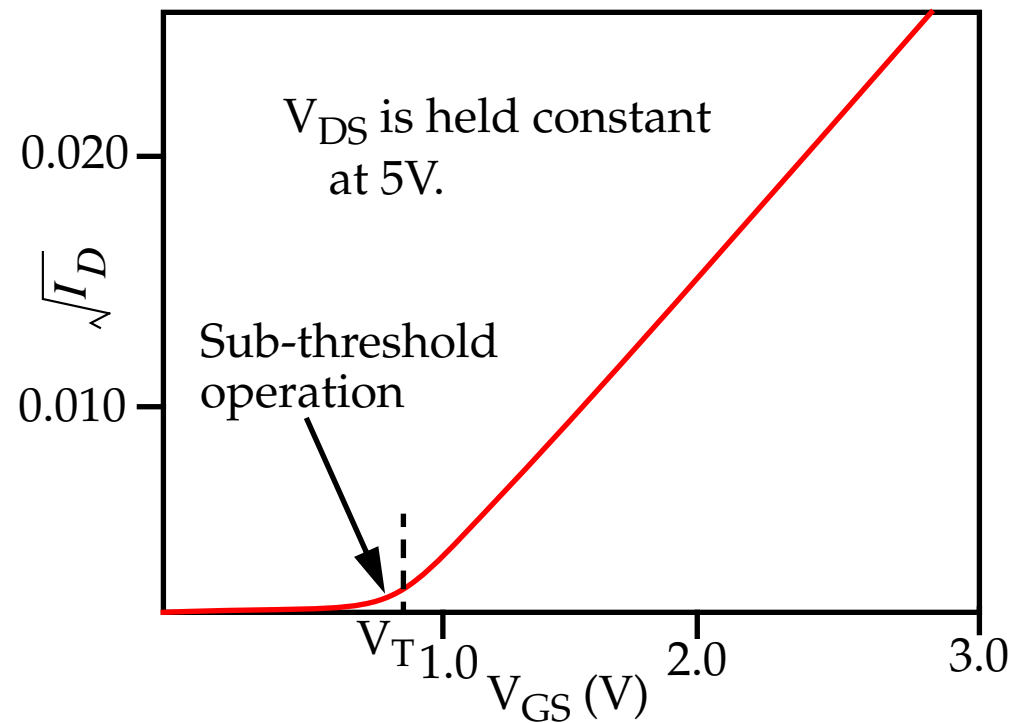
Current-Voltage Relation.

**Triode region:** The transistor behaves like a voltage-controlled resistor.

**Saturation region:** It behaves like a voltage-controlled current source (ignoring channel-length modulation effects).

Linear relationship  
for values:

$$V_{GS} \gg V_T$$

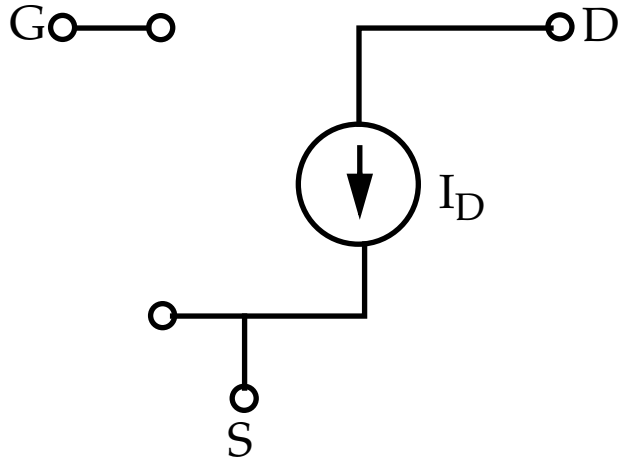


Note: Analytical expressions of  $\lambda$  have proven inaccurate.

Device experiments indicate that  $\lambda$  varies  $\sim 1/\text{channel length}$ .

**Static Behavior**

Manual Analysis Model:



$$V_{DS} > V_{GS} - V_T$$

$$I_D = \frac{k_n'}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$V_{DS} < V_{GS} - V_T$$

$$I_D = k_n' \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

with

$$V_T = V_{T0} + \gamma (\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$$