

## Homework # 1, Due Feb. 6th, 2007

### Problem 1 *Fourier Transforms for Real Signals.*

In what follows, assume the Fourier Transform definition

$$X(\Omega) = \int_R x(t) \exp[-j\Omega t] dt. \quad (1)$$

Assume that  $x(t)$  is real. Show that:

1(a) The real and imaginary parts of the spectrum are

$$X_R(\Omega) = \int_R x(t) \cos(\Omega t) dt, \quad (2)$$

where  $X_R(\Omega)$  is even and

$$X_I(\Omega) = - \int_R x(t) \sin(\Omega t) dt, \quad (3)$$

where  $X_I(\Omega)$  is odd. **Please correct your notes by adding the minus sign to the definition of  $X_I(\Omega)$ .** Furthermore, show that:

$$X(-\Omega) = X^*(\Omega). \quad (4)$$

1(b) Assume that  $x(t)$  is also even:  $x(t) = x(-t)$ . Prove the expressions for  $X_R(\Omega)$ ,  $X_I(\Omega)$ ,  $x(t)$  as given on the top of page 2.8. **Note that the expressions for the odd signals are actually correct!**

**Problem 2.** Show the time-scaling property of page 2.10.

**Problem 3.** Show the time shifting property of page 2.11.

**Problem 4.** Derive the Fourier Transform of  $2p_1(t) + 3p(t-1) - 4p_1(t-2)$  (page 2.27).

**Problem 5.** Derive the Fourier Transform of  $\exp(-a|t|)$  (page 2.29).