EMP Interaction Notes

Note XI
25 July 1967

The Current Induced on a Finite, Perfectly Conducting, Solid Cylinder in Free Space by an Electromagnetic Pulse

Richard W. Sassman
Northrop Corporate Laboratories
Pasadena, California

Abstract

This paper describes the development of the equations and computer programs for calculating the current induced on a finite, perfectly conducting, solid cylinder in free space by an electromagnetic plane-wave pulse polarized parallel to the axis of the cylinder. Calculations were made for three different cylinders having length to diameter ratios of 10, 100 and 1000 and three different pulses: the unit step function and two pulses of the form $e^{-c_1 t}-e^{-c_2 t}$.

Acknowledgements

This work was funded partly by the Air Force Weapons Laboratory and partly by the Defense Atomic Support Agency under contract F29601-67-C-0016. Guidance and encouragement were generously given by Capt. Carl Baum of AFWL. Dr. K. S. H. Lee and Mr. R. W. Latham made many valuable suggestions in the development of the equations. Mr. Dale Milligan made important contributions to the presentation of the results.

I INTRODUCTION

In the investigation of the vulnerability of missile systems to EMP it is frequently necessary to have a knowledge of the currents induced on the surface of the missile by an electromagnetic pulse. The results presented here are a first step toward obtaining a knowledge of the magnitude and distribution of these currents.

We shall idealize the problem by treating the case of the finite, perfectly conducting, solid cylinder in a simple non-conducting medium and calculate the total axial current induced on the cylinder. The solution of the problem is divided into two parts. First, the current induced by harmonic plane waves polarized with the electric vector parallel to the axis of the cylinder is calculated by solving numerically an integral equation for the current. Second, the time history of the current induced by three different transient waves is obtained by Fourier integration of the frequency-domain results.

Note that rationalized MKS units and $e^{-i\omega t}$ time dependence are used throughout.

II FORMULATION OF THE INTEGRAL EQUATION

The integral equation derived here is equivalent to the one reported in Sensor and Simulation Note XLV except that the form of the equation in that report did not indicate the nature of the kernel at the intersection of the side and end surfaces of the cylinder[1]. The form derived herein is better suited to the development of the quadrature formulas in the next section.

We begin with the integral equation for the current density on a perfectly conducting surface[1,2]

$$\underline{K}(\underline{r}) = \underline{K}^{ex}(\underline{r}) + \hat{n}(\underline{r}) \times \lim_{\underline{p}\to\underline{r}} \int_S \nabla G(|\underline{p}-\underline{r}'|) \times \underline{K}(\underline{r}')ds \qquad (1)$$

where

$\underline{r}$ = the position vector of a point on the surface S

$\hat{n}(\underline{r})$ = unit outward normal to S at $\underline{r}$

$\underline{p}$ = the position vector of a point above the surface S.

$\underline{K}(r)$ = surface current density at $\underline{r}$

$\underline{K}^{ex}(\underline{r}) = \hat{n}(\underline{r}) \times \underline{H}^{ex}(\underline{r})$

$\underline{H}^{ex}(\underline{r})$ = magnetic field at $\underline{r}$ due to external sources

$$G(|\underline{p}-\underline{r}'|) = \frac{e^{ik|\underline{p}-\underline{r}'|}}{4\pi|\underline{p}-\underline{r}'|}$$

$k = \omega/c$

and the time dependence $e^{-i\omega t}$ has been suppressed.

For a circular cylinder, equation (1) becomes the coupled set of equations

$$\underline{K}(z,a,\theta) = \underline{K}^{ex}(z,a,\theta) + \hat{n}(\theta) \times \lim_{\delta \to 0} \int_0^{2\pi} \int_{-h}^{h} \nabla G(z-z', a+\delta, a, \theta-\theta') \times \underline{K}(z', a, \theta') a\, dz'\, d\theta$$

$$+ \hat{n}(\theta) \times \lim_{\delta \to 0} \int_0^{2\pi} \int_0^{a} \nabla G(z-h, a+\delta, \rho', \theta-\theta') \times \underline{K}(h, \rho', \theta') \rho'\, d\rho'\, d\theta'$$

$$+ \hat{n}(\theta) \times \lim_{\delta \to 0} \int_0^{2\pi} \int_0^{a} \nabla G(z+h, a+\delta, \rho', \theta-\theta') \times \underline{K}(-h, \rho', \theta') \rho'\, d\rho'\, d\theta' \qquad (2a)$$

$$\underline{K}(h,\rho,\theta) = \underline{K}^{ex}(h,\rho,\theta) + \hat{z} \times \lim_{\delta \to 0} \int_0^{2\pi} \int_{-h}^{h} \nabla G(h+\delta-z', \rho, a, \theta-\theta') \times \underline{K}(z', a, \theta')\, a\, dz'\, d\theta'$$

$$+ \hat{z} \times \lim_{\delta \to 0} \int_0^{2\pi} \int_0^{a} \nabla G(\delta, \rho, \rho', \theta-\theta') \times \underline{K}(h, \rho', \theta') \rho'\, d\rho'\, d\theta'$$

$$+ \hat{z} \times \int_0^{2\pi} \int_0^{a} \nabla G(2h, \rho, \rho', \theta-\theta') \times \underline{K}(-h, \rho', \theta') \rho'\, d\rho'\, d\theta' \qquad (2b)$$

$$\underline{K}(-h,\rho,\theta) = \underline{K}^{ex}(-h,\rho,\theta) - \hat{z} \times \lim_{\delta \to 0} \int_0^{2\pi} \int_{-h}^{h} \nabla G(-h-\delta-z', \rho, a, \theta-\theta') \times \underline{K}(z', a, \theta')\, a\, dz'\, d\theta'$$

$$- \hat{z} \times \int_0^{2\pi} \int_0^{a} \nabla G(-2h, \rho, \rho', \theta-\theta') \times \underline{K}(h, \rho', \theta') \rho'\, d\rho'\, d\theta'$$

$$- \hat{z} \times \lim_{\delta \to 0} \int_0^{2\pi} \int_0^{a} \nabla G(-\delta, \rho, \rho', \theta-\theta') \times \underline{K}(-h, \rho', \theta') \rho'\, d\rho'\, d\theta' \qquad (2c)$$

where

$\hat{n}(\theta) = \hat{x}\cos\theta + \hat{y}\sin\theta$

$\hat{x}$ = unit vector in the x direction

$\hat{y}$ = unit vector in the y direction

$\hat{z}$ = unit vector in the z direction

a = radius of the cylinder

2h = length of the cylinder

$$G(z-z',\rho,\rho',\theta-\theta') = \frac{e^{ikR}}{4\pi R}$$

$$R^2 = (z-z')^2 + \rho^2 + \rho'^2 - 2\rho\rho'\cos(\theta-\theta') .$$

Now if we dot multiply equation (2a) by $a\hat{z}$ and equations (2b) and (2c) by $\rho\hat{\rho}(\theta)$ and integrate with respect to θ we get after expanding and simplifying

$$\tfrac{1}{2}I_1(z) = I_1^{ex}(z) - \int_0^{2\pi}\int_{-h}^{h} a^2(1-\cos\psi)\, F(z-z',a,a,\psi)\, I_1(z')dz'd\psi$$

$$+ a(h-z)\int_0^{2\pi}\int_0^{a} \cos\psi\, F(z-h,a,\rho',\psi)\, I_2(\rho')d\rho'd\psi$$

$$+ a(h+z)\int_0^{2\pi}\int_0^{a} \cos\psi\, F(z+h,a,\rho',\psi)\, I_3(\rho')d\rho'd\psi \qquad (3a)$$

$$\tfrac{1}{2}I_2(\rho) = I_2^{ex}(\rho) - \int_0^{2\pi}\int_{-h}^{h} \rho(\rho-a\cos\psi)\, F(h-z',\rho,a,\psi)\, I_1(z')dz'd\psi$$

$$+ 2h\rho\int_0^{2\pi}\int_0^{a} \cos\psi\, F(2h,\rho,\rho',\psi)\, I_3(\rho')d\rho'd\psi \qquad (3b)$$

$$\tfrac{1}{2}I_3(\rho) = I_3^{ex}(\rho) - \int_0^{2\pi}\int_{-h}^{h} \rho(\rho - a\cos\psi)\, F(-h-z',\rho,a,\psi)\, I_1(z')\,dz'\,d\psi$$

$$+\, 2h\rho \int_0^{2\pi}\int_0^{a} \cos\psi\, F(-2h,\rho,\rho',\psi)\, I_2(\rho')\,d\rho'\,d\psi \;,$$

$$(-h < z < h)\;, \quad (\rho < a)\;, \tag{3c}$$

$$\tfrac{3}{4}I_1(h) = \tfrac{3}{4}I_2(a) = I_1^{ex}(h) - \int_0^{2\pi}\int_{-h}^{h} a^2(1-\cos\psi)\, F(h-z',a,a,\psi)\, I_1(z')\,dz'\,d\psi$$

$$+\, 2ha \int_0^{2\pi}\int_0^{a} \cos\psi\, F(2h,a,\rho',\psi)\, I_3(\rho')\,d\rho'\,d\psi \tag{3d}$$

$$\tfrac{3}{4}I_1(-h) = \tfrac{3}{4}I_3(a) = I_1^{ex}(-h) - \int_0^{2\pi}\int_{-h}^{h} a^2(1-\cos\psi)\, F(-h-z',a,a,\psi)\, I_1(z')\,dz'\,d\psi$$

$$+\, 2ha \int_0^{2\pi}\int_0^{a} \cos\psi\, F(-2h,\rho,\rho',\psi)\, I_2(\rho')\,d\rho'\,d\psi \tag{3e}$$

where

$$I_1(z) \equiv \int_0^{2\pi} \hat{z}\cdot\underline{K}(z,a,\theta)\,a\,d\theta$$

$$I_2(\rho) \equiv -\int_0^{2\pi} \hat{\rho}(\theta)\cdot\underline{K}(h,\rho,\theta)\,\rho\,d\theta$$

$$I_3(\rho) \equiv \int_0^{2\pi} \hat{\rho}(\theta)\cdot\underline{K}(-h,\rho,\theta)\,\rho\,d\theta$$

$$I_1^{ex}(z) \equiv \int_0^{2\pi} \hat{z}\cdot\underline{K}^{ex}(z,a,\theta)a\,d\theta \equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(z,a,\theta)a\,d\theta$$

$$I_2^{ex}(\rho) \equiv -\int_0^{2\pi} \hat{\rho}(\theta)\cdot\underline{K}^{ex}(h,\rho,\theta)\rho\,d\theta \equiv -\int_0^{2\pi} \hat{n}(\theta)\cdot\hat{z} \times \underline{H}^{ex}(h,\rho,\theta)\rho\,d\theta$$

$$\equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(h,\rho,a)\rho\,d\theta$$

$$I_3^{ex}(\rho) \equiv \int_0^{2\pi} \hat{\rho}(\theta)\cdot\underline{K}^{ex}(-h,\rho,\theta)\rho\,d\theta \equiv -\int_0^{2\pi} \hat{n}(\theta)\cdot\hat{z} \times \underline{H}^{ex}(-h,\rho,\theta)\rho\,d\theta$$

$$\equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(-h,\rho,\theta)\rho\,d\theta$$

$$F(z-z',\rho,\rho',\psi) = \frac{(ikR-1)\; e^{ikR}}{4\pi R^3}$$

$$R^2 = (z-z')^2 + \rho^2 + \rho'^2\; 2\rho\rho' \cos\psi \; .$$

$I_1(z)$ is the total current flowing in the direction of the axis of the cylinder, $I_2(\rho)$ and $I_3(\rho)$ are the total currents crossing a circle of radius ρ on the ends. It should be noted that the integral equation is exact; no approximations have been made in the derivation. Further although there may be circumferential currents on the cylinder depending upon the external sources, these currents do not couple to the axial and radial currents.

III NUMERICAL QUADRATURE

We assume that the external magnetic field is in the positive x direction and is that of a plane wave propagating in the -y direction. That is

$$\underline{H}^{ex}(z,a,\theta) = \hat{x}H_x^{ex}\, e^{-iky} .$$

Hence

$$I_1^{ex}(z) \equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(z,a,\theta)a\,d\theta$$

$$\equiv -\int_0^{2\pi} H_x^{ex} \sin\theta\, e^{-ika\sin\theta}\, a\,d\theta$$

$$\equiv -aH_x^{ex} \int_0^{2\pi} \sin\theta\, \{\cos(-ka\sin\theta) + i\sin(-ka\sin\theta)\}d\theta$$

$$\equiv i2\pi a\, J_1(ka)\, H_x^{ex}$$

$$I_2^{ex}(\rho) \equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(h,\rho,\theta)\rho\,d\theta$$

$$\equiv i2\pi\rho\, J_1(k\rho)\, H_x^{ex}$$

$$I_3^{ex}(\rho) \equiv \int_0^{2\pi} \hat{z}\cdot\hat{n}(\theta) \times \underline{H}^{ex}(-h,\rho,\theta)\rho\,d\theta$$

$$\equiv i2\pi\rho\, J_1(k\rho)\, H_x^{ex} .$$

If the length of the cylinder is much greater than its diameter the current on the end will be much smaller than the current elsewhere (excluding minima) and the computational labor can be greatly reduced by assuming

$$I_2(\rho) = I_1(h)\, f(\rho) \ .$$

In reference 1, to satisfy the requirement imposed by the integral equation that $I_2(\rho)$ and its derivative be zero at $\rho = 0$, $f(\rho) = \rho^2/a^2$ was used. Here we will use

$$f(\rho) = 1 - (1-\rho^2/a^2)^{\frac{2}{3}}$$

which, in addition to satisfying the above requirements at $\rho = 0$, leads to the correct form for the charge density for $\rho \to a$. Later, when the results of the calculations using the two forms were compared, there was very little difference in the current even at the end.

From symmetry

$$I_1(z) = I_1(-z)$$

$$I_3(\rho) = I_2(\rho)$$

therefore, equations (3) become

$$\int_0^{2\pi}\int_0^{h} a^2(1-\cos\psi)\,[F(z-z',a,a,\psi) + F(z+z',a,a,\psi)]\, I(z')dz'd\psi$$

$$+ \tfrac{1}{2}I(z)-I(h)a\int_0^{2\pi}\int_0^{a} \cos\psi\,[(h-z)\,F(h-z,a,\rho',\psi) + (h+z)\,F(h+z,a,\rho',\psi)]$$

$$\cdot[1-(1-\frac{\rho'^2}{a^2})^{\frac{2}{3}}]d\rho'd\psi = 12\pi a\, J_1(ka)\, H_x^{ex}\ , \qquad (0 \le z < h)\ ; \qquad (4a)$$

$$\int_0^{2\pi}\int_0^{h} a^2(1-\cos\psi)\,[F(h-z',a,a,\psi) + F(h+z',a,a,\psi)]\, I(z')dz'd\psi$$

$$+ I(h)\{\tfrac{3}{4} - 2ah \int_0^{2\pi}\int_0^{a} \cos\psi\, F(2h,a,\rho',\psi)[1-(1-\rho'^2/a^2)^{\frac{3}{2}}]d\rho' d\psi\}$$

$$= i2\pi a\, J_1(ka)\, H_x^{ex} \qquad (4b)$$

Now, referring to figure 1, let the cylinder be divided into N_p zones and the last zone divided into $N_s + 1$ zones according to the formulae

$$\frac{h}{N_p-1} = \Delta'$$

$$2N_1 = N_p-1$$

$$z_j = 2j\Delta' \;; \qquad 0 \le j < N_1$$

$$d' = \Delta'$$

$$d^{(j+1)} = d^{(j)}/3^j$$

$$z_{N_1+j-1} = h-\tfrac{1}{2}(d^{(j)} + d^{(j+1)})$$

$$2\Delta^{(j+1)} = d^{(j)} - d^{(j+1)}$$

for $(1 \le j \le N_s)$

and $z_{N_1+N_s} = h$.

Also assume the current is constant over a zone, that is

$$I(z) = I(z_j) = I_j$$

for z within zone j .

Then equation (6) becomes

$$\sum_{n=0}^{N_u} C_{m,n} I_n = Y_m \ , \qquad 0 \le m \le N_u$$

where

$$N_u = N_1 + N_s$$

$$Y_m = i2\pi a \ J_1(ka) \ H_x^{ex}$$

$$C_{m,n} = \begin{cases} U(\xi_a,\delta) + U(\xi_b,\delta) \ ; & 0 < n < N_u \\ U(\xi_a,\delta) \ ; & n = 0 \\ U(\xi_a-\delta/2,\delta/2) + U(\xi_b-\delta/2,\delta/2) + T(\xi_a) + T(\xi_b); & m \neq n = N_u \\ \tfrac{1}{2}U(0,\delta) + \tfrac{1}{2} + U(2h-\delta/2,\delta/2) \ ; \ T(2\,\frac{h}{a}) \ ; & m = n = N_u \end{cases}$$

$$\xi_a = \frac{|z_m - z_n|}{a} = |m-n|\,\delta$$

$$\xi_b = \frac{z_m + z_n}{a} = (m+n)\delta$$

$$\delta = \frac{\Delta^{(j)}}{a}$$

$$U(\xi,\delta) = \frac{2}{\pi} \int_0^{\pi/2} \int_{-\delta}^{\delta} \sin^2\psi\,(ikaR_u - 1)\,\frac{e^{ikaR_u}}{R_u^3}\, dt\, d\psi + \tfrac{1}{2}\delta(\xi)$$

$$R_u^2 = (t+\xi)^2 + 4\sin^2\psi$$

$$\delta(\xi) = \begin{cases} 0, & \xi \neq 0 \\ 1, & \xi = 0 \end{cases}$$

$$T(\xi) = -\frac{\xi}{2\pi}\int_0^{\pi}\int_0^{1} \cos\psi\,[1-(1-t^2)^{\frac{3}{2}}]\,\frac{(ikaR_T-1)\,e^{ikaR_T}}{R_T^3}\,dt\,d\psi$$

$$R_T^2 = \xi^2 + 1 + t^2 - 2t\cos\psi$$

Numerical Integration

$U(0,\delta)$ cannot be evaluated in its present form because the integrand is infinite for $t = \psi = 0$. To handle the singularity we add and subtract $1 + \frac{1}{2}k^2a^2R_u^2$ to the integrand. Then

$$U(0,\delta) = \tfrac{1}{2} + \frac{4}{\pi}\int_0^{\pi/2}\int_0^{\delta} \sin^2\psi\{[1+\tfrac{1}{2}k^2a^2R_u^2 + (ikaR_u-1)e^{ikaR_u}] - [1+\tfrac{1}{2}k^2a^2R_u^2]\}\,\frac{dt\,d\psi}{R_u^3}.$$

$$\tfrac{1}{2} - \frac{4}{\pi}\int_0^{\pi/2}\int_0^{\delta}\frac{\sin^2\psi\,dt\,d\psi}{(t^2+4\sin^2\psi)^{\frac{3}{2}}} = \tfrac{1}{2} - \frac{\delta}{\pi}\int_0^{\pi/2}\frac{d\psi}{\sqrt{\delta^2+4\sin^2\psi}}$$

$$= \tfrac{1}{2}\left\{1 - \frac{2}{\pi}\int_0^{\pi/2}\frac{d\psi}{\sqrt{1+\frac{4}{\delta^2}\sin^2\psi}}\right\}$$

$$= \tfrac{1}{2}\left\{1 - \frac{2}{\pi}\frac{\delta}{\sqrt{\delta^2+4}}\int_0^{\pi/2}\frac{d\psi}{\sqrt{1-\frac{4}{\delta^2+4}\cos^2\psi}}\right\}$$

$$= \tfrac{1}{2}\left\{1 - \frac{2}{\pi}\frac{\delta}{\sqrt{\delta^2+4}}\,K\right\}$$

where the modulus of the elliptic integral K is $\frac{2}{\sqrt{\delta^2+4}}$. If $\frac{4}{\delta^2} \le 0.1$, $[1 + \frac{4}{\delta^2}\sin^2\psi]^{-\frac{1}{2}}$ is expanded by the binomial theorem and integrated term by term with the result

$$1 - \frac{2}{\pi}\int_0^{\pi/2} \frac{d\psi}{\sqrt{1 + \frac{4}{\delta^2}\sin^2\psi}} = \frac{1^2}{2^2}\left(\frac{4}{\delta^2}\right)^2 - \frac{1^2\cdot 3^2}{2^2\cdot 4^2}\left(\frac{4}{\delta^2}\right)^4 + \frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}\left(\frac{4}{\delta^2}\right)^6 - \dots .$$

Also

$$-\frac{2}{\pi}k^2a^2\int_0^{\pi/2}\int_0^{\delta}\frac{\sin^2\psi\, dt d\psi}{\sqrt{t^2+4\sin^2\psi}} = -\frac{2}{\pi}k^2a^2\int_0^{\pi/2}\sin^2\psi\log\left(\frac{\delta + \sqrt{\delta^2+4\sin^2\psi}}{2\sin\psi}\right)d\psi$$

$$= -\frac{2}{\pi}k^2a^2\{-\int_0^{\pi/2}\sin^2\psi\log\sin\psi\, d\psi + \log\frac{\delta}{2}\int_0^{\pi/2}\sin^2\psi\, d\psi$$

$$+\int_0^{\pi/2}\sin^2\psi\log[1 + \sqrt{1 + (\tfrac{2}{\delta})^2\sin^2\psi}]d\psi\}$$

$$= -\tfrac{1}{2}k^2a^2\{\frac{4}{\pi}\int_0^{\pi/2}\sin^2\psi\log[1 + \sqrt{1 + (\tfrac{2}{\delta})^2\sin^2\psi}]d\psi + \log\delta - \tfrac{1}{2}\} .$$

Now define

$$U_1(t) = [1 + \tfrac{1}{2}k^2a^2R^2 + (ikaR-1)e^{ikaR}]/R^3$$

with $R^2 = t^2 + 4\sin^2\psi$, and

$$\int_0^{\delta} U_1(t)dt = \frac{\delta}{2}\sum_{n=1}^{N} w_n U_1(\delta t_n)$$

where

$$t_n = \tfrac{1}{2}(1 + x_n)$$

and w_n and x_n are the weights and abcissas of the N point Gaussian quadrature, and

$$U_2(\psi) \equiv \{\int_0^{\delta} U_1(t)dt - \frac{k^2a^2}{2} \log[1 + \sqrt{1 + (\frac{2}{\delta})^2 \sin^2\psi}]\} \sin^2\psi ,$$

$$\frac{4}{\pi}\int_0^{\pi/2} U_2(\psi)d\psi = \sum_{n=1}^{N} w_n U_2(\psi_n)$$

where

$$\psi_n = \frac{\pi}{4}(1 + x_n) .$$

Then

$$U(0,\delta) = \tfrac{1}{2}U_0(\delta) + \sum_{n=1}^{N} w_n U_2(\psi_n) - \tfrac{1}{2}k^2a^2 [\log \delta - \tfrac{1}{2}]$$

where

$$U_0(\delta) \equiv 1 - \frac{2}{\pi}\int_0^{\pi/2} \frac{d\psi}{\sqrt{1 + \frac{4}{\delta^2}\sin^2\psi}} .$$

Next define

$$U_3(t) \equiv \frac{4 \sin^2\psi}{R^3} (ikaR-1)e^{ikaR}$$

where

$$R^2 = (\xi + t)^2 + 4 \sin^2\psi$$

$$U_4(\psi) \equiv \frac{1}{\delta}\int_{-\delta}^{\delta} U_3(t)dt = \sum_{n=1}^{N} w_n U_3(\delta x_1) .$$

Then

$$U(\xi,\delta) = \frac{\delta}{8} \sum_{n=1}^{N} w_n \, U_4(\psi_n) .$$

Next, adding and subtracting $1 + \frac{1}{2}k^2 a^2 R_T^2$ from the integrand of $T(x)$, we get

$$T(x) = -\frac{x}{2\pi} \int_0^{\pi} \int_0^{1} \{(ikaR_T - 1)e^{ikaR_T}[1-(1-t^2)^{\frac{3}{2}}] + 1 + \frac{1}{2}k^2 a^2 R_T^2\} \frac{dt \cos\psi \, d\psi}{R_T^3}$$

$$+ \frac{x}{2\pi} \int_0^{\pi} \int_0^{1} [1 + \frac{1}{2}k^2 a^2 R_T^2] \frac{dt \cos\psi \, d\psi}{R_T^3} , \text{ and}$$

$$\frac{x}{2\pi} \int_0^{\pi} \int_0^{1} \frac{dt \cos\psi \, d\psi}{R_T^3} = \frac{x}{2\pi} \int_0^{\pi} \frac{\cos\psi}{x^2 + \sin^2\psi} \left\{ \frac{1-\cos\psi}{\sqrt{x^2+2-2\cos\psi}} + \frac{\cos\psi}{\sqrt{x^2+1}} \right\} d\psi$$

$$= \frac{x}{2\pi} \int_0^{\pi/2} \frac{1}{x^2 + \sin^2\psi} \left\{ \left[\frac{1-\cos\psi}{\sqrt{x^2+2-2\cos\psi}} - \frac{1+\cos\psi}{\sqrt{x^2+2+2\cos\psi}} + \frac{2\cos\psi}{\sqrt{x^2+1}} \right] \cos\psi \right.$$

$$\left. - \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}} \right] \right\} d\psi + \frac{x}{2\pi} \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}} \right] \int_0^{\pi/2} \frac{d\psi}{x^2 + \sin^2\psi}$$

$$= \frac{x}{2\pi} \int_0^{\pi/2} \frac{1}{x^2 + \sin^2\psi} \left\{ \left[\frac{1-\cos\psi}{\sqrt{x^2+2-2\cos\psi}} - \frac{1+\cos\psi}{\sqrt{x^2+2+2\cos\psi}} + 2\frac{\cos\psi}{\sqrt{x^2+1}} \right] \cos\psi \right.$$

$$\left. - \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}} \right] \right\} d\psi + \frac{1}{4} \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}} \right] \frac{1}{\sqrt{x^2+1}} ,$$

$$\frac{k^2a^2x}{4\pi}\int_0^{\pi}\int_0^{1}\frac{dt\,\cos\psi\,d\psi}{R_T}=\frac{k^2a^2x}{4\pi}\int_0^{\pi}\log\left[\frac{1-\cos\psi+\sqrt{x^2+2-2\cos\psi}}{\sqrt{x^2+1}-\cos\psi}\right]\cos\psi\,d\psi\ .$$

Now define

$$f^-(t)\equiv\{(ikaR-1)e^{ikaR}[1-(1-t^2)^{\frac{2}{3}}]+1+\tfrac{1}{2}k^2a^2R^2\}/R^3\,;\quad R^2=x^2+1+t^2-2t\cos\psi$$

$$f^+(t)\equiv\{(ikaR-1)e^{ikaR}[1-(1-t^2)^{\frac{2}{3}}]+1+\tfrac{1}{2}k^2a^2R^2\}/R^3\,;\quad R^2=x^2+1+t^2+2t\cos\psi$$

$$T_1(t)\equiv f^+(t)-f^-(t)$$

$$G(\psi)\equiv 2\int_0^1 T_1(t)dt=\sum_{n=1}^{N}w_nT_1(t_n)\,;\qquad t_n=(1+x_n)$$

$$L(\psi)\equiv\log\frac{[1-\cos\psi+\sqrt{x^2+2-2\cos\psi}][\sqrt{x^2+1}+\cos\psi]}{[\sqrt{x^2+1}-\cos\psi][1+\cos\psi+\sqrt{x^2+2+2\cos\psi}]}$$

$$B(\psi)\equiv\frac{1}{x^2+\sin^2\psi}\left\{\left[\frac{1-\cos\psi}{\sqrt{x^2+2-2\cos\psi}}-\frac{1+\cos\psi}{\sqrt{x^2+2+2\cos\psi}}+2\frac{\cos\psi}{\sqrt{x^2+1}}\right]\cos\psi\right.$$

$$\left.-\left[\frac{2}{\sqrt{x^2+1}}-\frac{2}{\sqrt{x^2+4}}\right]\right\}\ .$$

Then

$$T(x) = \frac{x}{2\pi} \int_0^{\pi/2} \{[\tfrac{1}{2}G(\psi)+\tfrac{1}{2}k^2a^2L(\psi)]\cos\psi + B(\psi)\}d\psi + \frac{1}{4\sqrt{x^2+1}} \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}}\right]$$

$$= \frac{x}{8} \{ \sum_{n=1}^{N} w_n\left[[\tfrac{1}{2}G(\psi_n)+\tfrac{1}{2}k^2a^2L(\psi_n)]\cos\psi_n+B(\psi_n)\right] + \frac{2}{x\sqrt{x^2+1}} \left[\frac{2}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+4}}\right]\} .$$

If $x > 1$ define

$$T_3(t) \equiv \{(ikaR^+-1)\frac{e^{ikaR^+}}{(R^+)^3} - (ikaR^--1)\frac{e^{ikaR^-}}{(R^-)^3}\} [1-(1-t^2)^{\frac{2}{3}}]$$

where

$$(R^+)^2 = x^2 + t^2 + 1 + 2t\cos\psi$$

$$(R^-)^2 = x^2 + t^2 + 1 - 2t\cos\psi \text{ , and}$$

$$T_4(\psi) \equiv 2\cos\psi \int_0^1 T_3(t)dt = \cos\psi \sum_{n=1}^{N} w_n T_3(t_n) .$$

Then

$$T(x) = \frac{x}{16} \sum_{n=1}^{N} w_n T_4(\psi_n) .$$

IV FOURIER INVERSION

The method of obtaining an inverse Fourier transform numerically is thoroughly discussed by Sulkowski[3] and will not be repeated here. It is only necessary to add that the frequency response of the dipole between data points is obtained by Lagrangian interpolation.

V COMPUTER PROGRAMS

Three computer programs were developed to do the computations required to obtain the current on the cylinder induced by an electromagnetic pulse: (1) DIPOLE, (2) DIPLOTK, (3) FORGE. DIPOLE calculates the current induced by a plane wave incident with the electric field parallel to the axis of the cylinder as a function of distance along the axis for a fixed value of the wave number k . DIPLOTK converts the data from DIPOLE into current as a function of angular frequency ω at a fixed distance along the axis. FORGE performs an inverse Fourier transform on the data from DIPLOTK and prints the current as a function of time at a fixed axial distance. All of these programs are written in FORTRAN for the CDC-6600 computer. The information on memory requirements and running time was obtained from checkout runs on the CDC-6600 computer at the Los Angeles Data Center.

Program DIPOLE

Introduction

DIPOLE is a computer program consisting of a main program deck DIPOLE and twelve subroutine decks. It requires as inputs the number of primary and the number of secondary zones into which the dipole length has been divided, the radius of the dipole and its height or half-length and the complex wave-number of the incident wave. DIPOLE produces as output a table of the complex amplitudes of the current flowing in the direction of the axis of the dipole as a function of the distance along the axis.

Operation

The input parameters are read from a punched card. Any number of problems can be run with one loading of the deck since upon completion of one problem the program will go back to the beginning and read another input card. The program stops upon reading an end-of-file card.

Input Parameters

1. NP TYPE INTEGER. This is the number of primary zones into which the dipole is divided and must be odd so that the midpoint of the dipole is also the midpoint of the middle zone. The value of NP must be limited to 25 maximum.
2. NS TYPE INTEGER. This is the number of secondary zones into which the last zone is divided less one. The value of NS must be limited to 5 maximum.
3. A TYPE REAL. This is the radius of the dipole.
4. HITE TYPE REAL. This is the height or half-length of the dipole.
5. KC TYPE COMPLEX. This is the complex wave-number of the incident plane wave.

Output

The output from DIPOLE is in two forms: (1) a printed output of the real and imaginary parts of the complex amplitude of the current and zone length vs. z for the upper half of the dipole and (2) a punched card or BCD tape output of the direction cosines, x and y coordinates and zone lengths in addition to the currents and z coordinates for the whole length of the dipole.

Accuracy

The accuracy of the current values depends upon the ratio of the zone length divided by the diameter which in turn depends upon the number of zones and the height to radius ratio of the dipole. Calculations have been made with HITE/A ratios of 10, 100 and 1000.

For HITE/A = 10, NP was 65 and NS was 4. For HITE/A = 100, NP was 129 and NS was 5. For HITE/A = 1000, NP was 257 and NS was 5. Using these values of current the electric field at the surface of the dipole was calculated and compared to the correct value of zero. From the results, it is believed that the error is less than 1% for HITE/A ratios of 10 and 100 and less than 5% for a ratio of 1000.

Memory Requirements

The program requires a field length of 136700_8 for loading and 132600_8 during execution.

Running Time

The central processor time for HITE/A = 10 is about 25 seconds; for HITE/A = 100 about 13 seconds and for HITE/A = 1000 about 32 seconds.

Program DIPLOTK

Introduction

The main purpose of DIPLOTK is to convert the output of program DIPOLE which is dipole currents as a function of axial distance for a set of wave numbers to the input required for program FORGE which is dipole currents as a function of angular frequency for a set of axial distances. It also multiplies the current by a phase correction factor to make the phase reference the first element of the cylinder hit by the incident wave. In addition DIPLOTK prints or plots the magnitude of the dipole current as a function of wave number at the users option. The plot is useful in establishing the range of interpolation of the current for wave numbers between data points of the input.

Operation

The input data is read from punched cards and/or magnetic tape. The current vs. angular frequency output can easily be accommodated by punched cards but magnetic tape may be used if desired. The plot output is in the form of a magnetic tape which produces a plot on a Calcomp plotter.

Input Parameters

1. INU — TYPE INTEGER. This is the logical unit number of the unit from which the input currents are to be read. A 60 indicates the card reader. A 0 indicates that the current is to be the output from a previous run.
2. IØ1 — TYPE INTEGER. This is the logical unit number of the unit on which the output currents are to be written. A 62 indicates the card punch. A 0 indicates that this output is not desired.
3. IØ2 — TYPE INTEGER. This is the logical unit number of the unit on which the current magnitude vs. wave number is to be written. A 0 indicates that this output is not desired.
4. IPL — TYPE INTEGER. This is the logical unit number of the plot tape. A 0 indicates that a plot is not desired.
5. NZØ — TYPE INTEGER. This is the number of values of axial distance z for which output currents are desired.
6. NWØ — TYPE INTEGER. This is the number of values of angular frequency ω or wave number k .
7. VU — TYPE REAL. This is the scale factor for the plot ordinate.
8. HU — TYPE REAL. This is the scale factor for the plot abcissa.
9. ID — HOLLERITH ARRAY. This is a title for the printed output. It is contained on two punched cards or equivalent magnetic tape records.

10. KL(N) TYPE INTEGER. For interpolation between the N-1 and Nth value of ω sets the lower limit to the KLth value of ω .

11. KU(N) TYPE INTEGER. For interpolation between the N-1 and Nth value of ω sets the upper limit to the KUth value of ω .

12. ZØ(N) TYPE REAL. The Nth value of z for which the current vs. ω is to be outputted.

Input Data

The input data will be on punched cards or magnetic tape--the output from program DIPOLE.

Accuracy

The accuracy of the output current values depends upon the separation of the input data points and the interpolation range. The Lagrangian interpolation scheme used in the program is equivalent to approximating the current as a function of ω by a polynomial. If this approximation is attempted over too great a range of ω there can be large errors in the result. If the interpolation range parameters KL and KU are chosen to produce a smooth plot of current magnitude vs. k and the data points are not too far apart there will be no degradation of accuracy of the currents.

Memory Requirements

The program requires a field length of $53{,}500_8$ for loading and 47400_8 during execution.

Running Time

The central processor time is about 15 seconds.

Program FORGE

Introduction

FORGE is an adaptation of the program FORPLEX by Sulkowski[3]. It performs the inverse Fourier transform on the dipole currents as a function of frequency to obtain dipole currents as a function of time. The major differences between FORGE and FORPLEX are: the angular frequency range has been arbitrarily fixed at 6 decades with the upper frequency limit set at 4×10^9 therefore the parameter sigma determines only the frequency increment; only one pass is made through subroutine CALCLT for each value of sigma; CALCLT has been modified to correspond to the harmonic time dependence $e^{-i\omega t}$; the multiplicity of subroutines has been reduced to two SET and CALCLT and one subroutine RCVCOM to calculate $F(\omega)$ has been added.

Operation

The number of time ranges, sigma, time interval and time range are read from punched cards by the main program FORGE. The time values are calculated and stored in array T. Then subroutine SET is called which calculates the ω values and stores them in array ØM. Subroutine RCVCOM is now called which calculates $F(\omega)$ and stores it in array RCVC. CALCLT is called now to calculate the inverse Fourier transform of $F(\omega)$ and store it in array V. After printing T and V, FORGE goes back and calls RCVCOM. This loop is repeated until the input of dipole current vs. ω is exhausted when the program stops.

Subroutine RCVCOM

A different subroutine RCVCOM is required for each incident waveform. Two versions have been written: one for the unit step function and one for the function $e^{-c_1 t} - e^{-c_2 t}$. The dipole current vs. ω data is read in this subroutine and values of current for values of ω in between the data points are calculated by Lagrangian interpolation over a range determined by the parameters KL and KU.

Input Parameters, Program FORGE

1. NT TYPE INTEGER. This is the number of values of DT and TIM.
2. SIG TYPE REAL. This is the accuracy parameter sigma which determines the frequency increment in the inverse Fourier transform integration.
3. DT(N) TYPE REAL. This is the Nth time increment.
4. TIM(N) TYPE REAL. This is the Nth time range.

Input Parameters, Subroutine RCVCOM

1. IU TYPE REAL. This is the logical unit number of the unit from which the current vs. frequency data are read.
2. NQ TYPE REAL. This is the number of data points in the current vs. frequency data.
3. KL(N) TYPE INTEGER. For interpolation between the N-1 and Nth value of ω sets the lower limit to the KLth value of ω.
4. KU(N) TYPE INTEGER. For interpolation between the N-1 and Nth value of ω sets the upper limit to the KUth value of ω.

Input Data, Subroutine RCVCOM

The input data will be on punched cards or magnetic tape--the output from program DIPLOTK.

Memory Requirements

The program requires a field length of 56200_8 for loading and 52600_8 during execution.

Running Time

The central processor time required is about 22 seconds for sigma = 0.01.

VI RESULTS

Geometrical relationships are shown in figure 1. The magnitude and phase of the currents as a function of wave number are presented in figures 2 through 7. Figures 8 through 13 exhibit the currents as a function of time with distance a parameter and current as a function of distance at the times where the current has a relative maximum or minimum. The incident pulse was the unit step function.

Figure 14 is a plot of the function $e^{-c_1 t} - e^{-c_2 t}$ which was used as the incident pulse in the succeeding current plots.

Figure 27 is a plot of the damping constant α as a function of a/h obtained by fitting a curve of the form $Ae^{-\alpha ct/h}$ to the envelope of the plot of current vs. time at the center of the cylinder.

The following symbols are used

- a = radius of cylinder [meters]
- h = length of cylinder/2 [meters]
- H_0 = unit magnetic field [1 ampere/meter]
- ω = 2π x frequency [hertz]
- k = ω/c = wave number
- c = speed of light in a vacuum [meters/second]

I = current [amperes]
t = time [seconds]
z = distance along the axis of the cylinder measured from the center [meters]
α = damping constant

REFERENCES

1 Sassman, R. W., Latham, R. W., and Berger, A. G., Electromagnetic Scattering from a Conducting Post, Sensor and Simulation Notes, Note XLV, June 1967.

2 Trulio, J. G., Electromagnetic Scattering and Radiation by the Integral Representation Method, Northrop Ventura Report 2798, August 1, 1963.

3 Sulkowski, Frank J., FORPLEX, A Program to Calculate Inverse Fourier Transforms, Mathematics Notes, Note II, 10 Nov. 1966.

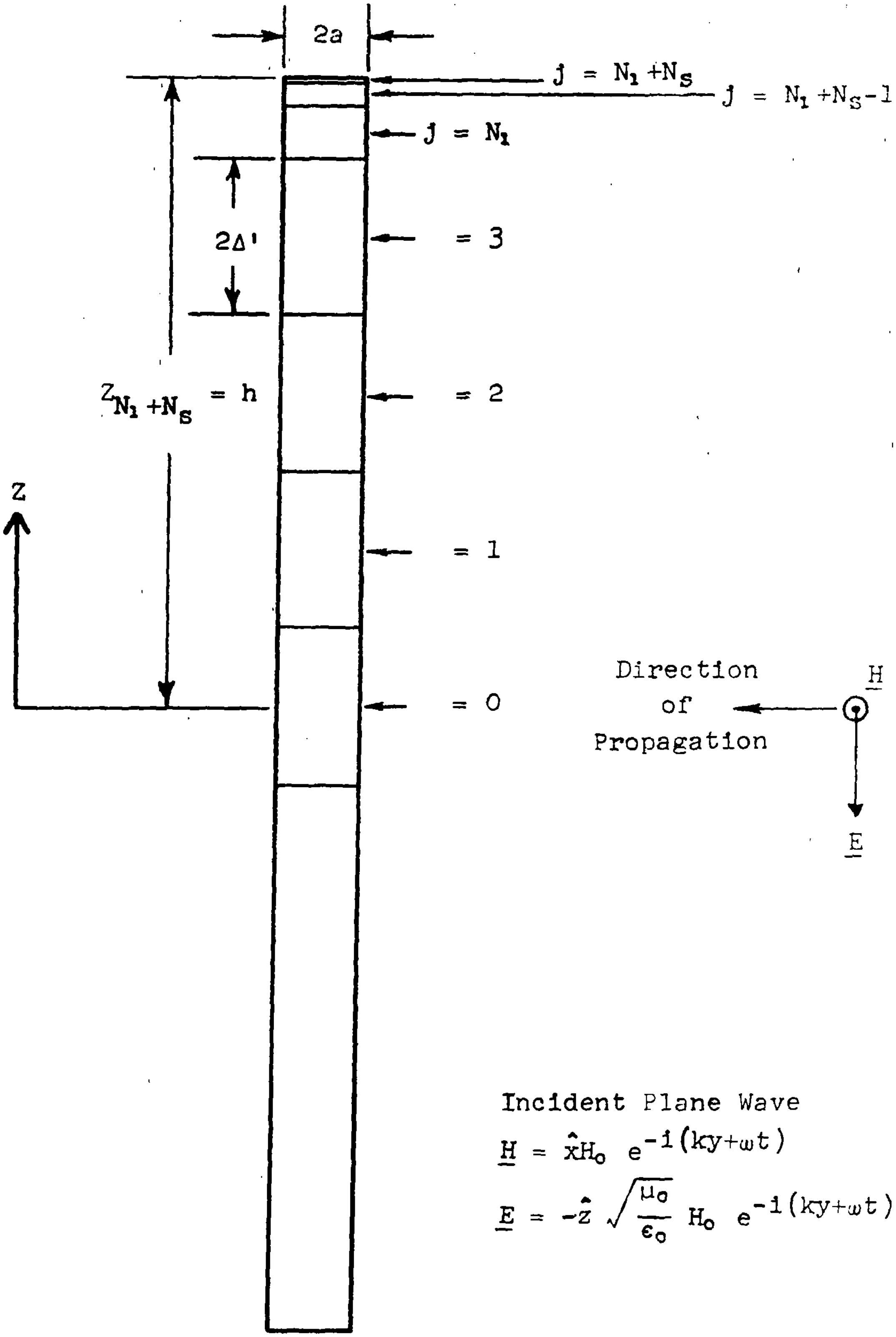

Figure 1. Geometry of the Cylinder

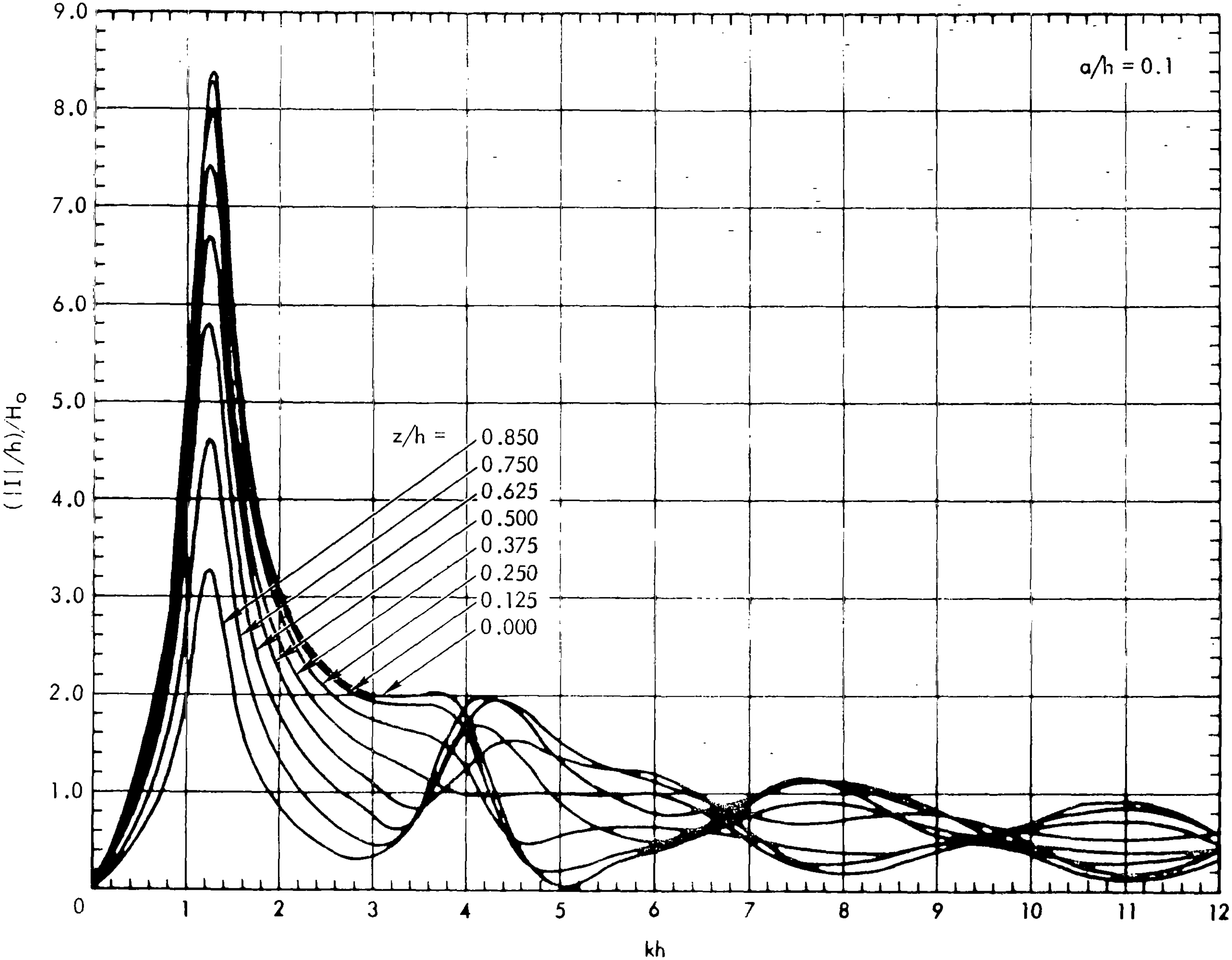

Figure 2 Magnitude of the Current on a Cylinder Scattering a Plane Wave versus kh

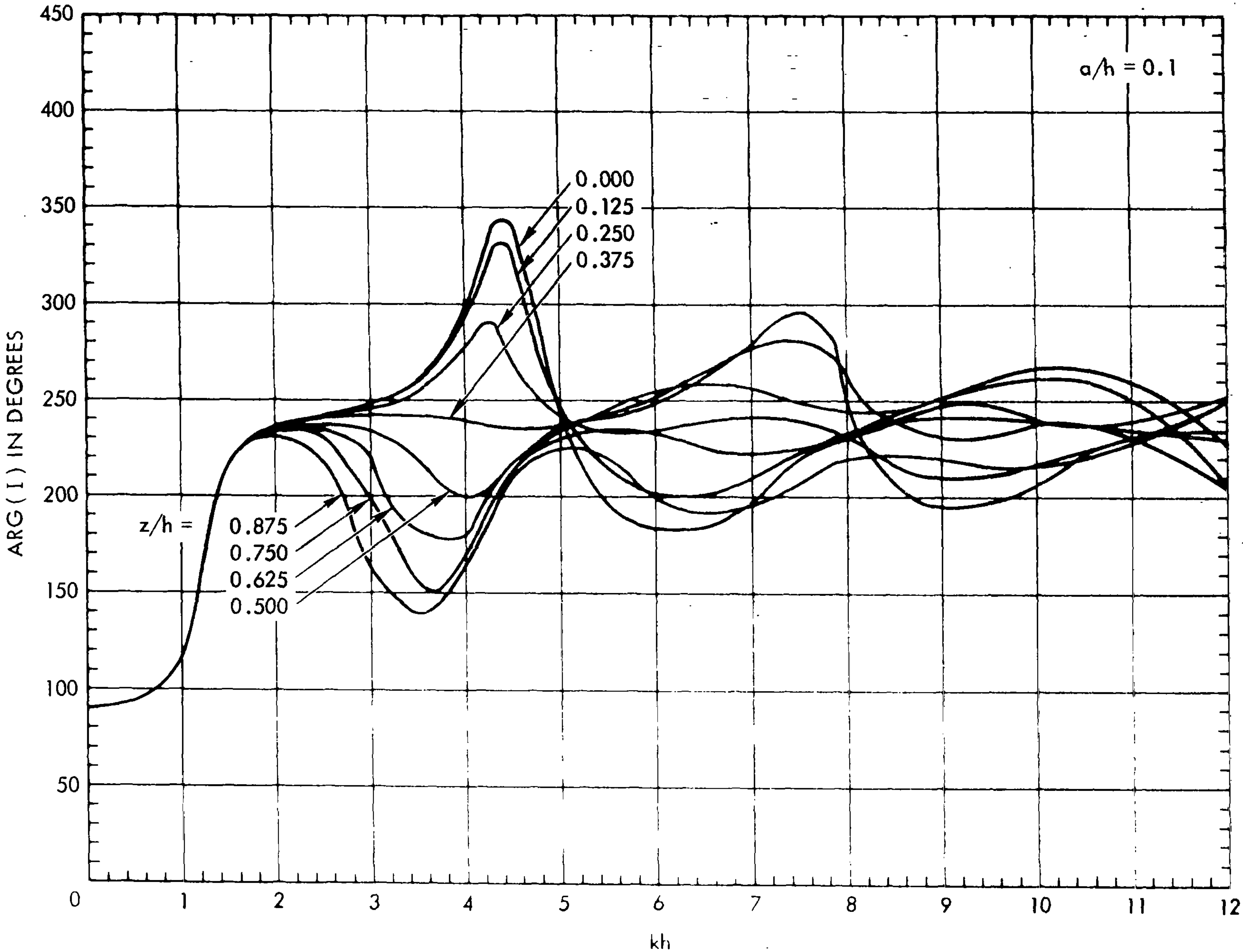

Figure 3 Phase of the Current on a Cylinder Scattering a Plane Wave versus kh

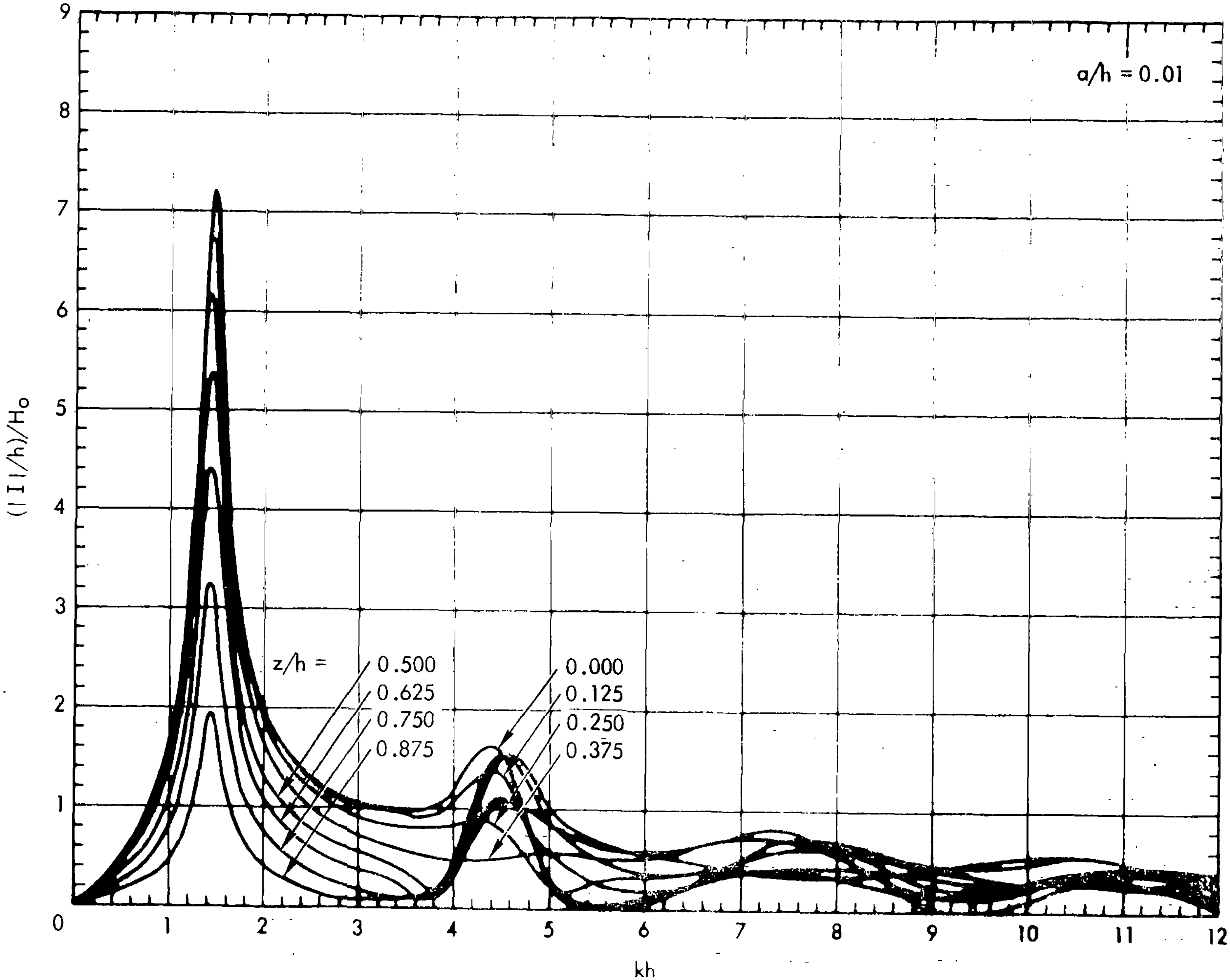

Figure 4 Magnitude of the Current on a Cylinder Scattering a Plane Wave versus kh

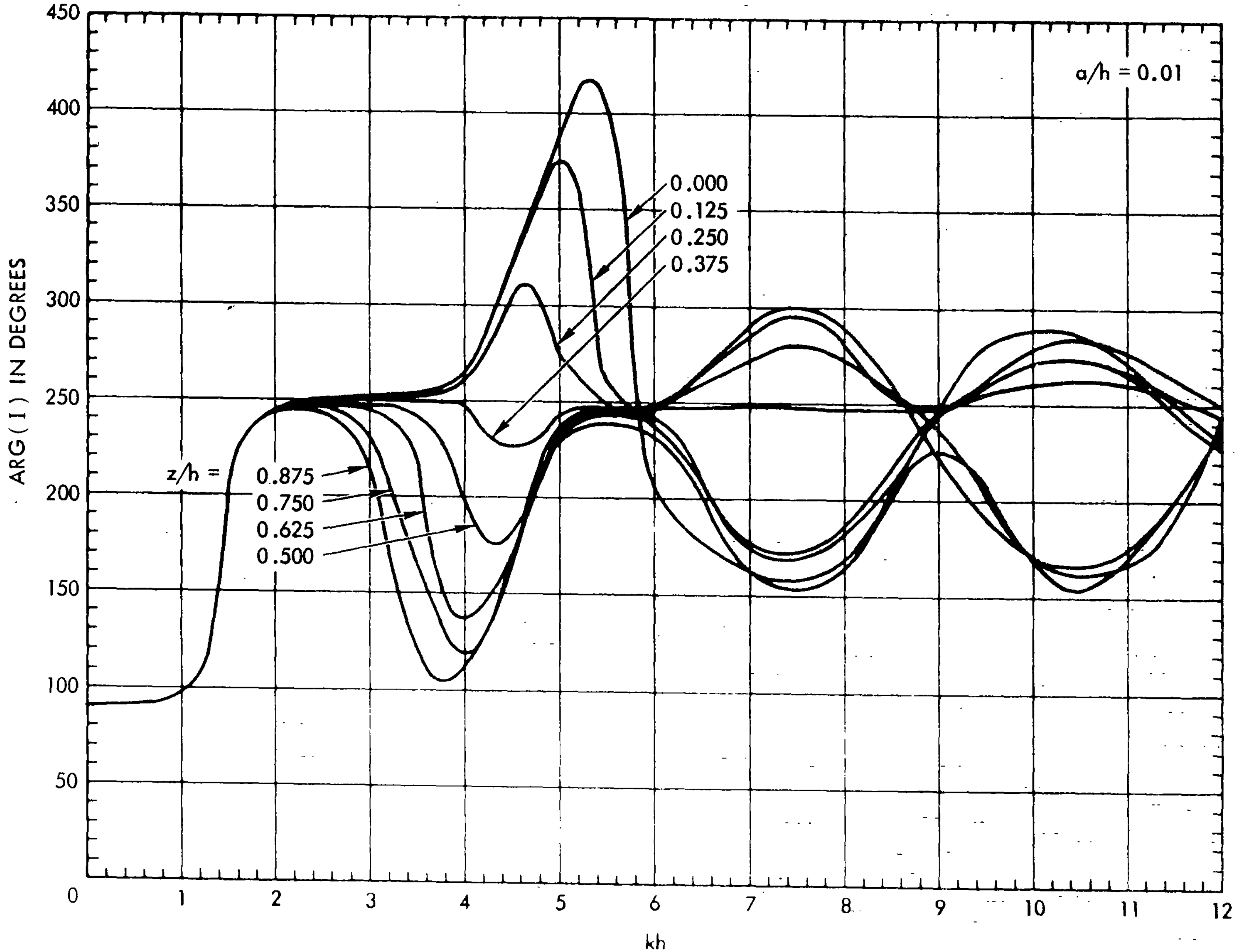

Figure 5 Phase of the Current on a Cylinder Scattering a Plane Wave versus kh

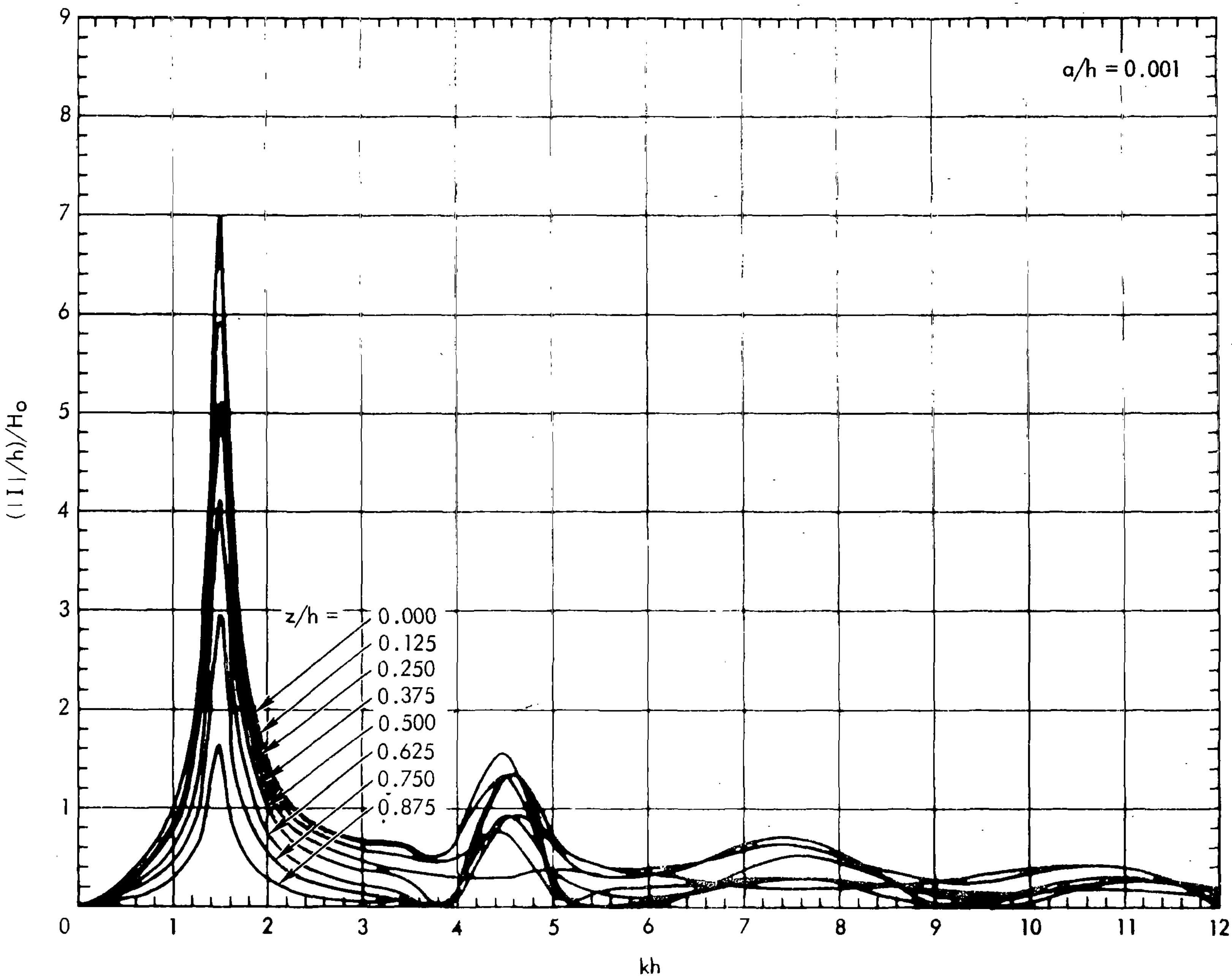

Figure 6 Magnitude of the Current on a Cylinder Scattering a Plane Wave versus kh

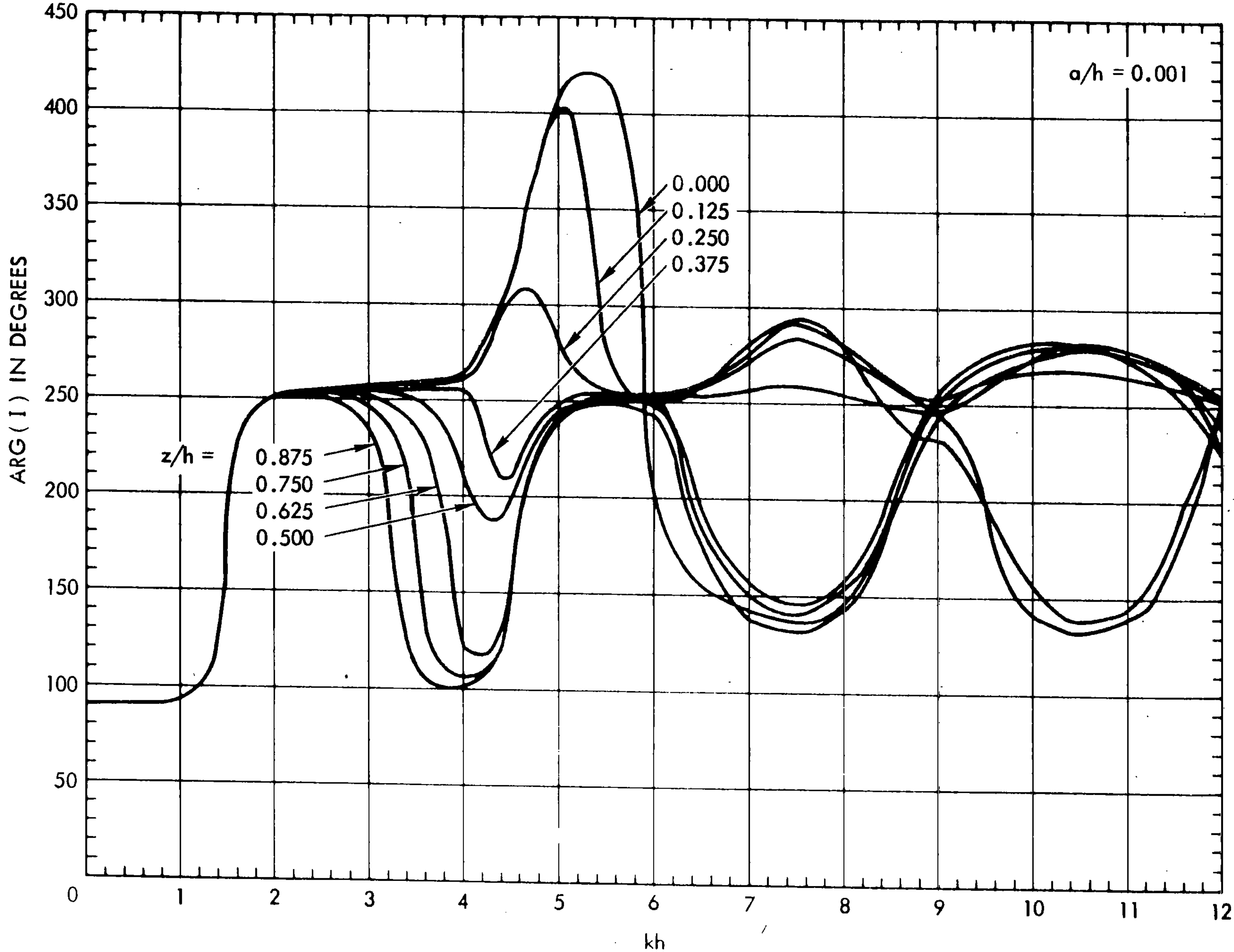

Figure 7 Phase of the Current on a Cylinder Scattering a Plane Wave versus kh

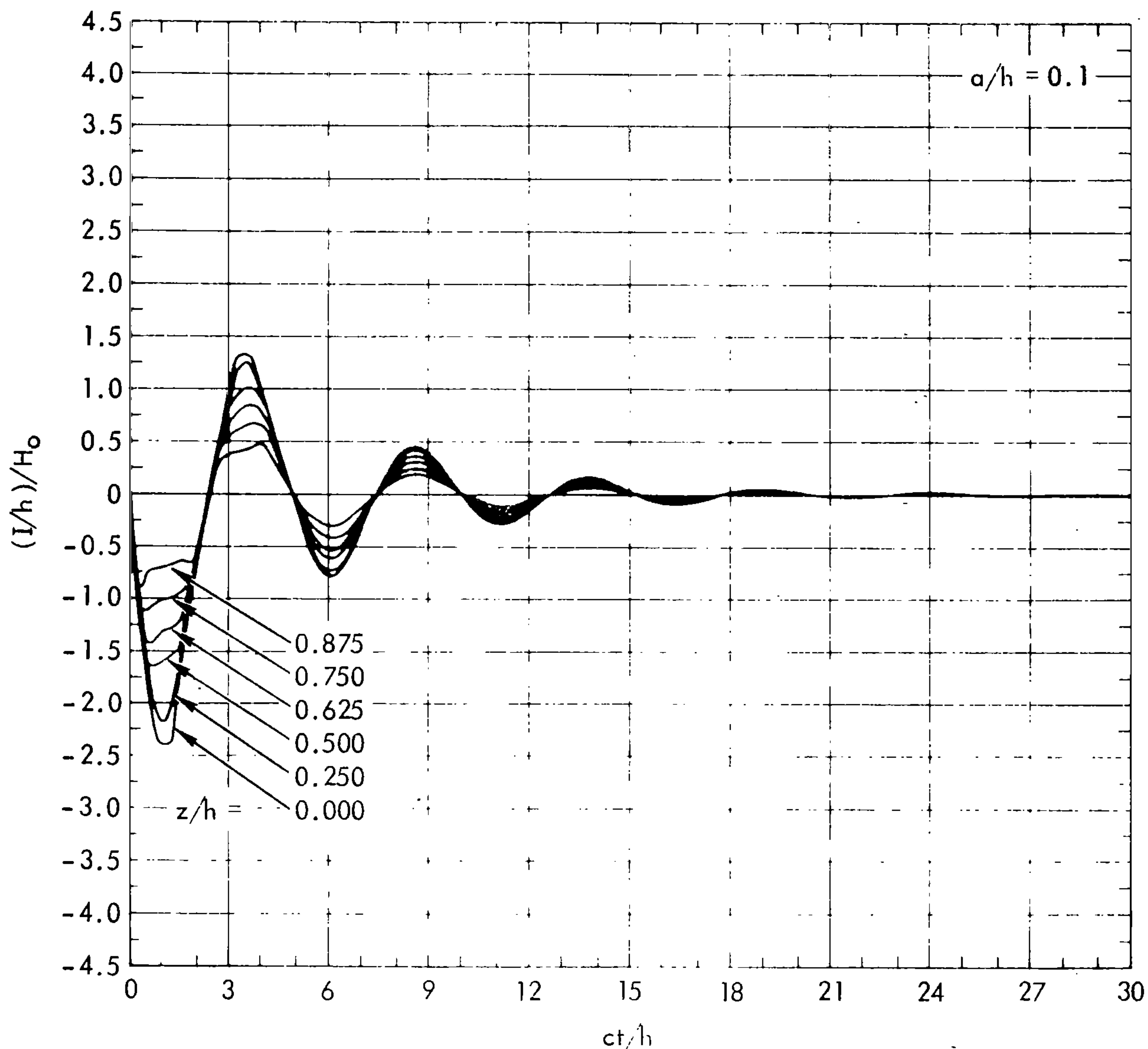

Figure 8 Current on a Cylinder Induced by a Unit Step of Magnetic Field verus ct/h

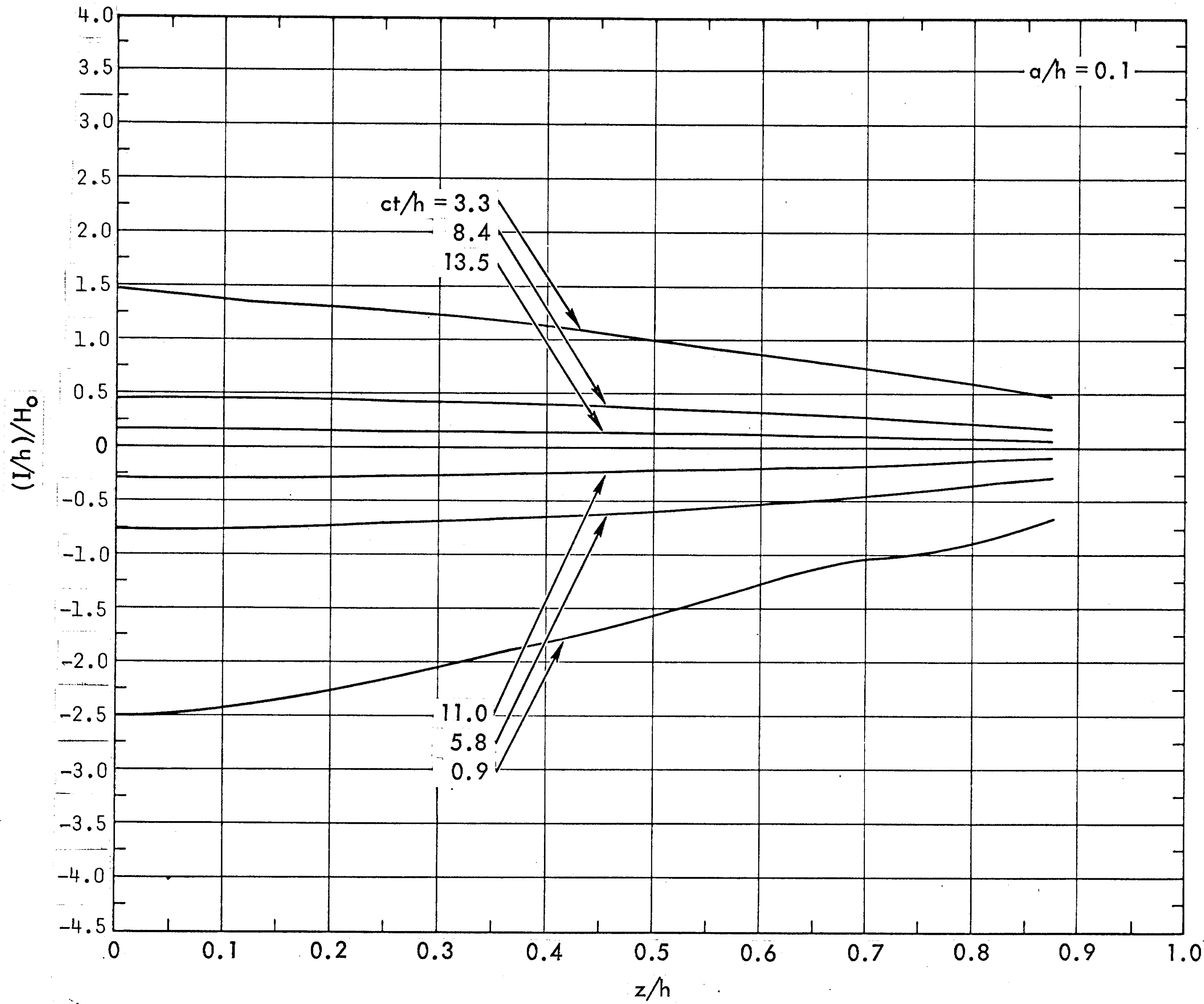

Figure 9 Current on a Cylinder Induced by a Unit Step of Magnetic Field versus z/h

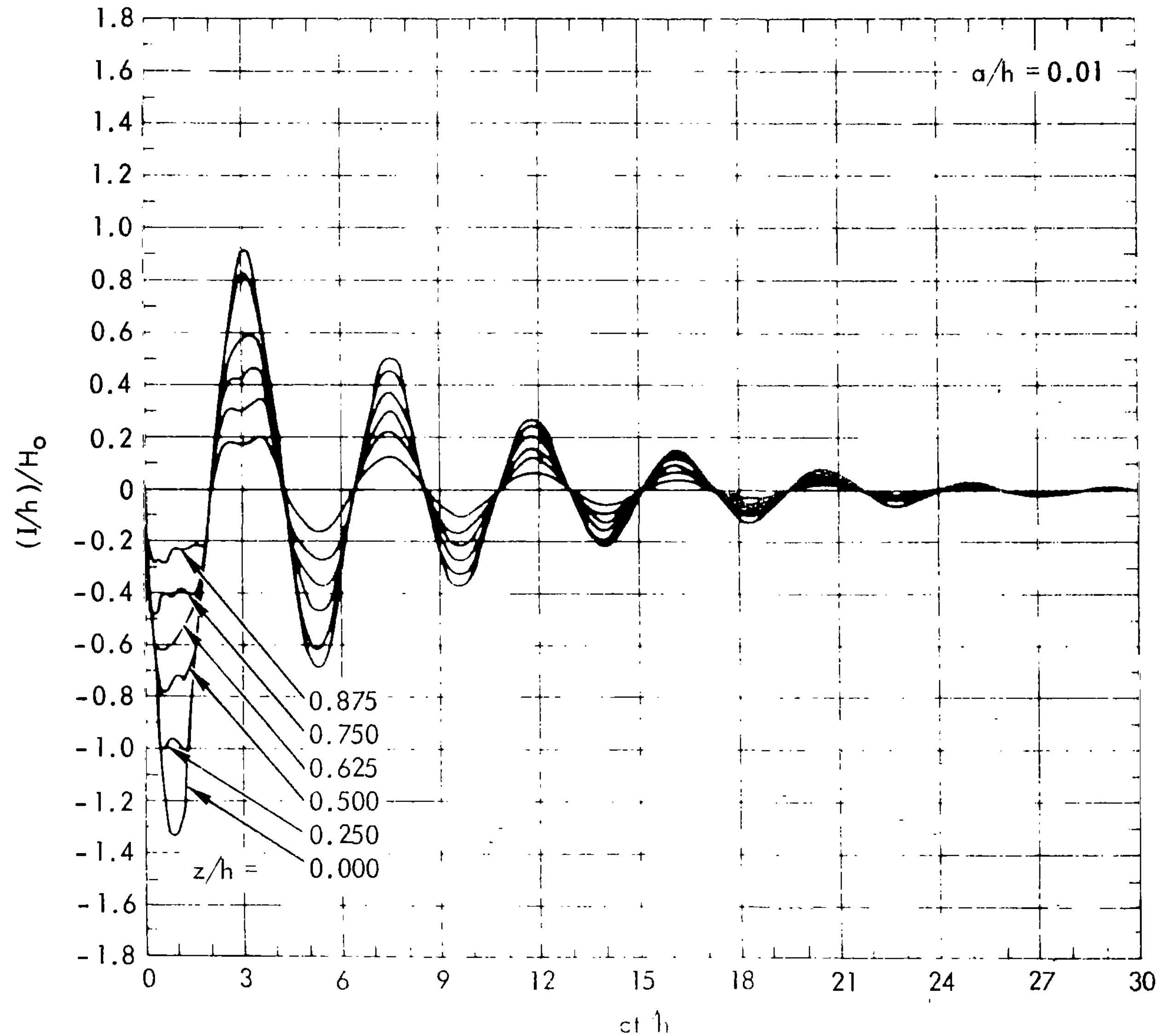

Figure 10 Current on a Cylinder Induced by a Unit Step of Magnetic Field versus ct/h

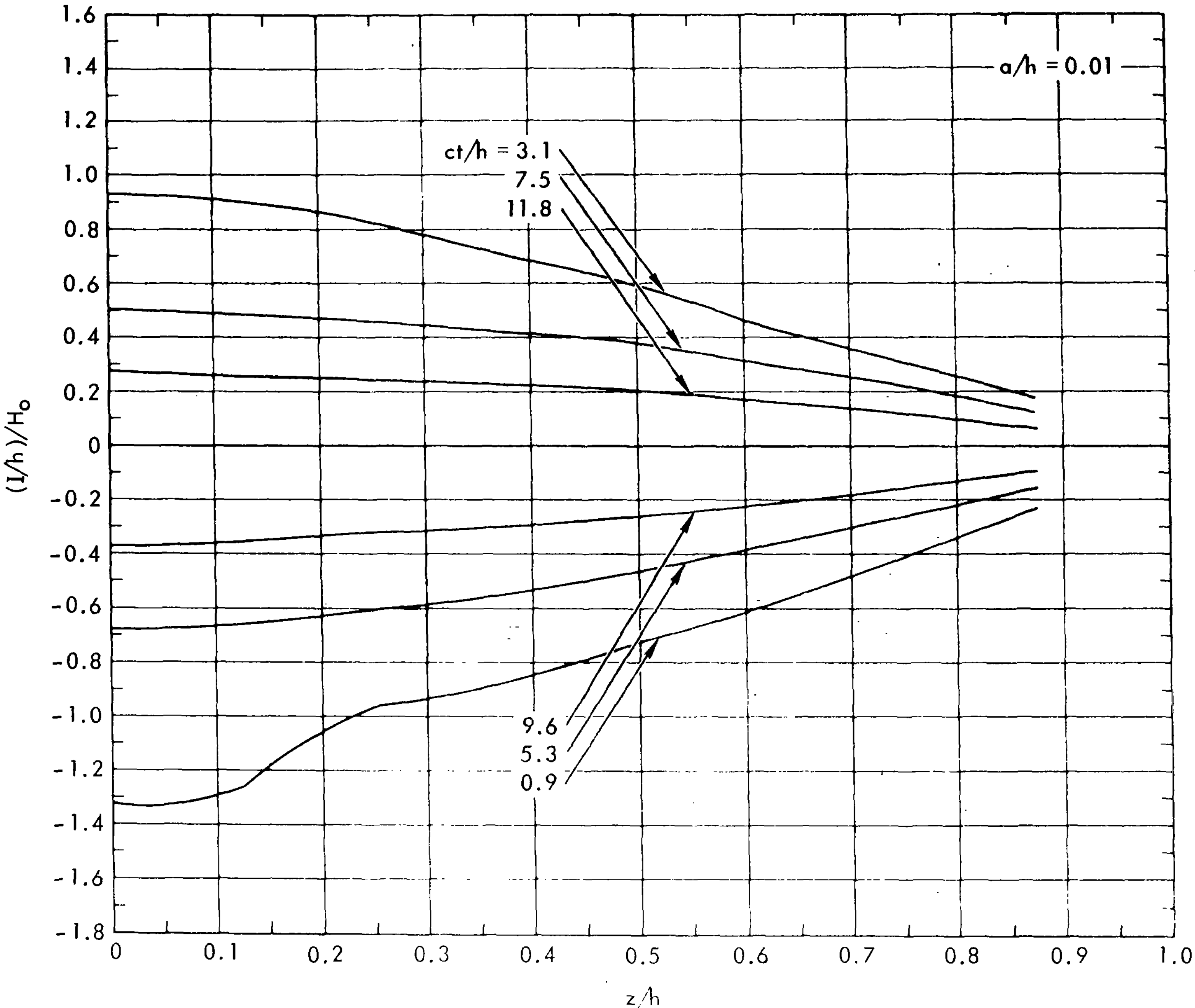

Figure 11 Current on a Cylinder Induced by a Unit Step of Magnetic Field versus z/h

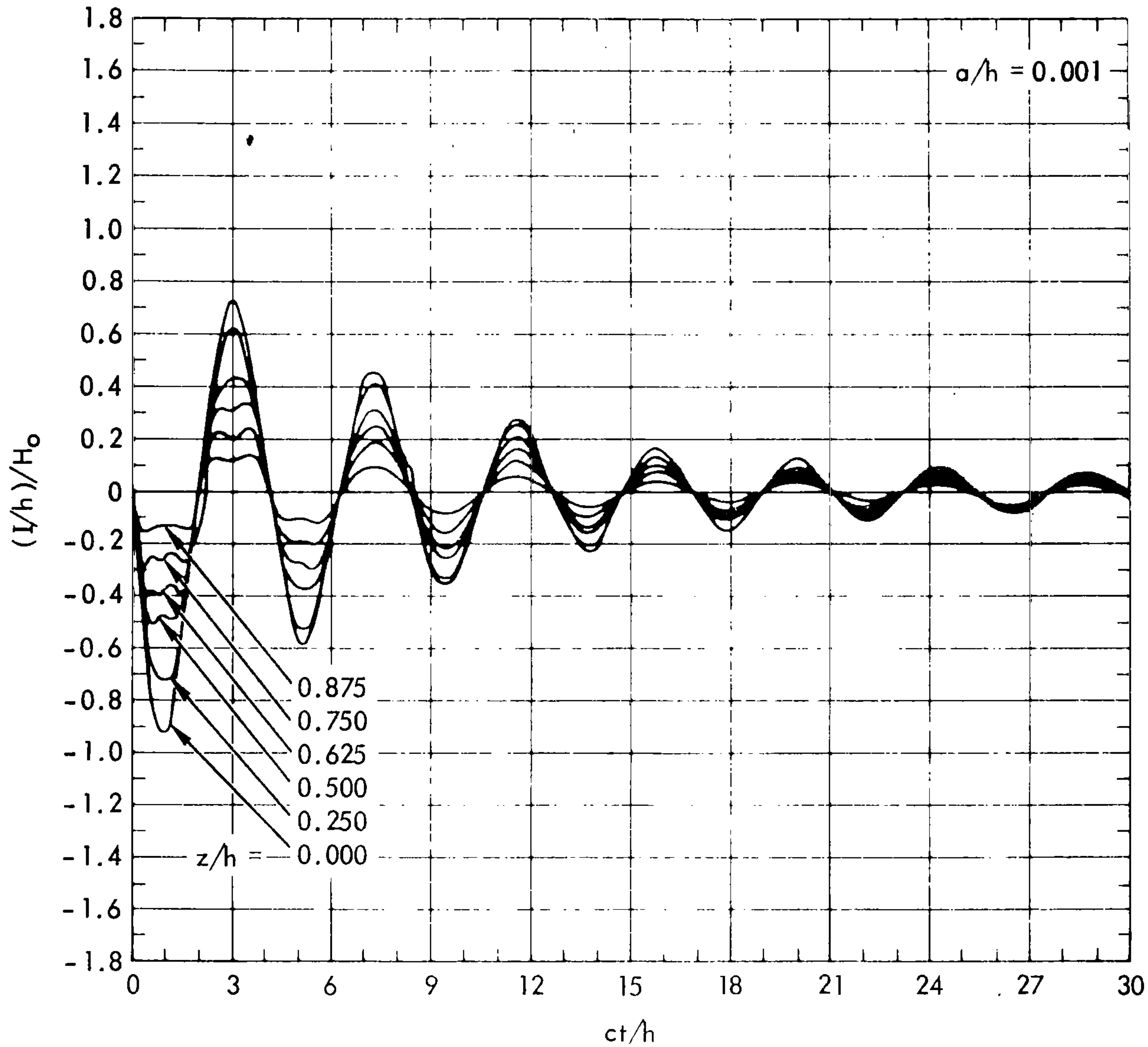

Figure 12 Current on a Cylinder Induced by a Unit Step of Magnetic Field versus ct/h

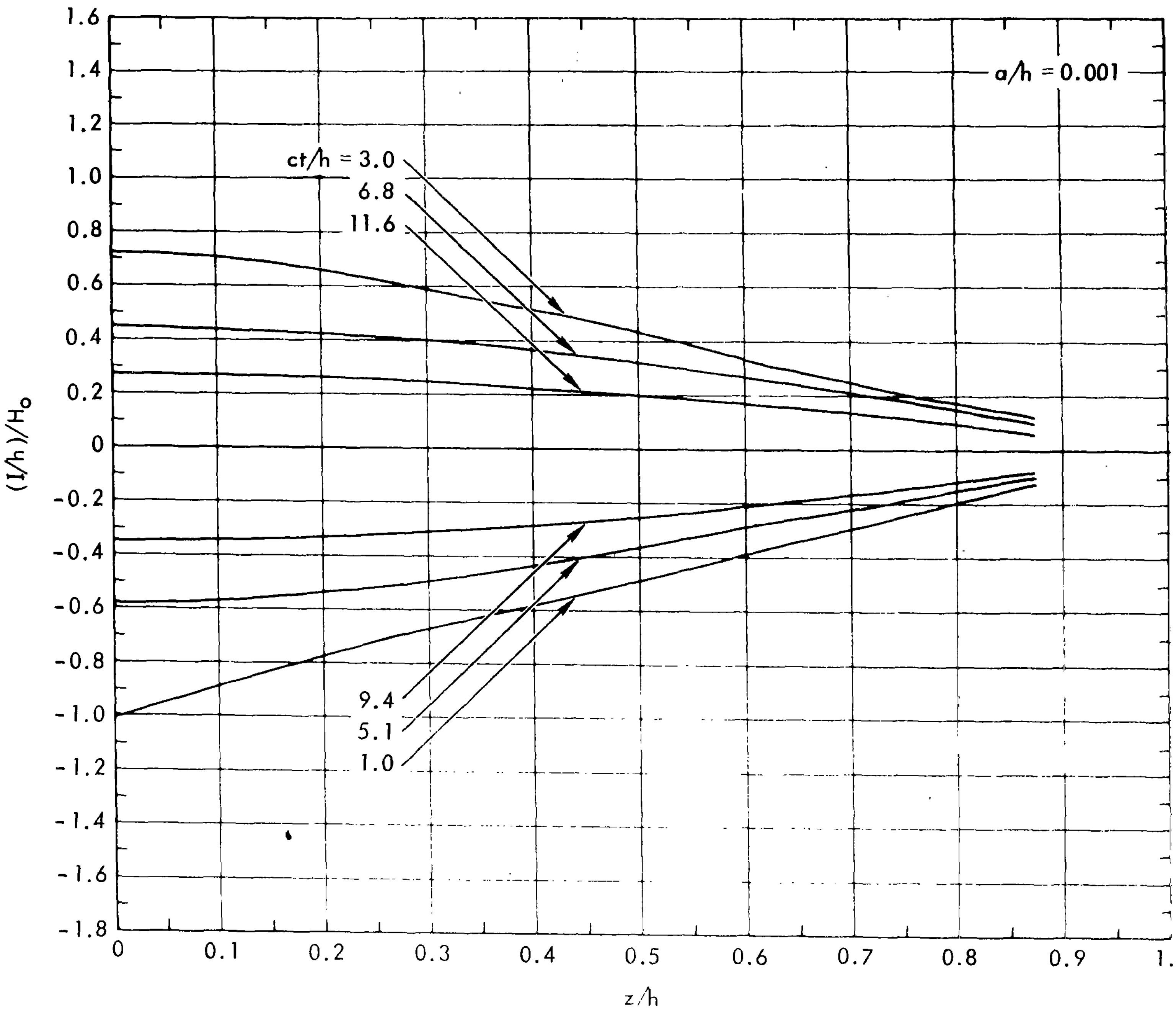

Figure 13 Current on a Cylinder Induced by a Unit Step of Magnetic Field versus z/h

changes to Int. note 11

C_1, C_2 = old nos/3×10^8

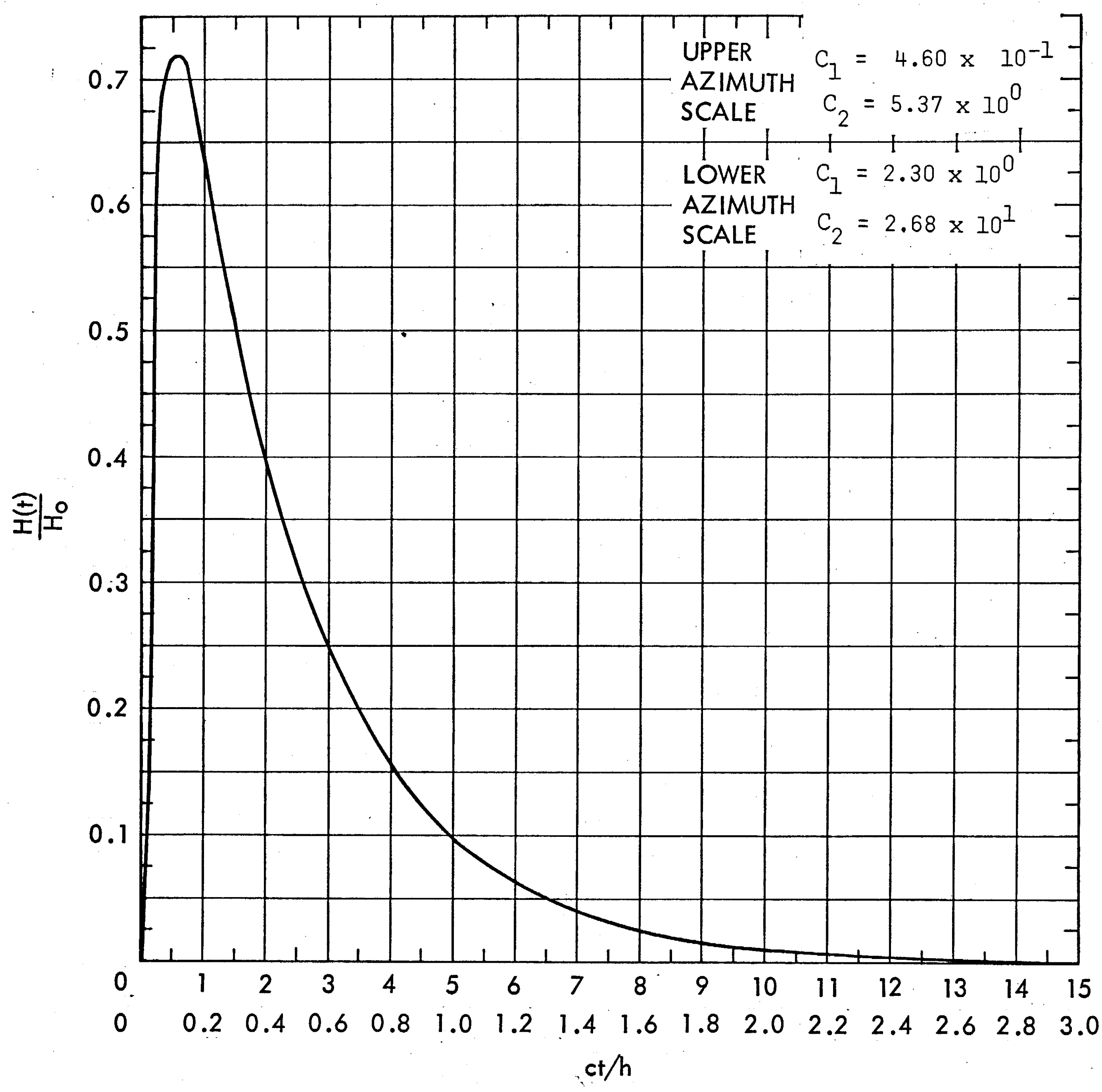

Figure 14 The Function $\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}}$ versus ct/h

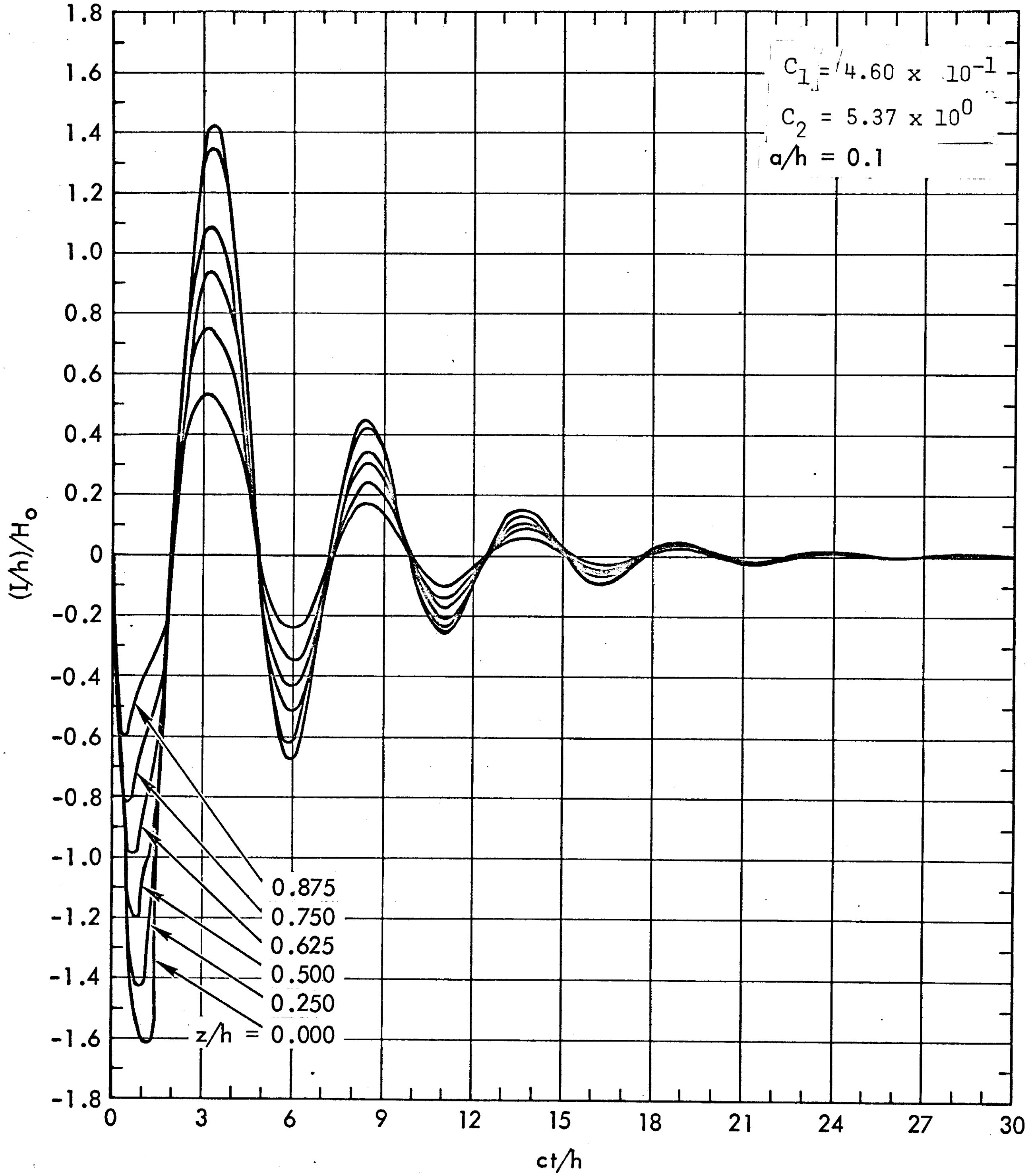

Figure 15 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}} \quad \text{versus } ct/h$$

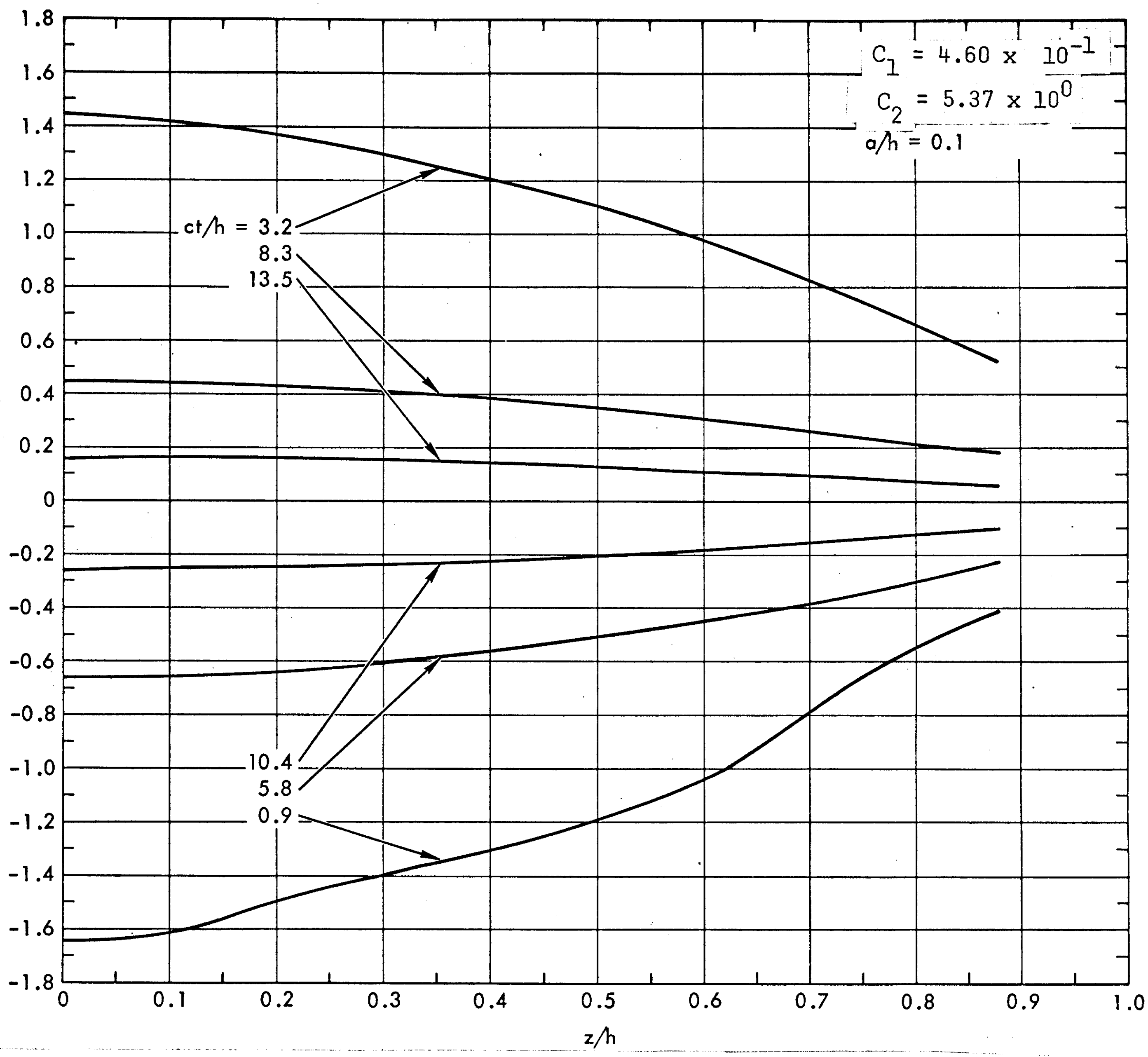

Figure 16 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}}$ **versus z/h**

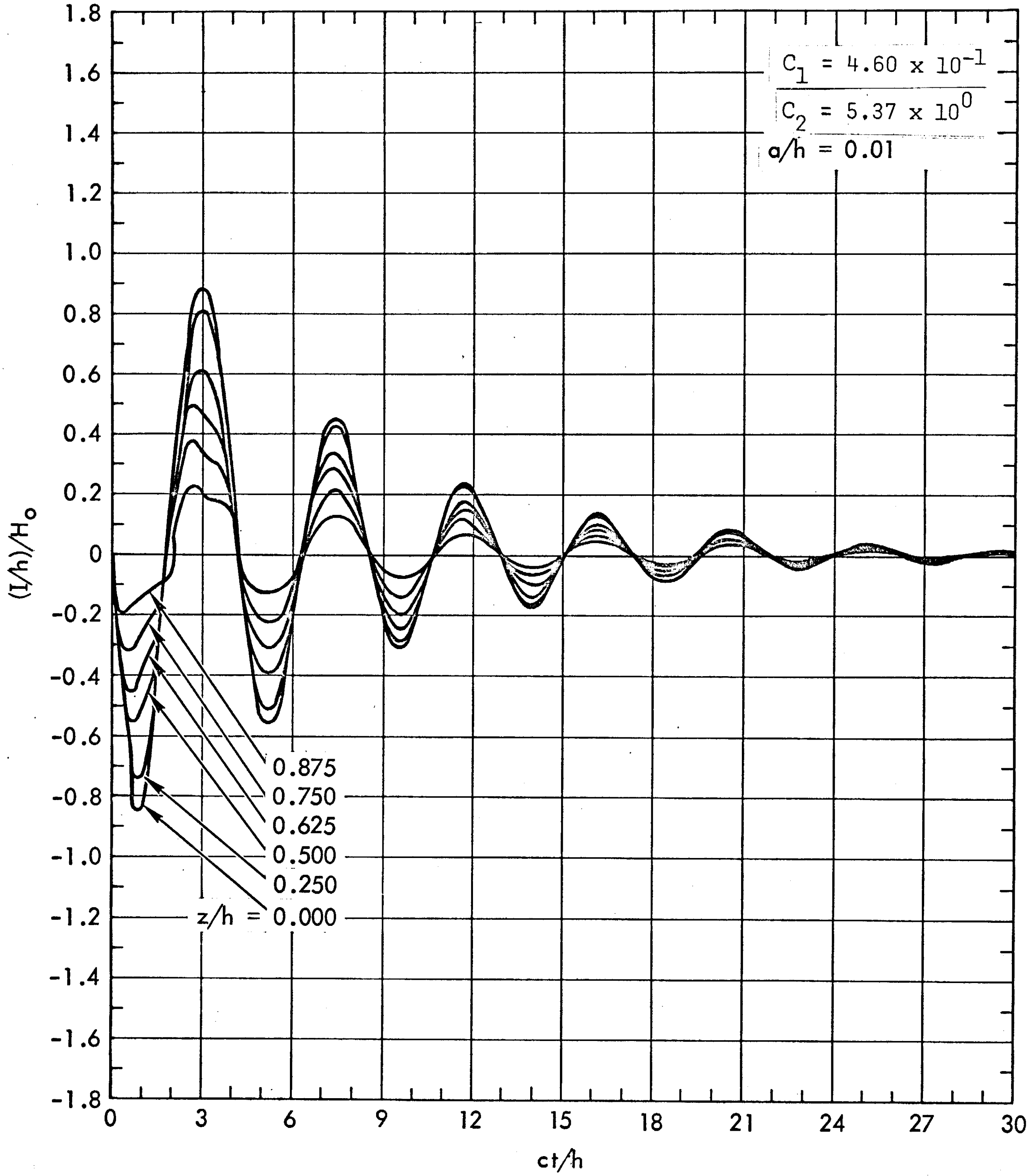

Figure 17 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}} \quad \text{versus } ct/h$$

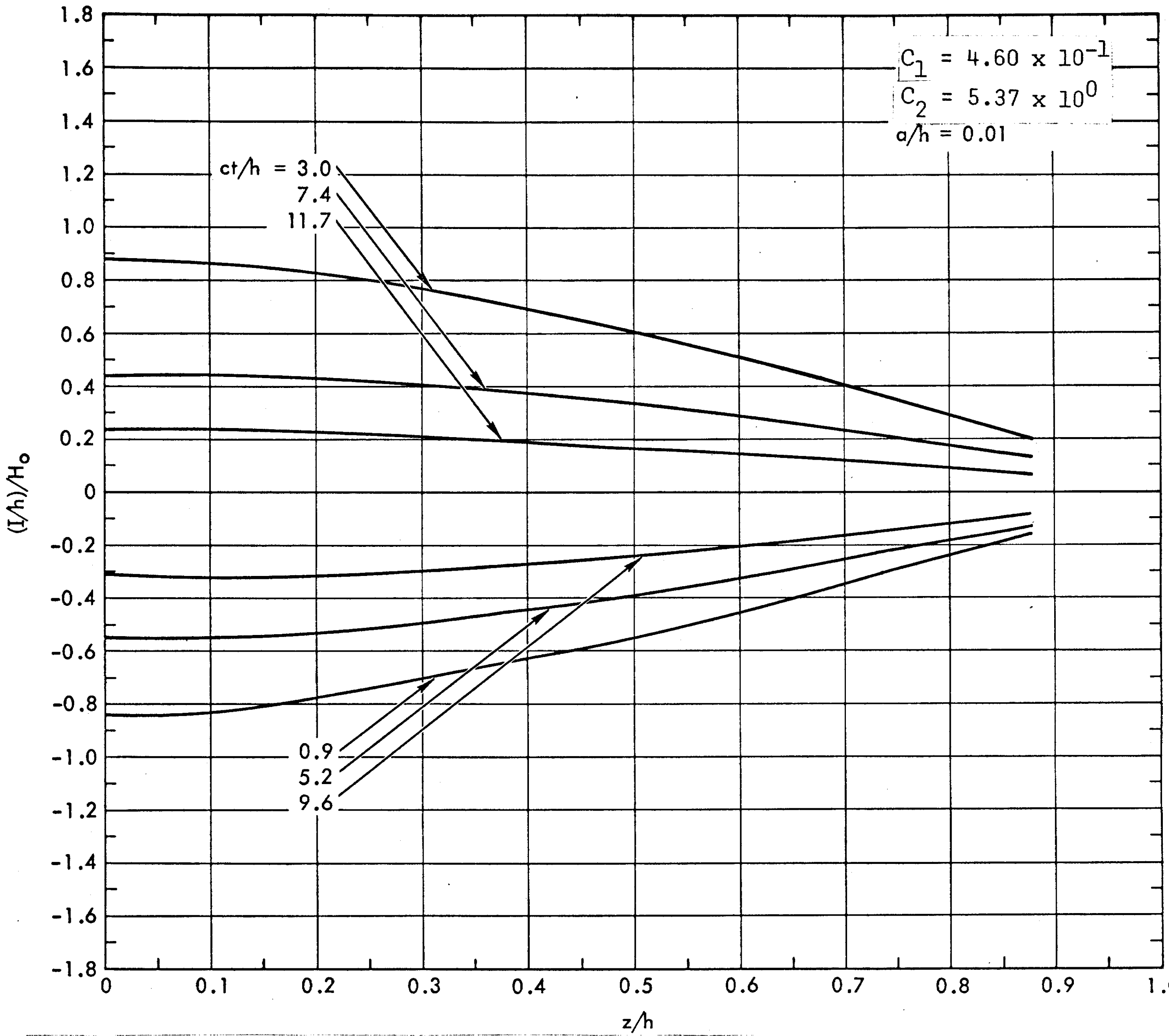

Figure 18 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1 \frac{ct}{h}} - e^{-C_2 \frac{ct}{h}}$ versus z/h

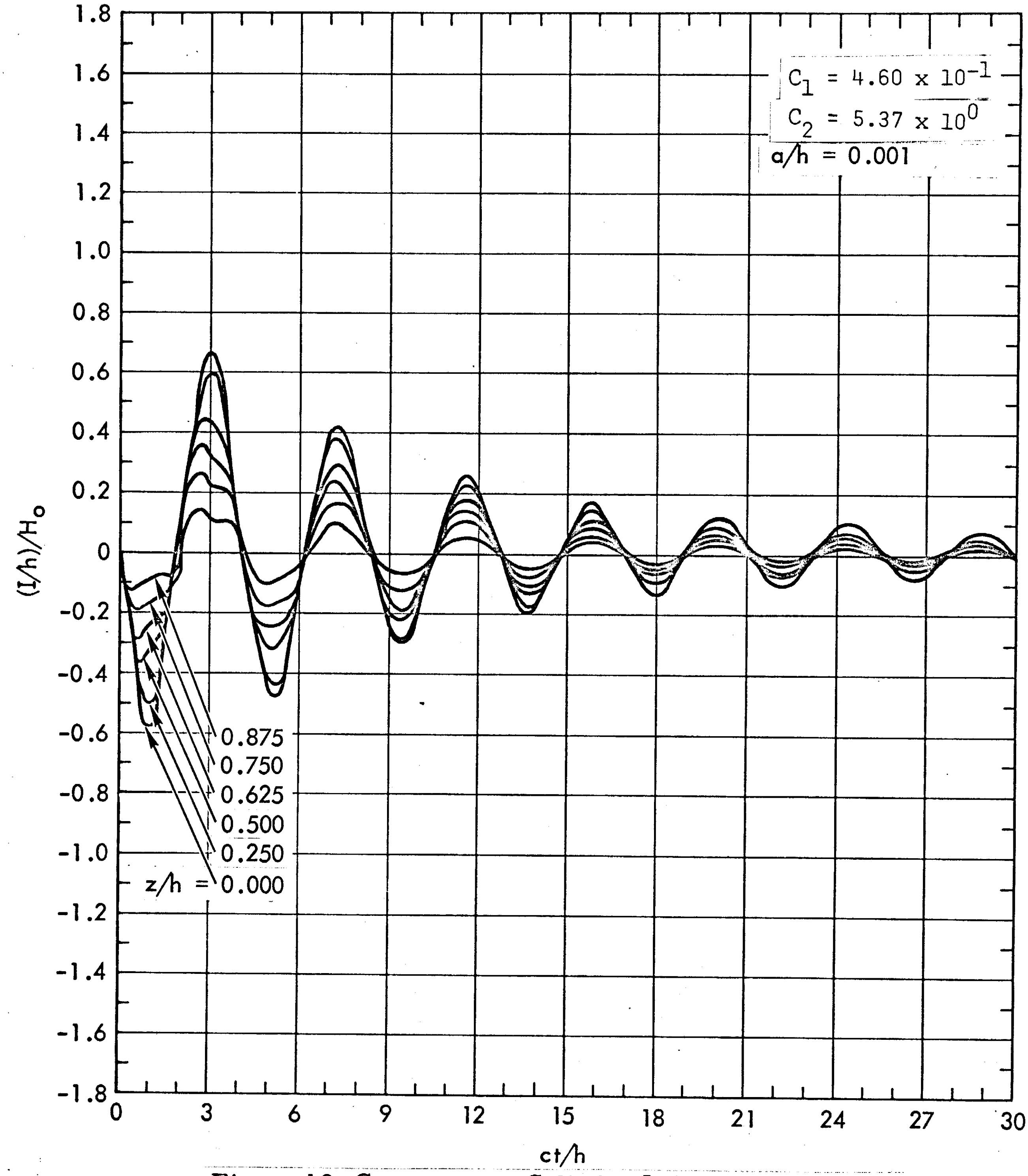

Figure 19 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}}$$ versus ct/h

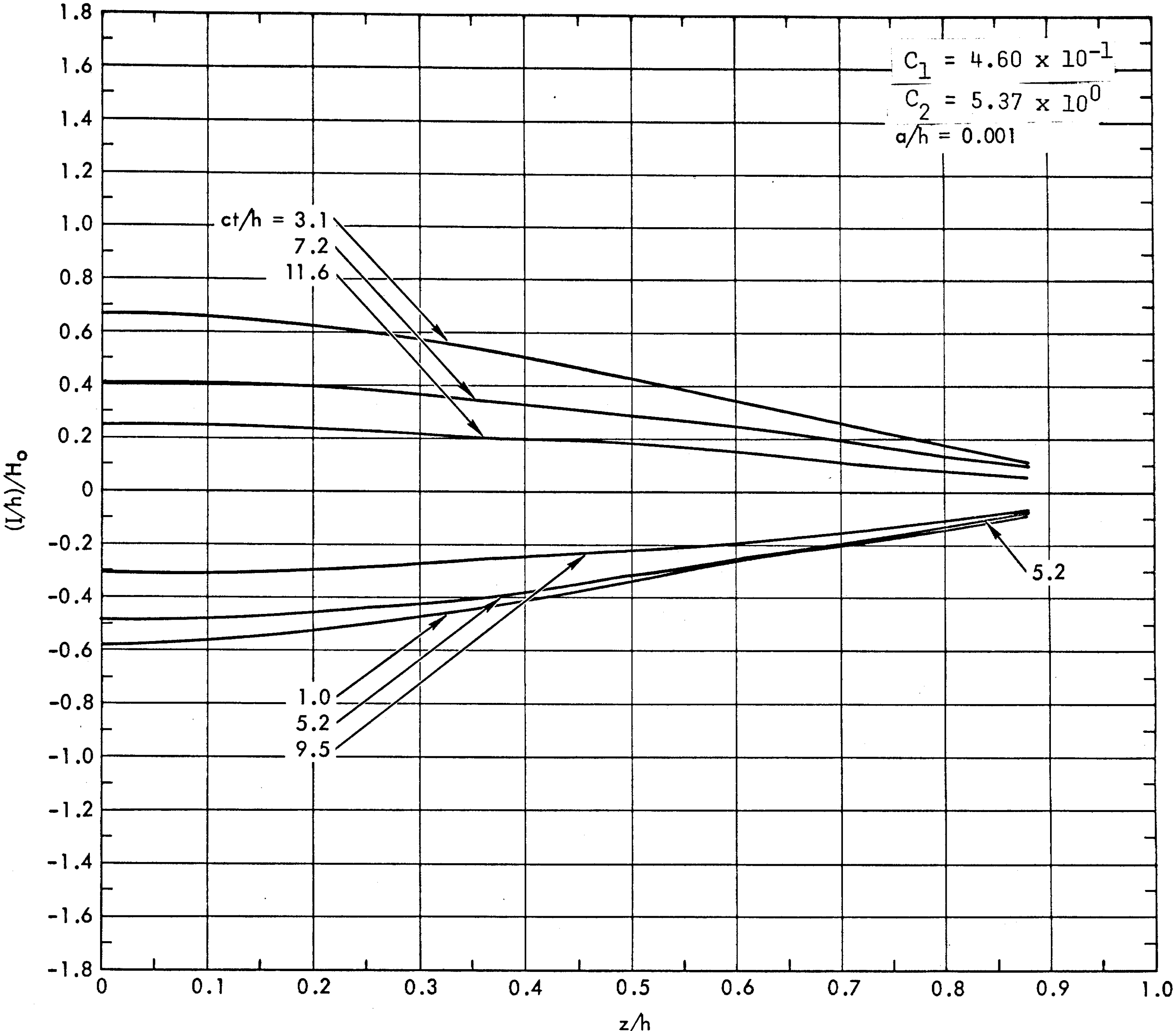

Figure 20 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1 \frac{ct}{h}} - e^{-C_2 \frac{ct}{h}}$ versus z/h

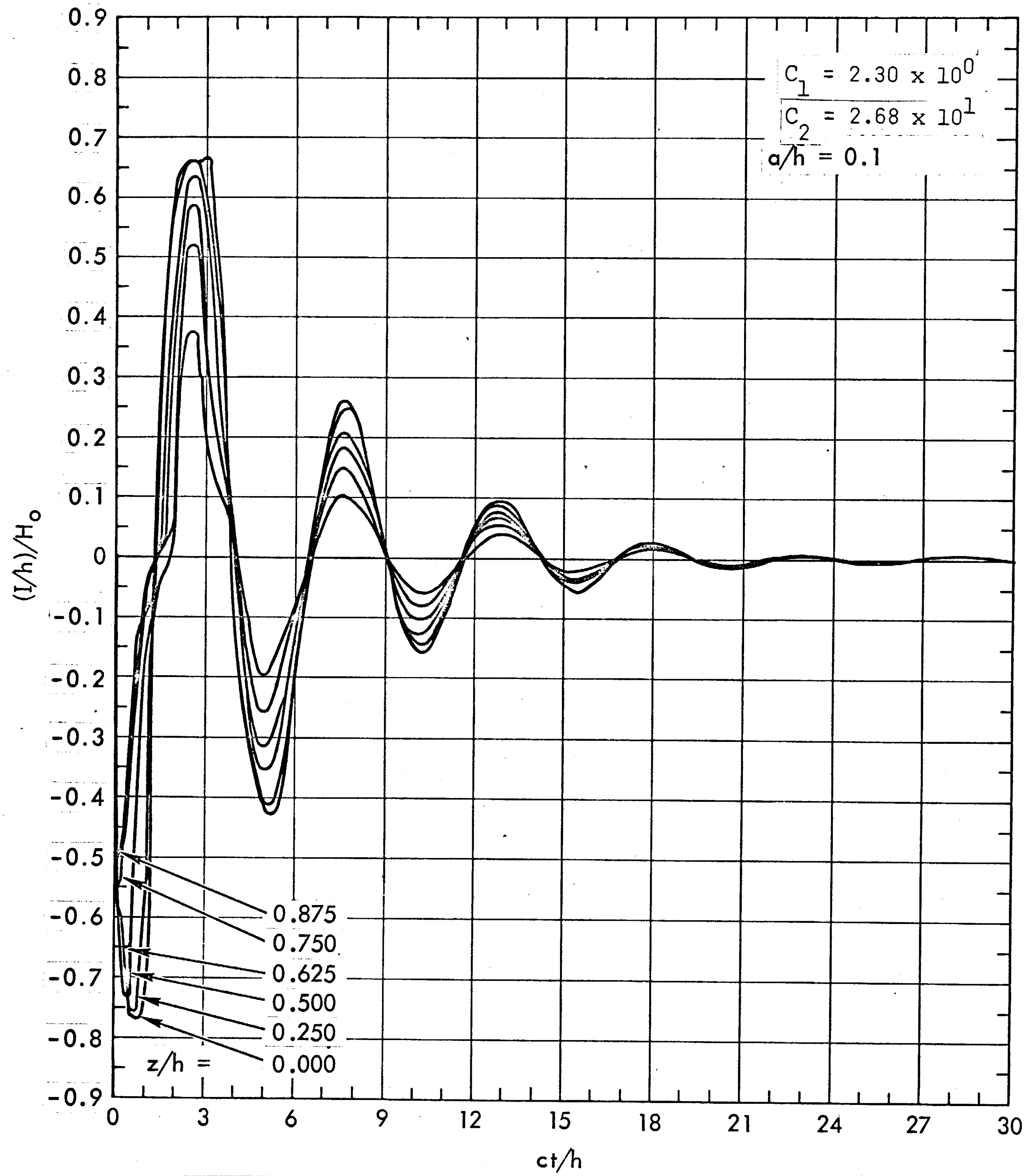

Figure 21 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1 \frac{ct}{h}} - e^{-C_2 \frac{ct}{h}} \quad \text{versus } ct/h$$

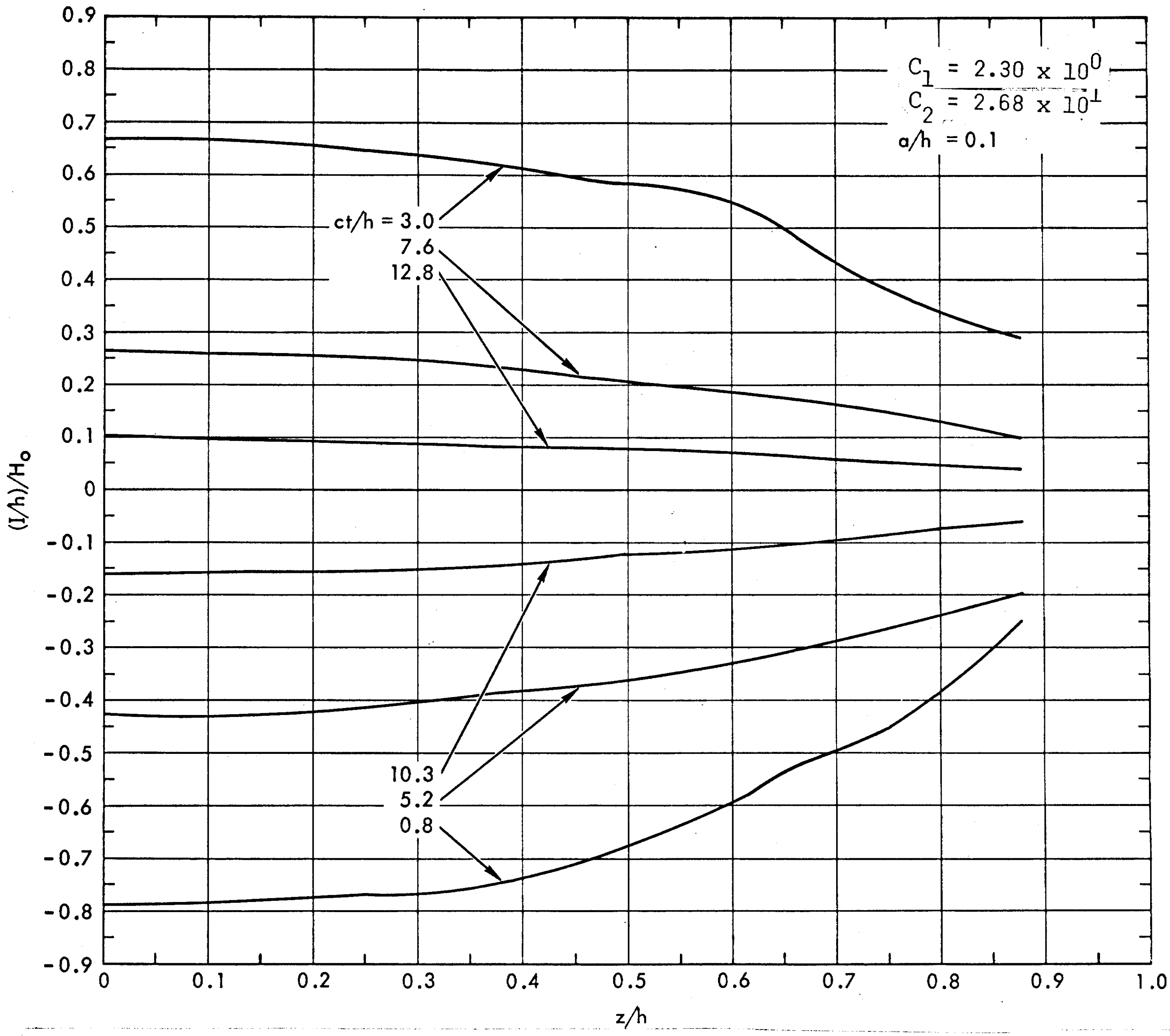

Figure 22 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}}$ versus z/h

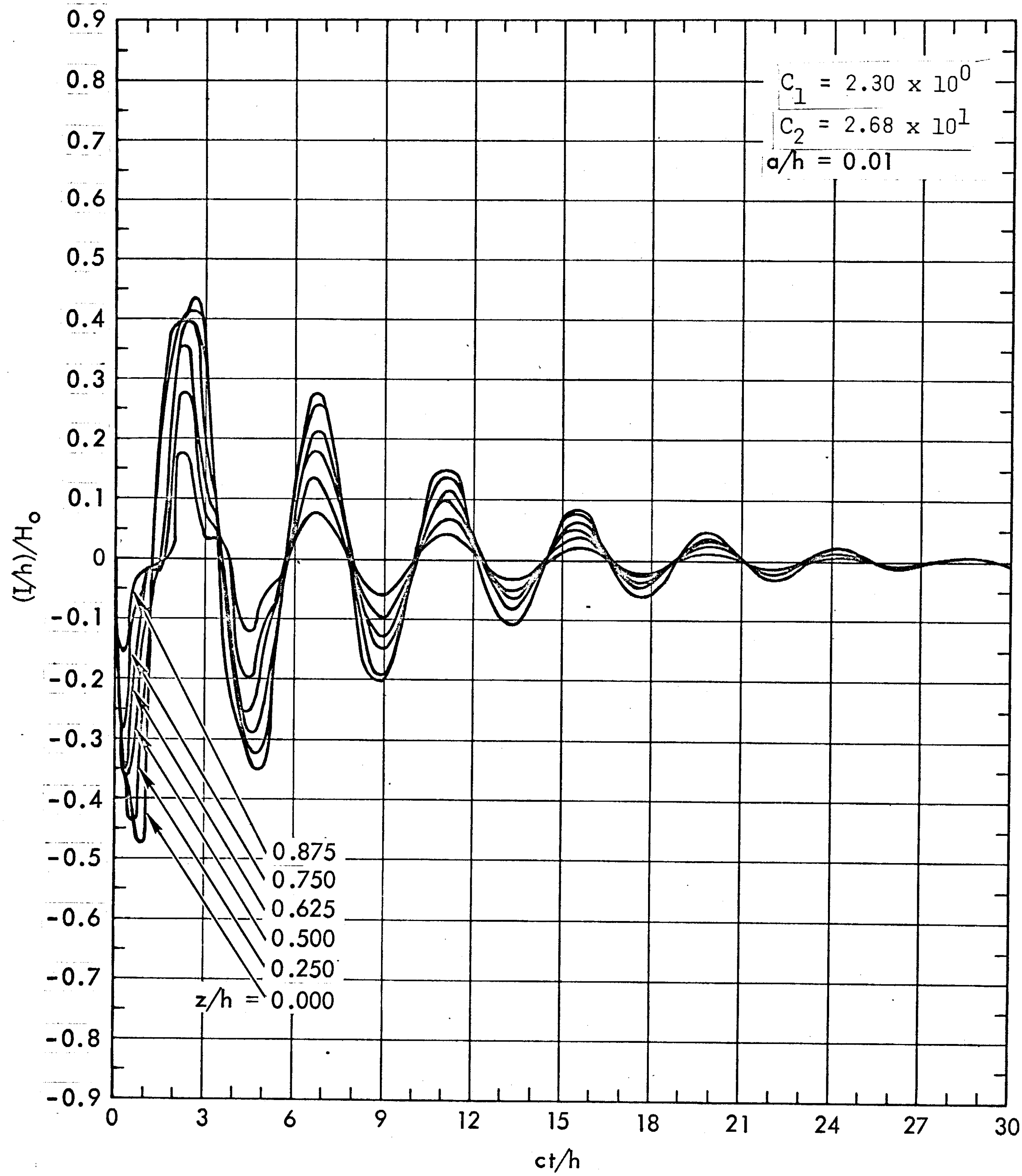

Figure 23 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}} \quad \text{versus } ct/h$$

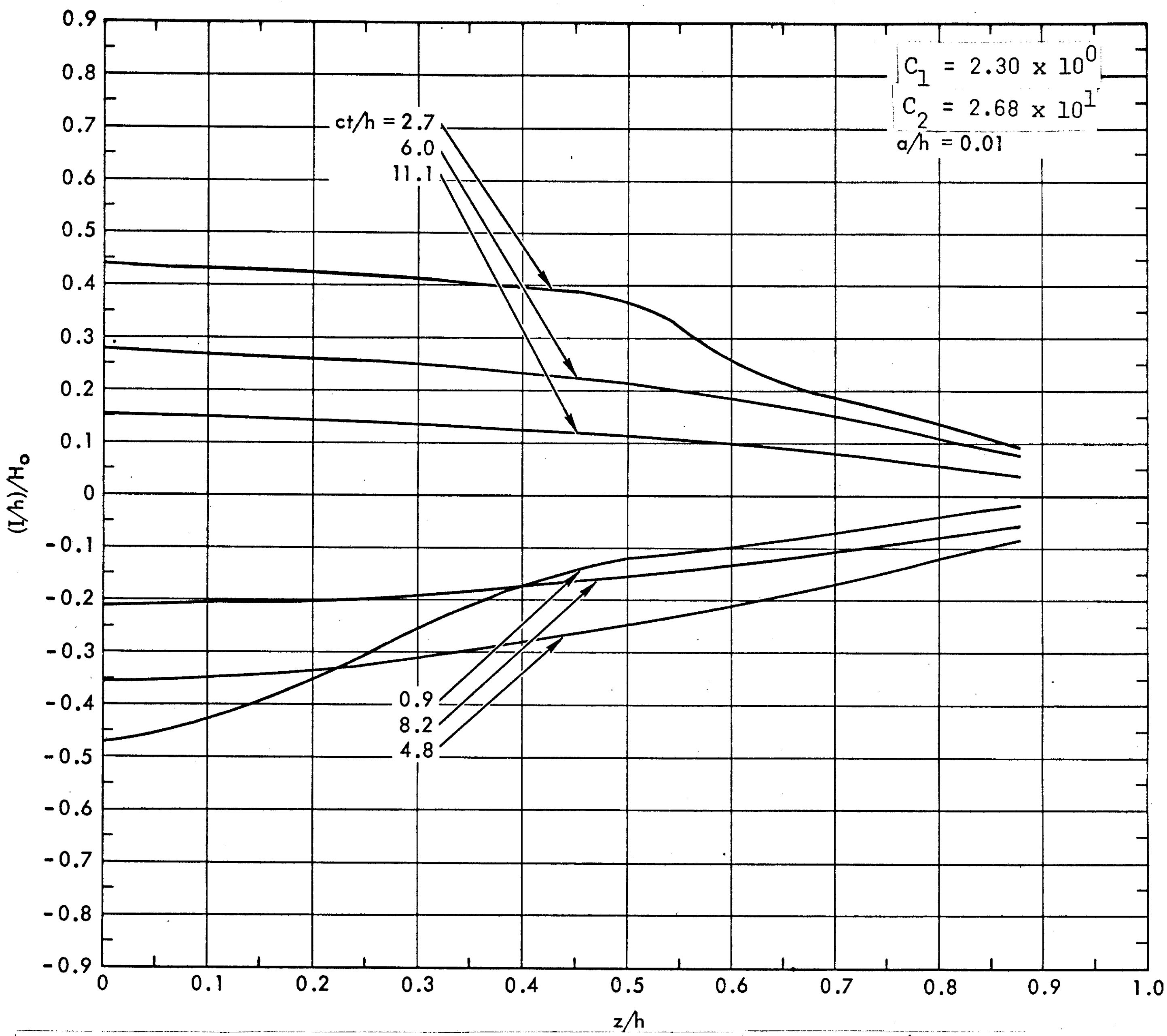

Figure 24 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1 \frac{ct}{h}} - e^{-C_2 \frac{ct}{h}}$ versus z/h

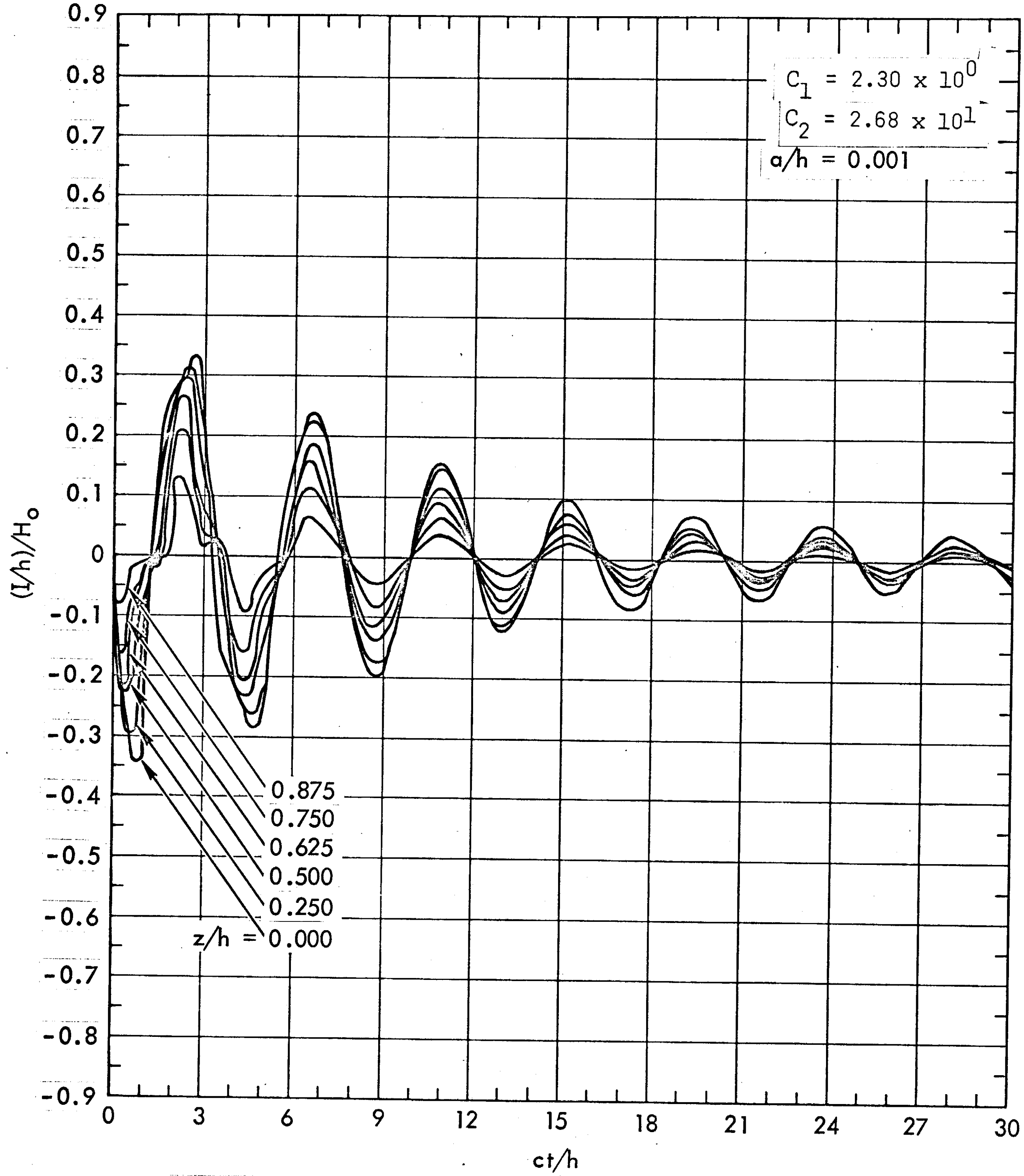

Figure 25 Current on a Cylinder Induced by the Pulse

$$\frac{H(t)}{H_o} = e^{-C_1 \frac{ct}{h}} - e^{-C_2 \frac{ct}{h}}$$ versus ct/h

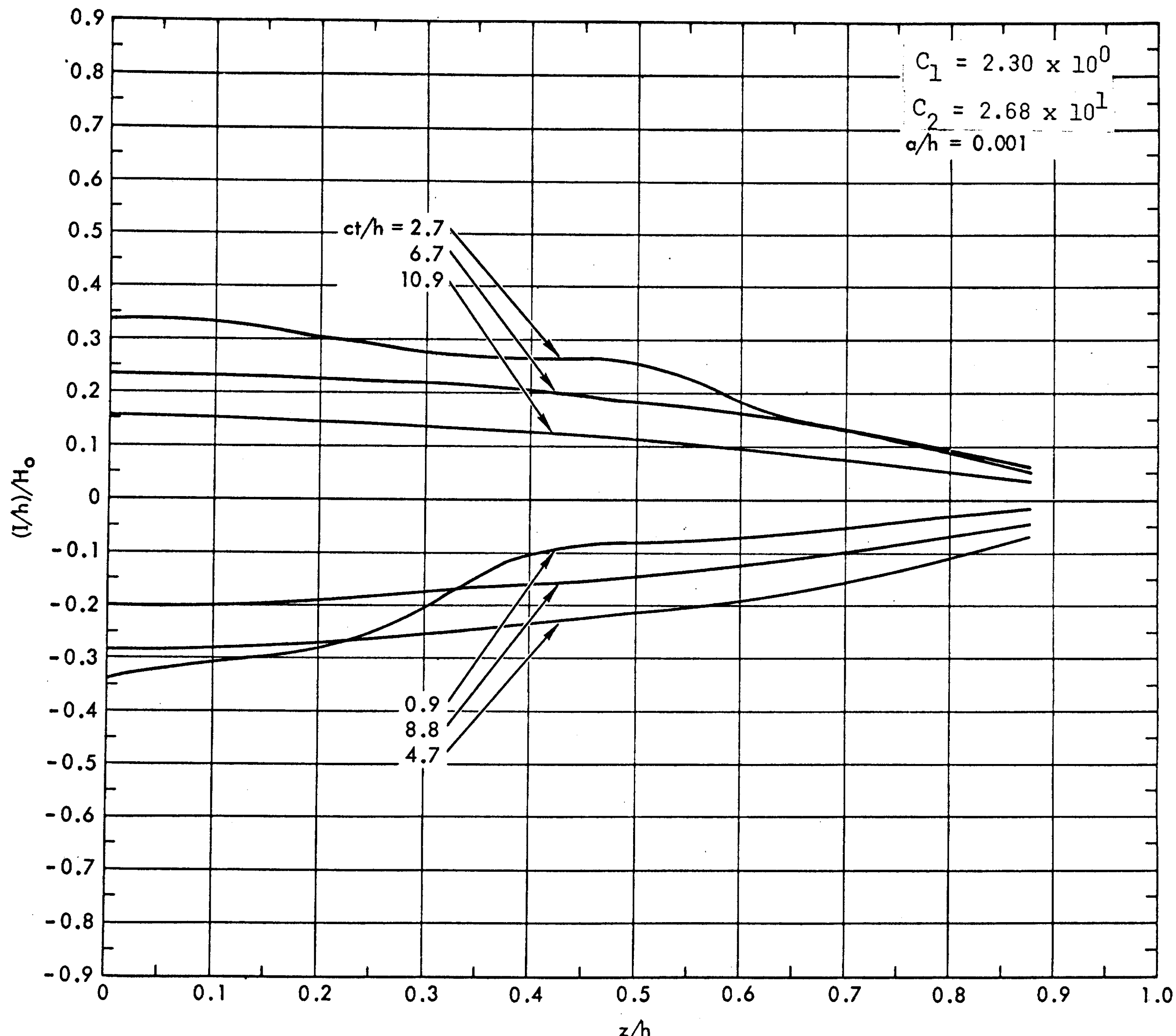

Figure 26 Current on a Cylinder Induced by the Pulse $\frac{H(t)}{H_o} = e^{-C_1\frac{ct}{h}} - e^{-C_2\frac{ct}{h}}$ versus z/h

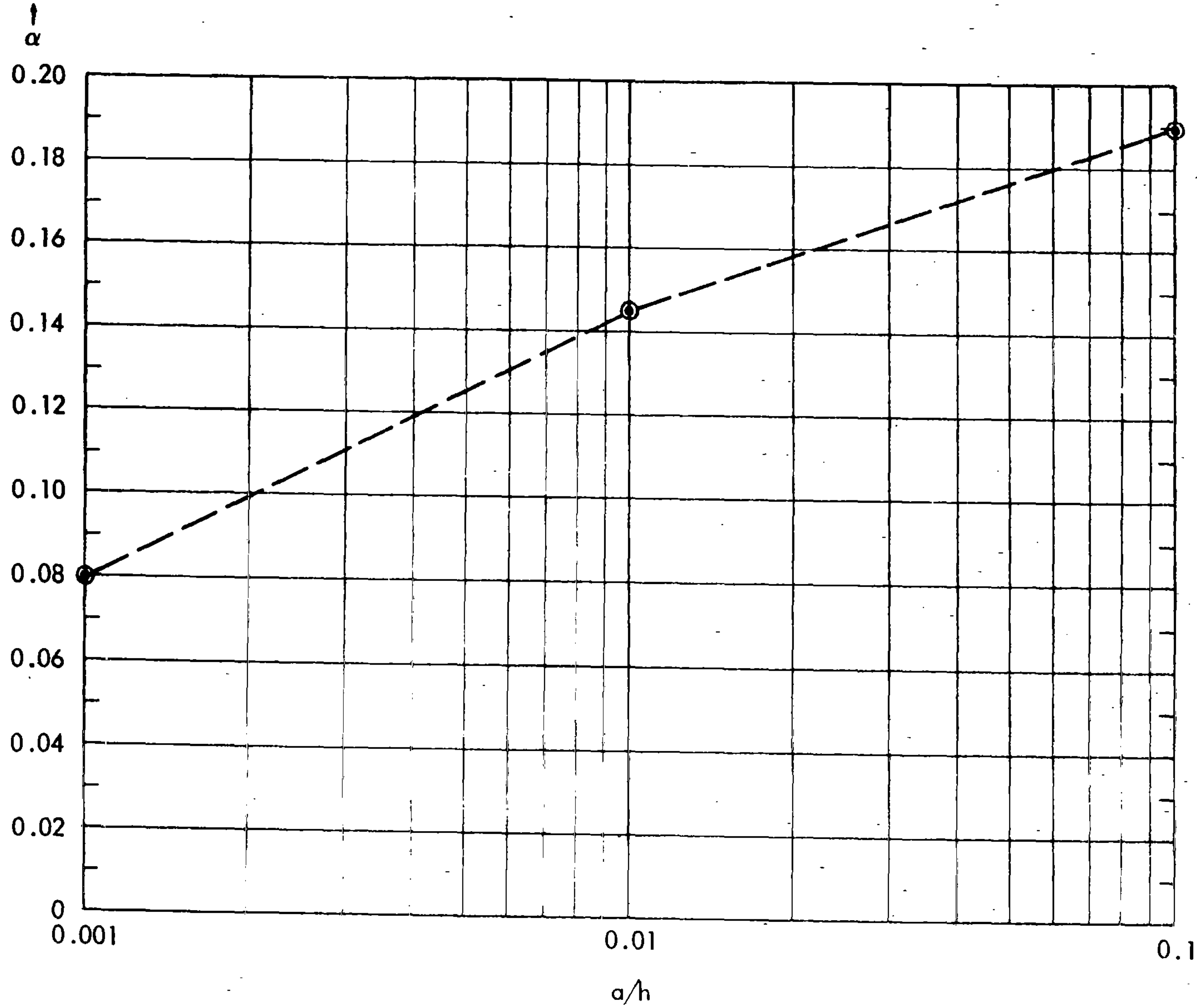

Figure 27 The Damping Constant α versus a/h