Interaction Notes

Note 44

August 1969

Finite Difference Methods for Electromagnetic Scattering Problems

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ABSTRACT

Electromagnetic pulse scattering is investigated with the use of numerical techniques. Two particular problems are considered. Investigated first is scattering from a finite-length cylinder inside a cylindrical waveguide filled with a time-varying, inhomogeneous medium. The pulse scattering from a cube in a homogeneous, isotropic medium is investigated. In both cases extensive numerical results are presented.

The numerical technique employed is the direct solution of Maxwell's equations with the use of finite difference method. The stability of the finite difference solution is discussed and stability criteria are obtained.

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CHAPTER I

INTRODUCTION

Maxwell's equations are the foundation of electromagnetic field theory. In general, every electromagnetic phenomenon is described by the solution of Maxwell's equations with appropriate boundary conditions. Since Maxwell's equations form a set of first order partial differential equations, some difficulty may be encountered in obtaining the solution. If an obstacle is present in the medium in which an electromagnetic wave is propagating or the medium is inhomogeneous or time-varying, effecting a solution of Maxwell's equations is extremely difficult.

Much work has been done recently in the development of numerical solutions to partial differential equations. The so-called finite-difference technique (1) is particularly advantageous in solving Maxwell's equations numerically. YEE (2) has used this technique in the investigation of scattering in isotropic media. In this thesis that technique is extended to inhomogeneous and time-varying media as well as scattering in three dimensions.

Basic considerations of finite difference solutions are discussed in Chapter II, where the differential equations to be solved are first introduced. The two curl equations of Maxwell's equations with some constitutive relations of the medium form the set of partial differential equations to be solved. Choosing an appropriate difference scheme, Maxwell's equations are replaced with a set of difference equations which can be solved iteratively if initial and boundary conditions are specified. The question as to whether such a solution would be a good approximation to that of the partial differential equations is answered in terms of the convergence and stability of the difference scheme. It has been shown that if proper restrictions are imposed on the grid size, it will always be possible to get a satisfactory solution (5).

In order to illustrate the technique, two specific electromagnetic scattering problems are considered. Scattering from a perfectly conducting finite-length cylinder which is embedded in a lossy medium with time-varying, inhomogeneous conductivity is presented in Chapter III. A cylindrical wave is considered to be incident upon the obstacle. The finite difference equations are used to examine the variation of the field components at time interval from $T = 0$ to $T = n\Delta t$ for an integer n and time increment Δt. The induced current on the surface of the cylinder is also obtained. In Chapter IV, a three dimensional scattering problem is considered. A perfectly conducting cube is considered to be illuminated by a plane wave pulse. This formulation is particularly significant in the study of short pulse radar returns.

CHAPTER II

GENERAL CONSIDERATIONS

Maxwell's equations are introduced in the early part of this chapter. It is shown that only the two curl equations with the constitutive relations of the medium sufficiently determine wave behavior. Proceeding to the numerical solution technique the finite difference equations are discussed briefly. Finally, conditions are described for which the difference equation approximation is satisfactory, i.e. the solutions of difference equations are bounded to that of the partial differential equations within a permissible error limit.

2.1 Maxwell's Equations

Electromagnetic wave phenomena in a particular medium are governed by Maxwell's equations(3):

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.1}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \tag{2.2}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{2.3}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \tag{2.4}$$

And for an isotropic nonferromagnetic medium ($\mu = \mu_o$) the following constitutive relations are satisfied:

$$\vec{J}(\vec{r},t) = \sigma(\vec{r},t)\vec{E}(\vec{r},t) \tag{2.5}$$

$$\vec{D}(\vec{r},t) = \varepsilon(\vec{r},t)\vec{E}(\vec{r},t) \tag{2.6}$$

$$\vec{B}(\vec{r},t) = \mu\vec{H}(\vec{r},t) \tag{2.7}$$

where σ is the conductivity, ε is the permittivity and μ is the permeability of the medium, for $\mu = \mu_o = 4\pi \times 10^{-7}$ henry/m.

Substituting (2.5), (2.6) and (2.7) into (2.1) and (2.2), a set of first order partial differential equations are obtained:

$$\mu_o \frac{\partial \vec{H}(\vec{r},t)}{\partial t} = -\nabla \times \vec{E}(\vec{r},t) \tag{2.8}$$

$$\varepsilon(\vec{r},t)\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = \nabla \times \vec{H}(\vec{r},t) - \sigma_e(\vec{r},t)\vec{E}(\vec{r},t) \tag{2.9}$$

where $\sigma_e(\vec{r},t) = \sigma(\vec{r},t) + \frac{\partial \varepsilon(\vec{r},t)}{\partial t}$

The solution of the simultaneous equations (2.8) and (2.9) will yield both $\vec{E}$ and $\vec{H}$, consequently, $\vec{D}$ and $\vec{B}$ can also be determined through (2.6) and (2.7). Thus all the field quantities are obtained. Hence the two curl equations (2.1) and (2.2) along with the constitutive relations sufficiently determine wave behavior.

2.2 Difference Equations

A difference equation is, in general, obtained by replacing the derivatives of the corresponding differential equation with their limiting definition. The unknown function is therefore not considered to be defined continuously over an infinite space but only defined at those grid points within some region. This is allowable if we are concerned with a finite time interval, i.e. with the limit $\Delta t \to 0$ for fixed t, not with the limit as $t \to \infty$ for fixed Δt.

Consider a set of first order partial differential equations with the form (4):

$$\frac{\partial u}{\partial t} = \sum_{s=1}^{n} A^s \frac{\partial u}{\partial x^s} + Bu \tag{2.10}$$

where u is a column matrix with m elements and x^s denote the n independent variables other than t. The coefficient matrix A^s and B may depend on both x^s and t. Note that if equations (2.8) and (2.9) are written in component form for a particular coordinate system and use matrix notation, a system of equations in the form of (2.10) is obtained.

The general expression for a one-level difference equation for (2.10) may be written in the form*:

$$u(x,t+\Delta t) = \sum_j C^j u(x+\Delta_j x,t) \tag{2.11}$$

where the $\Delta_j x$ are vectors with n components. The notation denotes that C^j multiplies the function u evaluated at some grid point a vector distance $\Delta_j x$ from x. Equation (2.11) expresses the value of u at the point x and at the time $t + \Delta t$ as a linear combination of values of u at time t and at points $x + \Delta_j x$.

The approximation of (2.11) to (2.10) is, in general, not unique. Various difference schemes are possible, e.g. forward difference, back-

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*The development and notation of this section is based on the work of Hahn (6), Lax (9) and Richtmyer (5), (8) .

ward difference or central difference formulas might be used [5]. The particular choice may depend upon the computing time available and the accuracy desired. In practice it usuallly works best to experiment with various schemes in order to fulfill these requirements.

2.3 Stability Consideration

The basic problem is whether the solutions of the set of difference equations (2.11) converge to the true solution of the initial-boundary value problem (2.10) as the grid is refined. For a particular problem, however, with some restrictions on the size and the type of the difference scheme, it is always possible to have a satisfactory solution. It is conventional to discuss this subject in terms of the convergence and stability of the difference scheme. Definitions are given as follows [6]:

CONVERGENCE: A difference scheme is called convergent if solution of the difference equation tends to that of the differential equation as Δt tends to zero.

STABILITY: A difference scheme is called stable if solutions of the difference equations are uniformly bounded functions of the initial data for all sufficiently small Δt and all $n\Delta t$ less than some finite value T. Thus, if the above criteria are satisfied, we then conclude that the solution of (2.11) tends to that of (2.10). Richtmyer's text [5] treats this subject in great detail. He has pointed out that Von Neumann's condition is a necessary and sufficient condition for convergence and stability provided that the coefficients of the partial differential equations are constants.

An amplification matrix $[M]$ for the difference equations is defined as:

$$[u(\vec{x}, \Delta t)] = [M] [u(\vec{x}, 0)]$$
$$[u(\vec{x}, 2\Delta t)] = [M] [u(\vec{x}, \Delta t)]$$
$$\vdots \qquad \qquad \vdots$$
$$[u(\vec{x}, n\Delta t)] = [M] [u(\vec{x}, (n-1)\Delta t)]$$

Thus, at $T = n\Delta t$, we have;

$$[u(\vec{x}, T)] = [M]^n [u(\vec{x}, 0)] \qquad (2.12)$$

where $(u(\vec{x}, 0))$ are the initial conditions.

$(u(\vec{x}, \Delta t))$ are the solutions of the difference equations at time Δt.

(M) is called the amplification matrix and can be calculated through the set of difference equations.

VON NEUMANN'S CONDITION:

Let $(M(\xi))$ be the amplification matrix of the difference scheme, and λ_i are the eigenvalues of $(M(\xi))$. Then, Von Neumann's condition for convergence requires that:

$|\lambda_i| \leq 1$ for all real ξ

The expression (2.12) shows that the solutions of the difference equations at time $T = n\Delta t$ are related to its initial values $(u(\vec{x},0))$ through the n^{th} power of the amplification matrix (M). Since the n^{th} power of a quantity greater than one grows without bound as n increases, Von Neumann's condition is clearly necessary for convergence.

Many authors have proved that convergence implies stability and vice versa. If Von Neumann's condition is used to verify that some proposed difference scheme converges, the eignevalues of the amplification matrix $[M(\xi)]$ must be checked to see that they do not exceed one in absolute value for all real value of ξ.

The discussion so far is restricted to those partial differential equations with constant coefficients, or ones for which the amplification matrix $[M]$ has only constant elements. When the coefficients of the partial differential equation are variables, no general theory has been developed to guarantee the convergence and stability of a proposed difference scheme.

A sufficient condition applicable to the case of variable coefficients has been given by Friedrichs [7], described by Lax and Richtmyer [8] and lately extended by Lax [9]. Friedrichs' condition states that if the matrices C^j satisfy

$$\sum_j C^j = I, \quad \text{where I is the identity matrix}$$

are symmetric and are Lipschitz continuous*, then the difference scheme is stable provided the C^j are nonnegative. Lax [10] has also predicted that if the condition

$$\left|\sum_j C^j(\vec{x},\xi)\right| \leq 1$$

is violated for any real $\vec{x}$ and ξ, then instability occurs. This would

*A matrix $[M(x)]$ is Lipschitz continuous at x_o if for a given $\delta>0$ there exists a constant K such that $||[M(x)] - [M(x_o)]|| < K|x-x_o|$ for $|x-x_o| < \delta$.

require, as pointed out by Richtmyer (5), the amplification matrix $(M(\vec{x},\xi))$ to be Lipschitz continuous and all the eigenvalues $\lambda(\vec{x},\xi)$ of $(M(\vec{x},\xi))$ to satisfy

$$|\lambda(\vec{x},\xi)| \leqslant 1$$

for all $\vec{x}$ and all real $\xi \neq 0 \bmod 2\pi$. When instability occurs in practice, it often first appears as a local disturbance in a region where the Von Neumann's condition is violated, hence Richtmyer suggests that it is necessary for stability that Von Neumann's condition be satisfied at every point and at every time t.

In a subsequent chapter an amplification matrix (M) with variable elements is obtained. The Von Neumann's condition is then used as a local condition to verify the stability of the difference scheme.

CHAPTER III

ELECTROMAGNETIC PULSE SCATTERING IN A TIME-VARYING, INHOMOGENEOUS MEDIA*

The electromagnetic scattering from a finite cylinder has been of great interest in recent years. Study of this problem requires that Maxwell's equations be solved with some appropriate boundary conditions. Unfortunately, the surface boundaries of a finite cylinder configuration do not coincide with the coordinate surfaces of a coordinate system in which the wave equation is separable. Hence it is very difficult to obtain a solution in closed or open form. Moreover, if the properties of the medium are inhomogeneous and time-varying, the analytic solution is impractical. Becuase of this and recent advances in high-speed digital computer capabilities, the finite-difference technique is used to solve the problem numerically.

3.1 Physical Geometry

Consider that a perfectly conducting, finite cylinder is embedded in a lossy medium where time-varying and spatially varying conductivity $\sigma(r,z,t)$ is assumed.Also the permittivity is considered to be a function of position and time, i.e. $\varepsilon(r,z,t) = \varepsilon_0 K(r,z,t)$, where K is the relative permittivity. Here the coordinates r and z are the usual

*Portions of this chapter are to be published in the IEEE Transactions on Antennas and Propagation., Sept. 1969 (11).

cylindrical coordinates. For simplicity of calculation, a very large cylindrical waveguide which containing the cylinder is introduced to approximate an infinite expanse of the medium. The geometry chosen for analysis is presented in Fig. 1.

3.2 Differential equations, Difference equations

It is convenient to use cylindrical coordinates where azimuthal symmetry is obtained. Since the field components are independent of the azimuth angle the magnetic field has only one component, H_ϕ. Expressing (2.8) and (2.9) in component form, the following set of first order partial differential equations is obtained:

$$
\begin{aligned}
\mu_o \frac{\partial H_\phi(r,z,t)}{\partial t} &= \frac{\partial E_z(r,z,t)}{\partial r} - \frac{\partial E_r(r,z,t)}{\partial z} \\
\varepsilon_o K(r,z,t)\frac{\partial E_r(r,z,t)}{\partial t} &= -\frac{\partial H_\phi(r,z,t)}{\partial z} - \sigma_e(r,z,t) \\
\varepsilon_o K(r,z,t)\frac{\partial E_z(r,z,t)}{\partial t} &= \frac{\partial H_\phi(r,z,t)}{\partial r} + \frac{H_\phi(r,z,t)}{r} - \sigma_e(r,z,t)
\end{aligned}
\tag{3.1}
$$

A one-level finite difference scheme as shown in Fig. 2 was chosen. The half-interval points have been used to reduce the truncation error. The set of first order partial differential equations (3.1) may be replaced by the set of finite difference equations:

$$
\mu_o \frac{H_\phi^{n+1}(I+\frac{1}{2},J+\frac{1}{2}) - H_\phi^{n}(I+\frac{1}{2},J+\frac{1}{2})}{\Delta t} = \frac{E_z^{n+\frac{1}{2}}(I+\frac{1}{2},J+1) - E_z^{n+\frac{1}{2}}(I+\frac{1}{2},J)}{\Delta r} - \frac{E_r^{n+\frac{1}{2}}(I+1,J+\frac{1}{2}) - E_r^{n+\frac{1}{2}}(I,J+\frac{1}{2})}{\Delta z}
$$

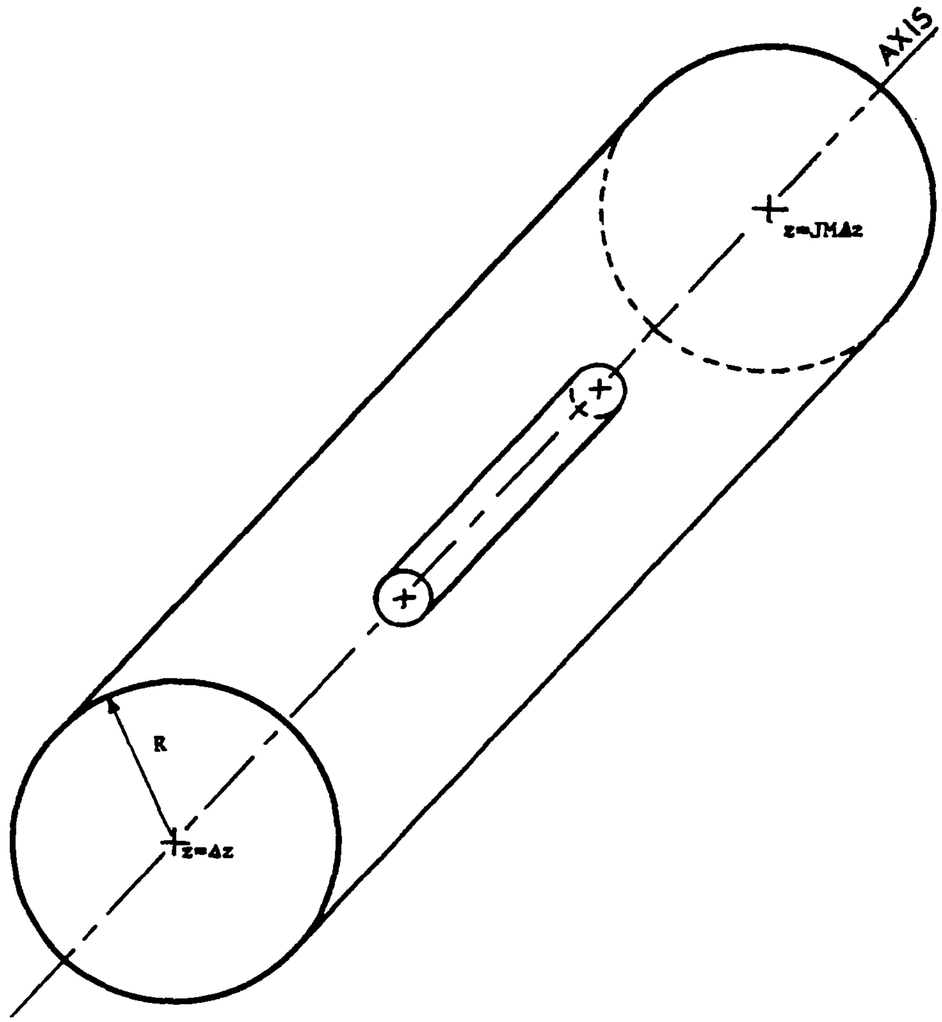

Figure 1: Cylindrical waveguide containing a solid cylindrical scatterer on axis of the waveguide

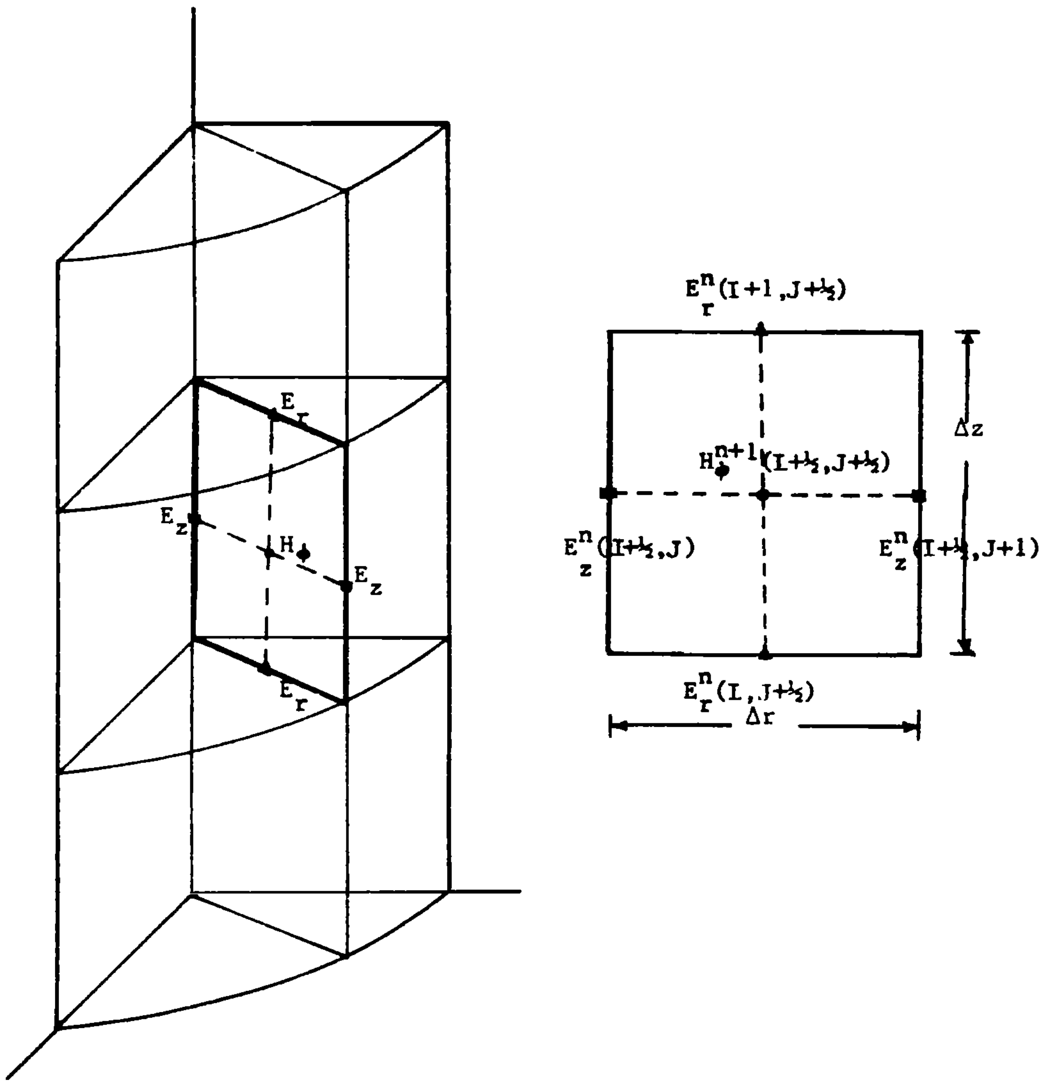

Figure 2: Finite-Difference scheme for cylindrical configuration.

$$\varepsilon_o \frac{E_r^{n+3/2}(I+1,J+\frac{1}{2})-E_r^{n+\frac{1}{2}}(I+1,J+\frac{1}{2})}{\frac{1}{K^{n+1}(I+1,J+\frac{1}{2})}\Delta t} = -\frac{H_\phi^{n+1}(I+\frac{3}{2},J+\frac{1}{2})-H_\phi^{n+1}(I+\frac{1}{2},J+\frac{1}{2})}{\Delta z} - \sigma_e^{n+1}(I+1,J+\frac{1}{2})E_r^{n+1}(I+1,J+\frac{1}{2}) \qquad (3.2)$$

$$\varepsilon_o \frac{E_z^{n+3/2}(I+\frac{1}{2},J+1)-E_z^{n+\frac{1}{2}}(I+\frac{1}{2},J+1)}{\frac{1}{K^{n+1}(I+\frac{1}{2},J+1)}\Delta t} = \frac{H_\phi^{n+1}(I+\frac{1}{2},J+\frac{3}{2})-H_\phi^{n+1}(I+\frac{1}{2},J+\frac{1}{2})}{\Delta r} + \frac{H_\phi^{n+1}(I+\frac{1}{2},J+1)}{(J+1)\Delta r} - \sigma_e^{n+1}(I+\frac{1}{2},J+1)E_z^{n+1}(I+\frac{1}{2},J+1)$$

where the following notation is used:

$$H_\phi^{n+1}(I,J) = H_\phi(I\Delta z, J\Delta r, (n+1)\Delta t)$$

$$H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2}) = H_\phi((I+\tfrac{1}{2})\Delta z, (J+\tfrac{1}{2})\Delta r, (n+1)\Delta t)$$

and so forth for other field components.

Let $\Delta\tau = c\Delta t$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

$$Z = \sqrt{\frac{\mu_o}{\varepsilon_o}} \simeq 120\pi \text{ ohms}$$

then (3.2) can be rearranged:

$$H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2}) = H_\phi^{n}(I+\tfrac{1}{2},J+\tfrac{1}{2}) + \frac{\Delta\tau}{Z\Delta r}\left[E_z^{n+\frac{1}{2}}(I+\tfrac{1}{2},J+1)-E_z^{n+\frac{1}{2}}(I+\tfrac{1}{2},J)\right] - \frac{\Delta\tau}{Z\Delta z}\left[E_r^{n+\frac{1}{2}}(I+1,J+\tfrac{1}{2})-E_r^{n+\frac{1}{2}}(I,J+\tfrac{1}{2})\right] \tag{3.3a}$$

$$E_r^{n+3/2}(I+1,J+\tfrac{1}{2}) = m_a E_r^{n+\frac{1}{2}}(I+1,J+\tfrac{1}{2}) - \frac{Z\Delta\tau}{\Delta z} m_b\left[H_\phi^{n+1}(I+3/2,J+\tfrac{1}{2}) - H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2})\right] \tag{3.3b}$$

$$E_z^{n+3/2}(I+\tfrac{1}{2},J+1) = m_c E_z^{n+\frac{1}{2}}(I+\tfrac{1}{2},J+1) + \frac{Z\Delta\tau}{\Delta r} m_d\left[H_\phi^{n+1}(I+\tfrac{1}{2},J+3/2) - H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2})\right] + dZH_\phi^{n+1}(I+\tfrac{1}{2},J+1) \tag{3.3c}$$

where

$$m_a = \frac{1 - m_1}{1 + m_1}$$

$$m_b = \frac{1}{K^{n+1}(I+1,J+\tfrac{1}{2})(1+m_1)}$$

$$m_c = \frac{1 - m_2}{1 + m_2}$$

$$m_d = \frac{1}{K^{n+1}(I+\tfrac{1}{2},J+1)(1+m_2)}$$

$$m_1 = \frac{\Delta t\sigma_e^{n+1}(I+1,J+\tfrac{1}{2})}{2\varepsilon_o K^{n+1}(I+1,J+\tfrac{1}{2})}$$

$$m_2 = \frac{\Delta t\sigma_e^{n+1}(I+\tfrac{1}{2},J+1)}{2\varepsilon_o K^{n+1}(I+\tfrac{1}{2},J+1)}$$

$$d = \frac{\Delta\tau}{\Delta r}\ \frac{1}{(J+1)K^{n+1}(I+\tfrac{1}{2},J+1)(1+m_2)}$$

and the following linear interpolation is used:

$$E_r^{n+1}(I+1,J+\tfrac{1}{2}) = \tfrac{1}{2}\left[E_r^{n+3/2}(I+1,J+\tfrac{1}{2}) + E_r^{n+\frac{1}{2}}(I+1,J+\tfrac{1}{2})\right]$$

$$E_z^{n+1}(I+\tfrac{1}{2},J+1) = \tfrac{1}{2}\left[E_z^{n+3/2}(I+\tfrac{1}{2},J+1) + E_z^{n+\frac{1}{2}}(I+\tfrac{1}{2},J+1)\right]$$

3.3 Amplification matrix and Stability Criteria

As discussed in the previous chapter, it is possible to obtain the amplification matrix $[M]$ and then use Von Neumann's condition as a local condition for stability. Let the initial fields be, in general, the form of an exponential function, i.e.

$$E_r^n(I,J) = E_r^n e^{jk_1 J\Delta r} e^{jk_2 I\Delta z}$$

$$E_r^{n+\frac{1}{2}}(I,J+\tfrac{1}{2}) = E_r^{n+\frac{1}{2}}(I,J)e^{jk_1\Delta r/2}$$

where k_1 & k_2 are components of the propagation constant, and $j = \sqrt{-1}$. The other notation is consistent with the foregoing.

With the foregoing expressions, the set of difference equations (3.3) can be written as

$$\begin{aligned}
H_\phi^{n+1}(I,J) &= H_\phi^n(I,J) + \frac{\Delta\tau}{Z\Delta r}\left(e^{jk_1\Delta r/2} - e^{-jk_1\Delta r/2}\right) E_z^{n+\frac{1}{2}}(I,J) \\
&\quad - \frac{\Delta\tau}{Z\Delta z}\left(e^{jk_2\Delta z/2} - e^{-jk_2\Delta z/2}\right) E_r^{n+\frac{1}{2}}(I,J) \\
E_r^{n+3/2}(I,J) &= m_a E_r^{n+\frac{1}{2}}(I,J) - m_b Z\frac{\Delta\tau}{\Delta z}\left(e^{jk_2\Delta z/2} - e^{-jk_2\Delta z/2}\right)H_\phi^{n+1}(I,J) \\
E_z^{n+3/2}(I,J) &= m_c E_z^{n+\frac{1}{2}}(I,J) + m_d Z\frac{\Delta\tau}{\Delta r}\left(e^{jk_1\Delta r/2} - e^{-jk_1\Delta r/2}\right)H_\phi^{n+1}(I,J) \\
&\quad + dZH_\phi^{n+1}(I,J)
\end{aligned} \qquad (3.4)$$

Let

$$a = 2\frac{\Delta\tau}{\Delta r}\sin(k_1\Delta r/2)$$

$$b = 2\frac{\Delta\tau}{\Delta z}\sin(k_2\Delta z/2)$$

The set of equations (3.4) can be written in a matrix form:

$$\begin{bmatrix} ZH_\phi^{n+1}(I,J) \\ E_r^{n+3/2}(I,J) \\ E_z^{n+3/2}(I,J) \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} ZH_\phi^{n}(I,J) \\ E_r^{n+1/2}(I,J) \\ E_z^{n+1/2}(I,J) \end{bmatrix}$$

where $[M]$ is the amplification matrix and is given by:

$$[M] = \begin{bmatrix} 1 & -jb & ja \\ -jm_b b & m_a - m_b b^2 & m_b ab \\ jm_d a + d & b(m_d a - jd) & m_c - m_d a^2 + jad \end{bmatrix} \qquad (3.5)$$

Note that $j = \sqrt{-1}$ is understood in (3.5).

Let λ_ℓ be the ℓth eigenvalue of the matrix $[M]$ given by (3.5), according to matrix theory, we have

$$\text{trace } [M] = \sum_{\ell=1}^{3} \lambda_\ell$$

thus

$$\sum_{\ell=1}^{3} \lambda_\ell = 1 + (m_a - m_b b^2) + (m_c - m_d a^2 + jad)$$

Since Von Neumann's condition requires that:

$$|\lambda_\ell| \leq 1 \qquad \text{for } \ell = 1,2,3$$

But since $\left|\sum_{\ell=1}^{3}\lambda_\ell\right| \leq \sum_{\ell=1}^{3}|\lambda_\ell|$

therefore $\left|1+(m_a - m_b b^2)+(m_c - m_d a^2 + jad)\right| \leq 3$ (3.6)

This is the condition which must be satisfied at every time step. For the time-varying, inhomogeneous medium, the grid size must be chosen such that (3.6) is always satisfied. However it should be pointed out that (3.6) is only a necessary condition, not a sufficient condition for stability and convergence.

3.4 Boundary Conditions and Initial Conditions

BOUNDARY CONDITIONS

The boundary conditions imposed on the perfectly conducting scattering surface are that tangential components of the electric field vanish. This boundary condition also implies that the normal component of the magnetic field is zero on the surface [2]. In subsequent numerical claculations, these are easily enforced in the programming by putting the corresponding field points on the surface of the scatterer equal to zero.

INITIAL CONDITIONS

The initial conditions are considered as those of a pulse propagating in a cylindrical waveguide which initially contains a lossless medium with constitutive parameters ε_o, μ_o. However, the difference equations (3.2) may treat more general constitutive properties for $t>0$. Since the field components of the TM_{01} mode have azimuthal symmetry, it is convenient to consider the initial electromagnetic pulse to be formed by this mode. The field components of the TM_{01} mode are:

$$\hat{E}_z(r,z,\omega) = \hat{E}_o(\omega)J_o(K_c r)e^{-j\beta z}$$

$$\hat{E}_r(r,z,\omega) = j\beta\hat{E}_o(\omega)J_1(K_c r)e^{-j\beta z}/K_c \tag{3.7}$$

$$\hat{H}_\phi(r,z,\omega) = jk\hat{E}_o(\omega)J_1(K_c r)e^{-j\beta z}/ZK_c$$

where $\hat{E}_o(\omega)$ is the complex amplitude of the mode at radian frequency ω.

$$\beta = \sqrt{k^2 - K_c^2}$$

$$k^2 = \omega^2\mu_o\varepsilon_o$$

$$Z = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$K_c = \frac{2.405}{R}$$, R is the radius of the waveguide.

Taking the Fourier transform of (3.1) with respect to time t for $\sigma(r,z,t)=0$ and $\varepsilon(r,z,t)=\varepsilon_o$, it may be shown that (3.7) is exactly a set of solutions of (3.1) in transform space.

The time histories of the field components forming the pulse are expressed as Fourier superpositions of the respective components given by (3.7). It is readily shown that:

$$E_r(r,z,t) = -\frac{J_1(K_c r)}{J_o(K_c r)}\frac{\partial}{\partial z}E_z(r,z,t)$$

$$H_\phi(r,z,t) = \frac{J_1(K_c r)}{ZK_c cJ_o(K_c r)}\frac{\partial}{\partial t}E_z(r,z,t) \tag{3.8}$$

where

$$E_z(r,z,t) = \frac{J_o(K_c r)}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{E}_o(\omega) e^{-j(\beta z-\omega t)} d\omega$$

It is convenient to choose R sufficiently large that h ≈ k for most of the frequency content of the pulse. The (3.8) yields

$$E_z(r,z,t) = J_o(K_c r)F(z\pm ct) \tag{3.9}$$

where $F(z\pm ct)$ is determined by the choice of $\hat{E}_o(\omega)$ or vice versa. The initial field components must be continuous functions in order for the difference formulas to apply, i.e. $F(z\pm ct)$ and $F'(z\pm ct)$ must be continuous for all $(z\pm ct)$ and the Fourier transform of $F(z\pm ct)$ must have negligible frequency content for $\omega \leq K_c c$. Thus, a suitable choice for $F(z\pm ct)$ is:

$$F(z\pm ct) = \begin{cases} \sin^2 A(z\pm ct) & 0 \leq A(z\pm ct) \leq \pi \\ 0 & \text{elsewhere} \end{cases} \tag{3.10}$$

3.5 Method of Computation

The computation of (3.3) is relatively simple in this case. For completeness, the procedure of calculation is described explicitly. The calculation process is illustrated in Figure 3. Those field components on the boundaries must be determined either by boundary conditions or by extrapolation. Since $E_z^{n+\frac{1}{2}}(I+\frac{1}{2},JM)$ is the tangential component of the electric field on the lateral boundary of the waveguide, it is since set equal to zero at these grid points. The other three boundaries include both ends and the axis of the waveguide. For these boundaries it is more convenient to use linear extrapolation. In this manner, all the grid points on the boundaries of the regions S_o; S_1;...S_n can be obtained for every time step Δt up to $T = n\Delta t$.

The initial fields for $H_\phi^o(I+\frac{1}{2},J+\frac{1}{2})$; $E_r^{\frac{1}{2}}(I+1,J+\frac{1}{2})$; and $E_z^{\frac{1}{2}}(I+\frac{1}{2},J+1)$ are given at the corresponding grid points on region S_o for $0 \leq z \leq Z$ where $z = (I-1)\ \Delta z$; $I = 1,2,\ldots$ IN with $(IN-1)\ \Delta z = Z$ and for $0 \leq r \leq R$ where $r = (J-1)\ \Delta r$; $J = 1,2,\ldots JM$ with $(JM-1)\Delta r = R$.

To compute the next time step Δt or $n = 1$:

(1) (3.3a) is used to compute $H_\phi^1(I+\frac{1}{2},J+\frac{1}{2})$ for $I = 1,2,\ldots IN-1$ and $J = 1,2,\ldots JM-1$. For example, $H_\phi^1(I+\frac{1}{2},J+\frac{1}{2})$ at points A_1 (Fig. 3) can be computed from these grid points D, E, F, G and H.

(2) (3.3b) is used to compute $E_r^{3/2}(I+1,J+\frac{1}{2})$ for $I = 1,2,\ldots IN-1$ and $J = 1,2,\ldots JM-1$. For example, $E_r^{3/2}(I+\frac{1}{2},J+\frac{1}{2})$ at point C can be computed from these grid points A_1, A_2 and D.

(3) (3.3c) is then used to compute $E_z^{3/2}(I+\frac{1}{2},J+1)$ for $I = 1,2,\ldots IN-1$ and $J = 1,2,\ldots JM-1$ in the same manner, however, an extrapolation must be used:

$$H_\phi^{n+1}(I+\tfrac{1}{2},J+1) = \left[H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{3}{2}) + H_\phi^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2})\right] /2$$

Thus, all the grid points at time $t = \Delta t$ have been obtained as shown on S_1. This computation is then repeated successively over the time interval $0 \leq t \leq T$ where $T = n\Delta t$.

To treat the boundary conditions that are imposed upon the scatterer, all grid points are first evaluated; and then the appropriate field components at the surfaces of the scatterer are set equal to zero.

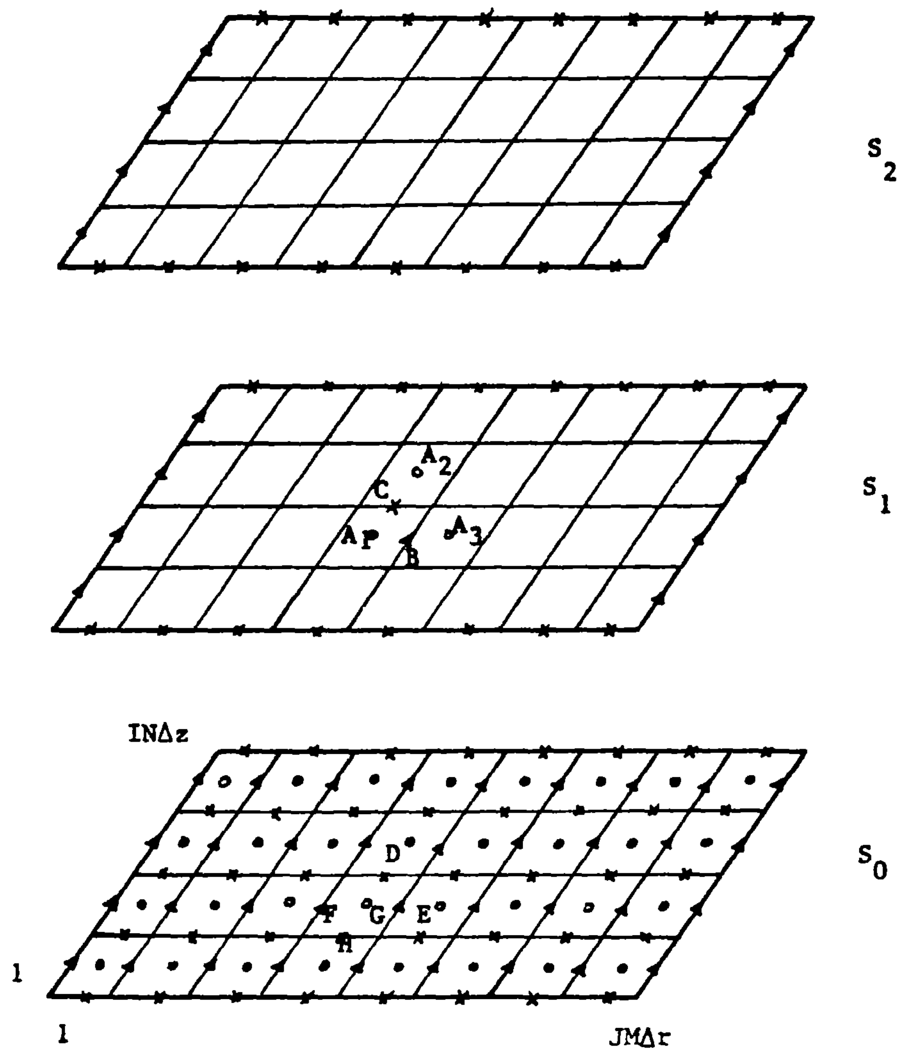

Figure 3: Illustration of calculation process. The initial field components are on the S_0 (n=0) surface. Note that H_ϕ, E_r and E_z are on different time level. Symbols are : x for E_r field; ▲ for E_z field ; o for H_ϕ field.

3.6 Numerical Results

The numerical results presented subsequently are obtained with the following parameters.

Size of the conducting finite cylinder:	radius r = 0.1 m
	length l = 0.75 m
For the outer waveguide :	radius R = 2405 m
Increments :	$\Delta r = \Delta z = 0.025$ m
	$\Delta\tau = 0.0125$ m

The incident wave is as indicated in (3.8), (3.9) and (3.10) where $A = (18\Delta z)^{-1}$. Three particular cases are considered.

Case I. Interior of the waveguide considered to be free space

To determine the accuracy of the finite difference equations, a lossless medium is considered first inside of the waveguide. Figure 4 shows the E_z component of the pulse which has propagated for 20 and 40 time cycles. Note that the pulse has propagated 10 and 20 spatial increments, respectively, as expected. Essentially no dispersion can be observed. Figure 5 shows the E_r component of the pulse where a slight disturbance is noted at the tail of the pulse. This evidently is the result of "round-off" error since only 8 significant figures in digital calculations have been used. Figure 6 shows the resulting buildup of current on the surface of the cylinder.

Case II. Scattering in a time-varying conducting medium

The waveguide is considered to be filled with a homogeneous, but time-varying, lossy medium. The conductivity is considered to vary in time as

$$\sigma^n = 2n \times 10^{-3} \text{ mhos/m} \qquad (3.11)$$

Figure 7 exhibits the attenuation of the pulse and dispersion. Again "round-off" error does not seem very pronounced in the computation of the E_z component. In Figure 8 the radial component of the electric field is shown for the time-varying conductivity. Figure 9 shows the buildup current on the cylinder.

Case III. Wave scattering in an inhomogeneous, time-varying medium

An inhomogemeous, time-varying medium with conductivity of the form

$$\sigma(z,t) = \sigma_o(t)f(z-ct)$$

is considered to be inside the waveguide. For t=0, the profile of the conductivity is shown in Figure 10 along with other initial field components of the pulse. The factore $\sigma_o(t)$ is assumed to vary in time exactly as (3.11) until it reaches a chosen maximum value (for the subsequent data this maximum is 0.1 mhos/meter). Figure 11 shows propagation of the E_z component of the pulse. It is noted that the attenuation at the tail is due to the conductivity existing in that region. In Figure 12 the radial component of the electric field is shown. The current distribution on the rod is shown in Figure 13.

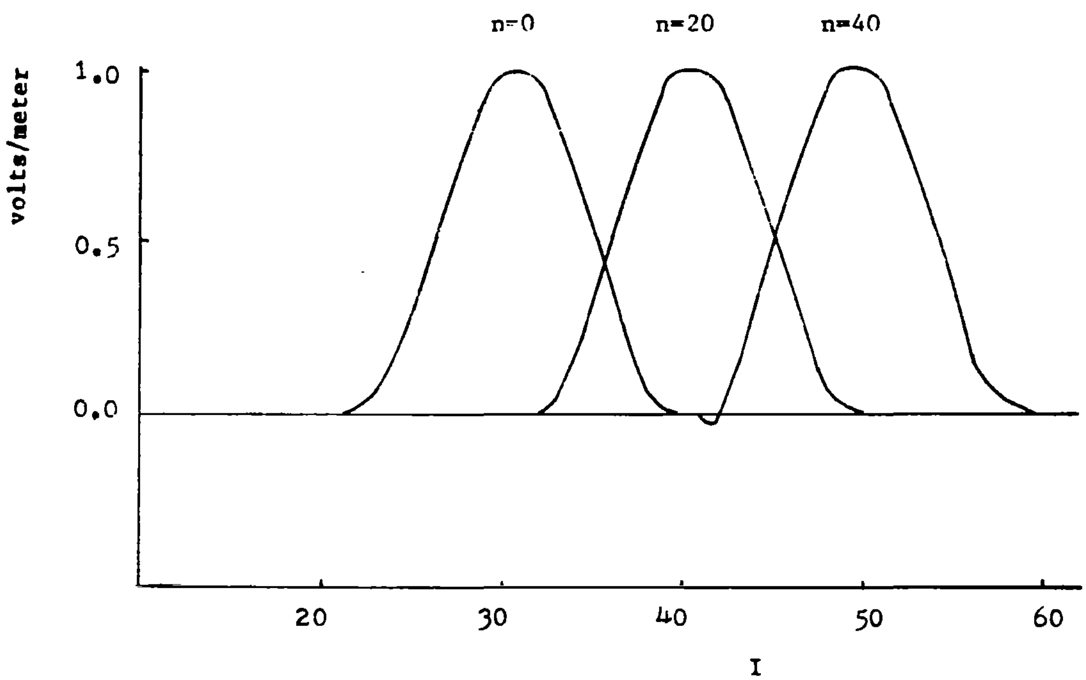

Figure 4: The propagation of the axial component of the electric field evaluated at J=20. Δz=0.025 m, $\Delta\tau$=0.0125 m, R=2405 m.

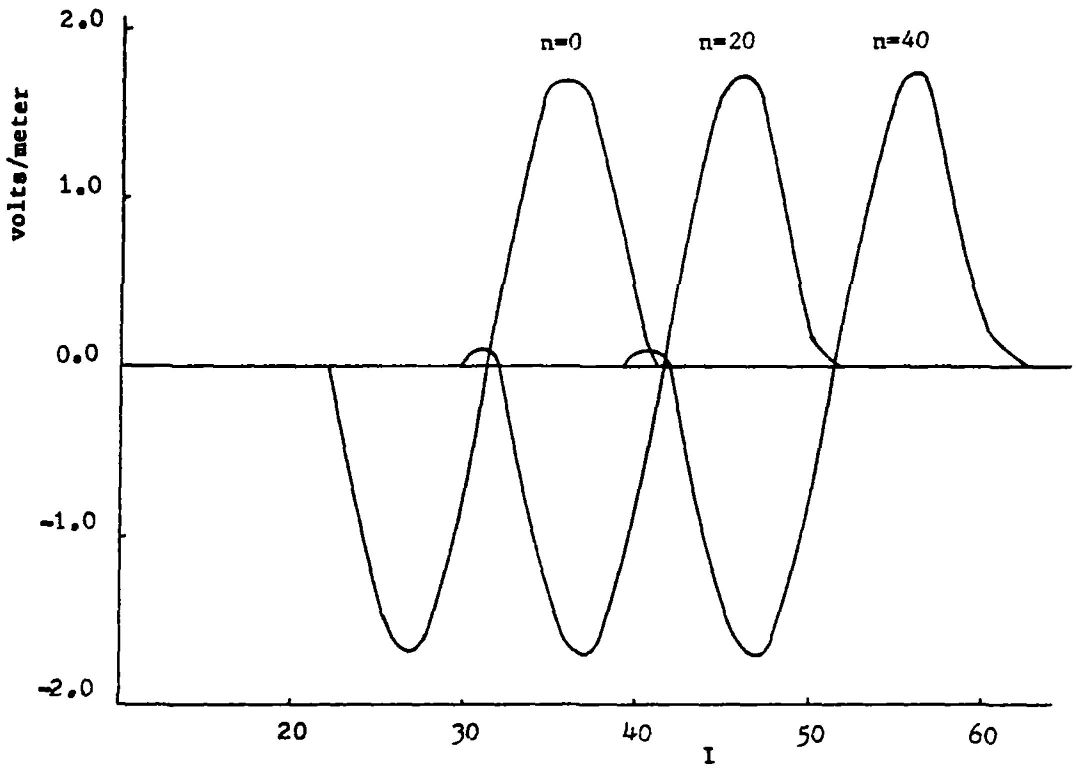

Figure 5: The propagation of the radial component of the electric field evaluated at J=20.

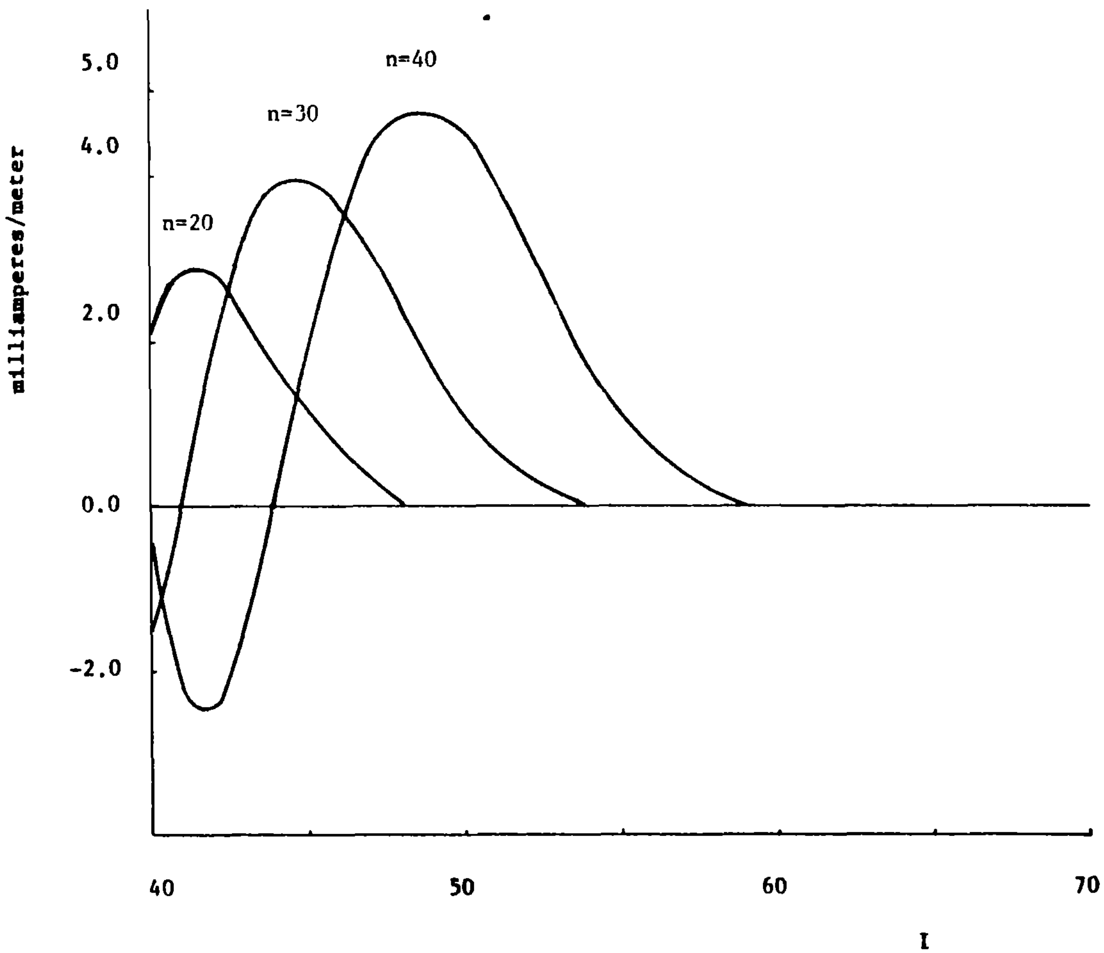

Figure 6: The current distribution induced on the cylindrical scatterer. Region of the scatterer is $40 \leq I \leq 70$ and $1 \leq J \leq 4$.

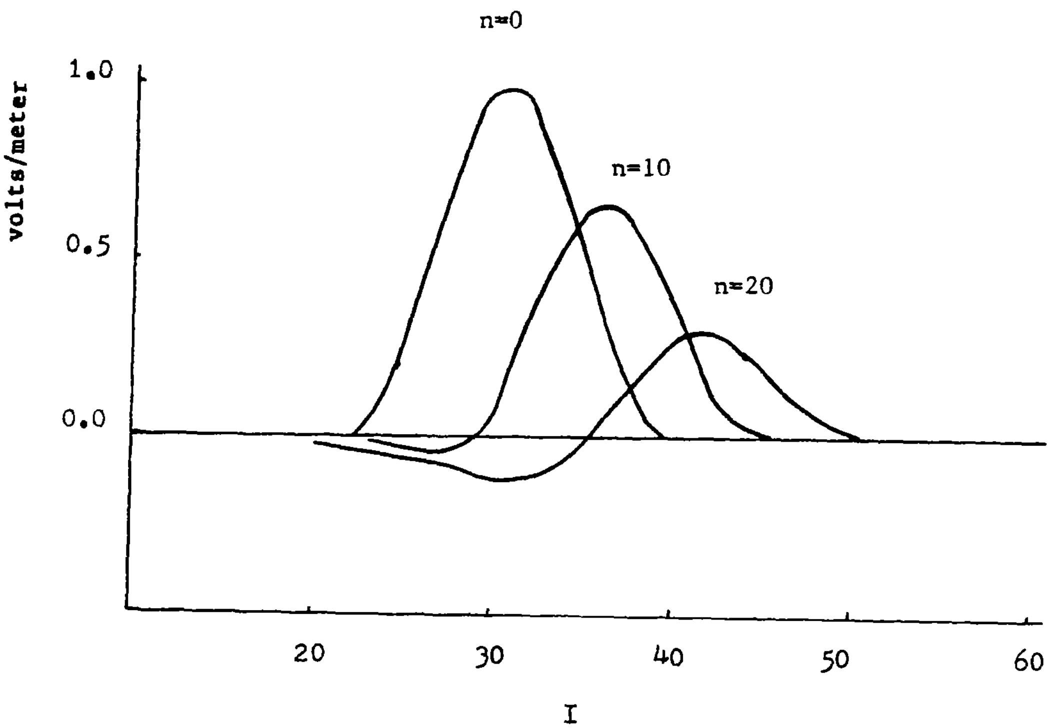

Figure 7: Propagation of the axial component of the electric field in a time-varying homogeneous medium at J=20.

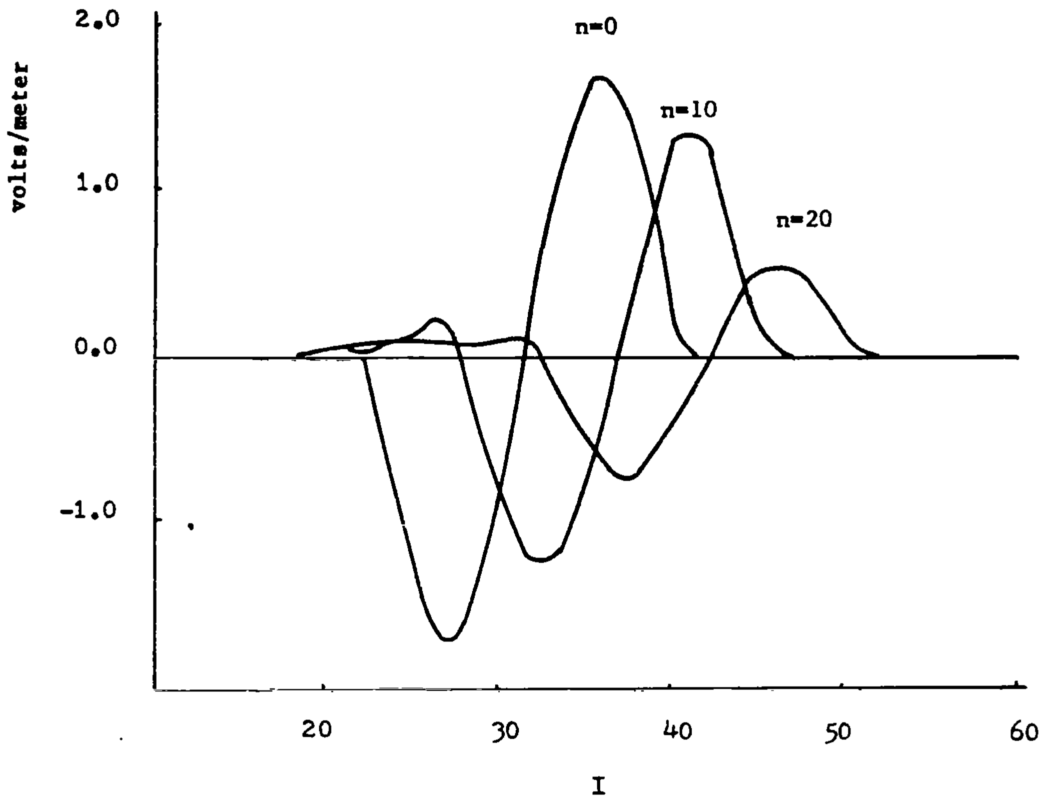

Figure 8: Propagation of the radial component of the electric field in a time-varying homogeneous medium at J=20.

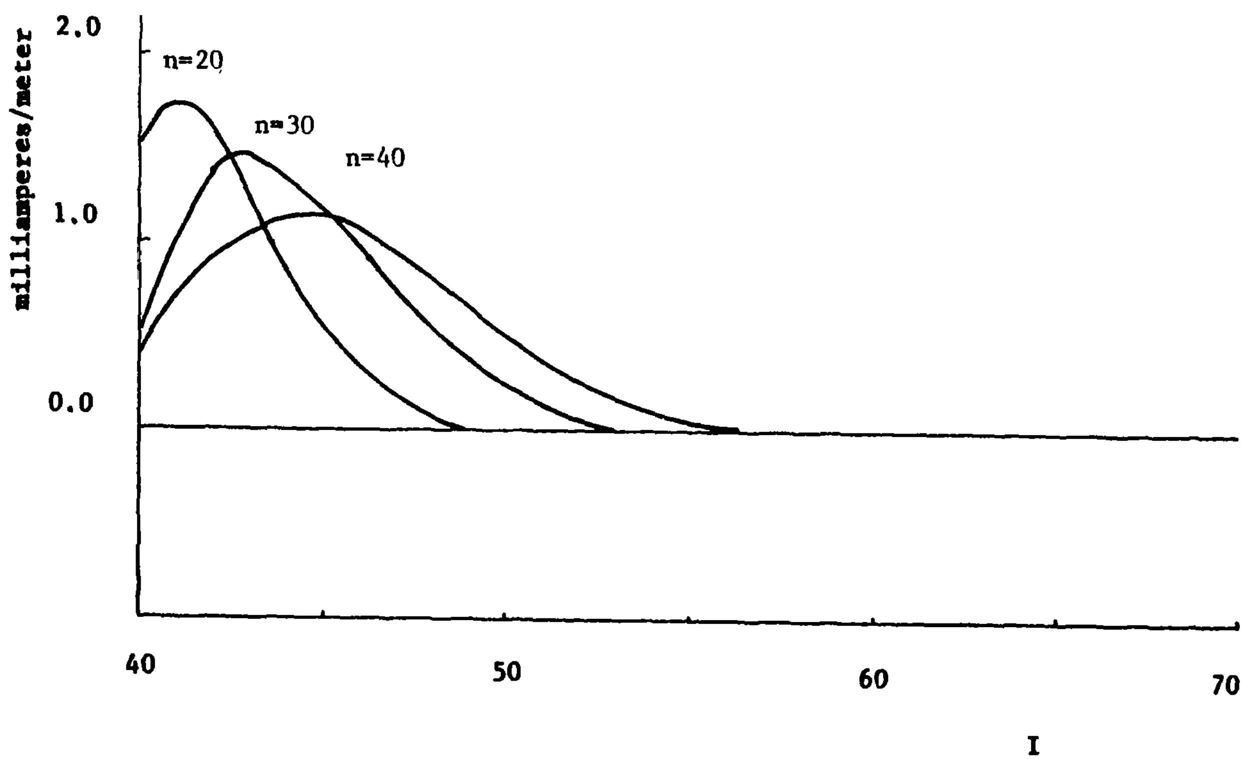

Figure 9: Current distribution induced on the cylindrical scatterer. The region of the scatterer is $40 \leq I \leq 70$ and $1 \leq J \leq 4$.

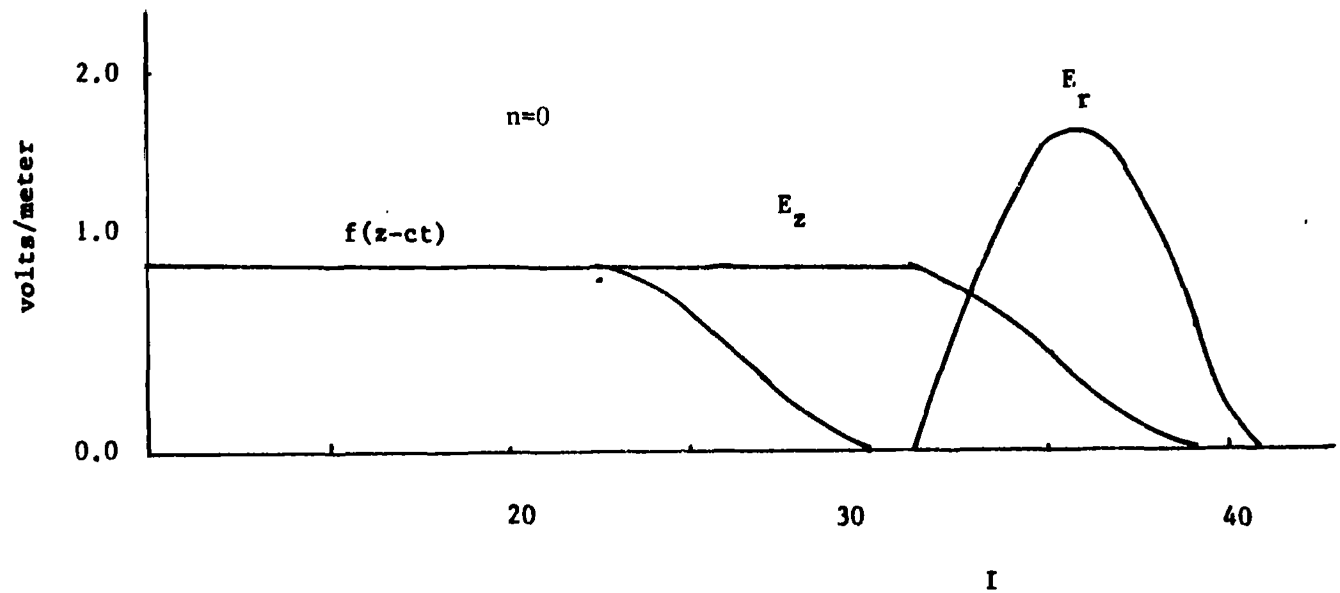

Figure 10: The electric field components at J = 20. The unitless quantity f(z-ct) is the spatial-time variation of the conductivity.

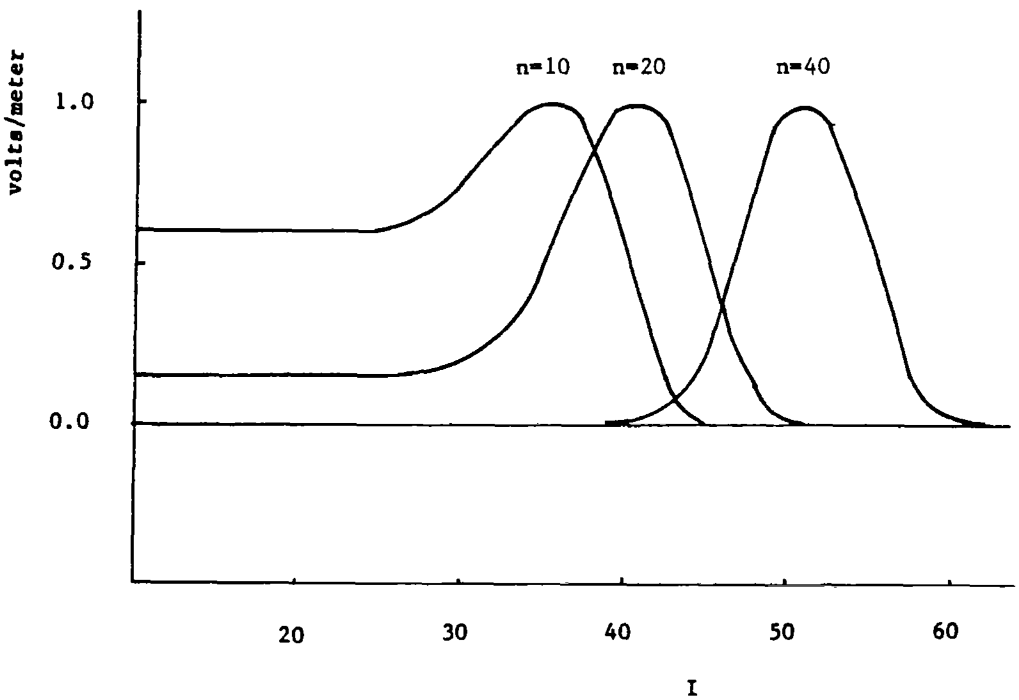

Figure 11: propagation of the axial component of the electric field at J = 20 in a time-varying, inhomogeneous medium.

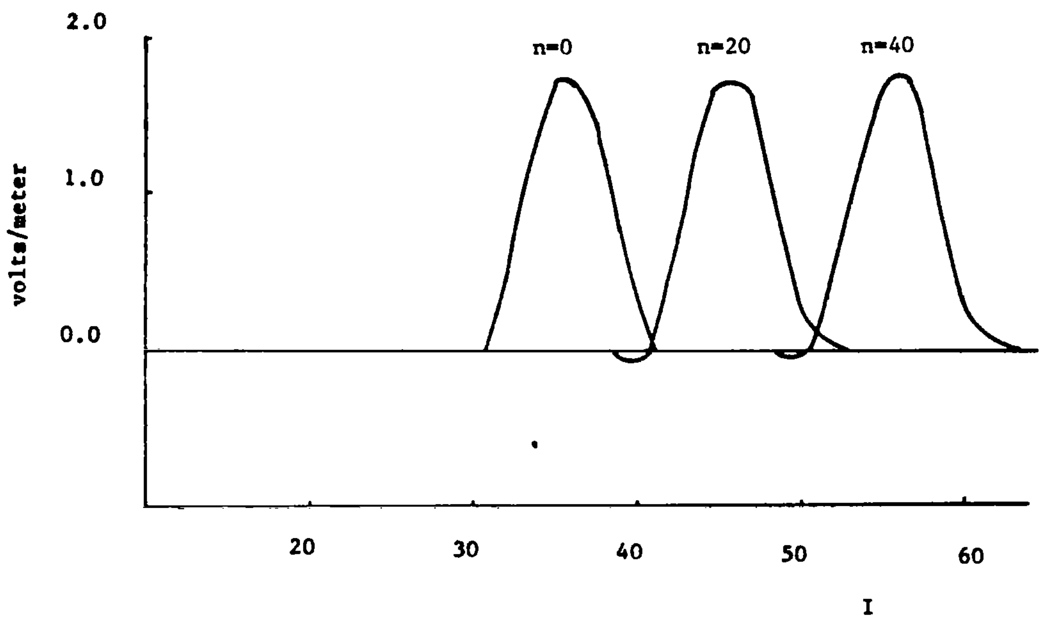

Figure 12: Propagation of the radial component of the electric field at J = 20 in a time-varying inhomogeneous medium.

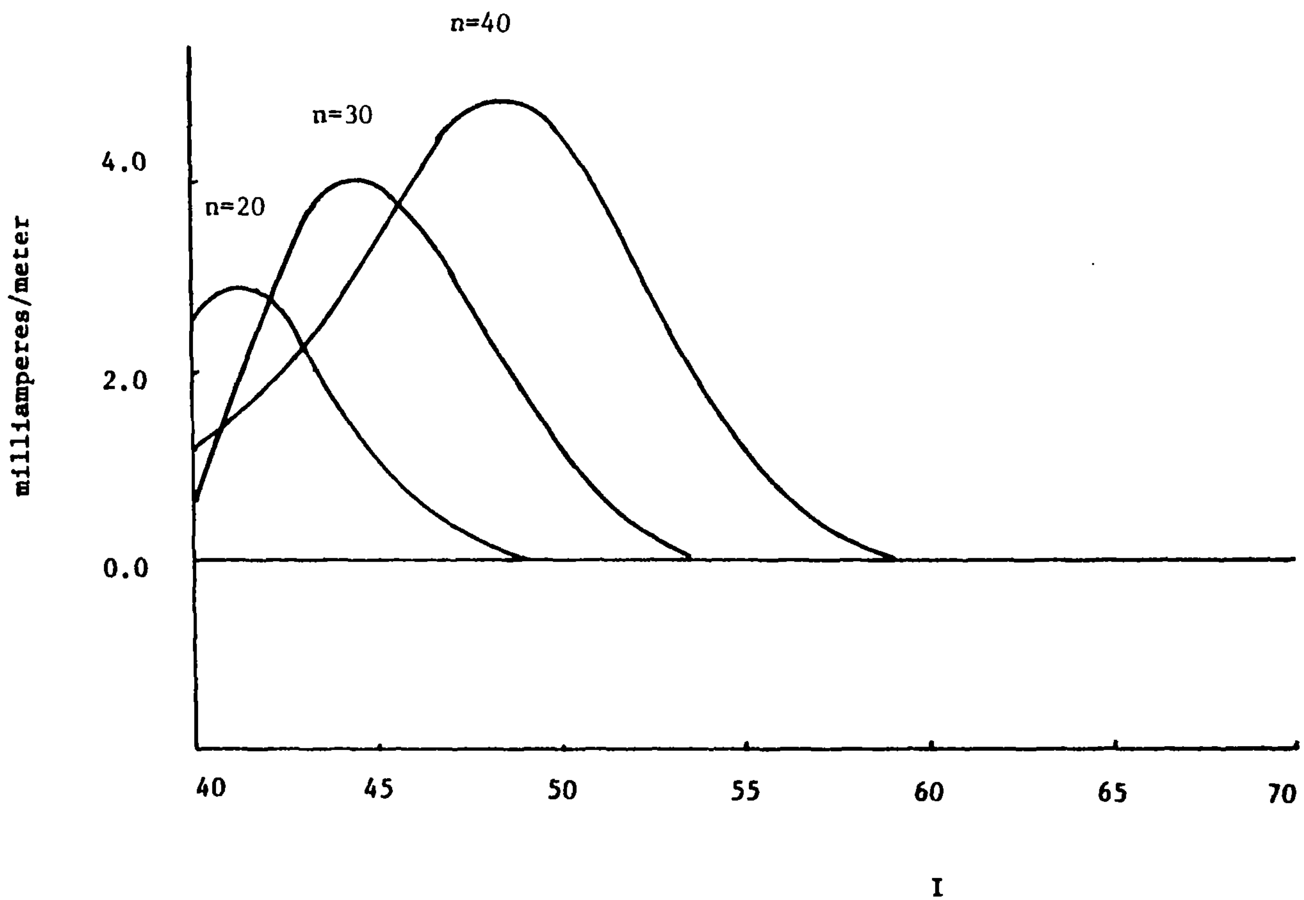

Figure 13: Current distribution induced on the cylindrical scatterer. The region of the scatterer is $40 \leq I \leq 70$ and $1 \leq J \leq 4$.

CHAPTER IV

ELECTROMAGNETIC PULSE SCATTERING FROM A CUBE IN THREE DIMENSIONS

The finite-difference technique discussed in Chapter II, in principle can be applied for any number of dimensions. However, for any physical problem at most only three dimensions would be involved. This chapter is thus devoted to the electromagnetic pulse scattering effect on an obstacle in three dimensions.

Consider a perfectly conducting cube of length L oriented in free space, i.e. a lossless medium in which $\mu=\mu_o$, $\varepsilon=\varepsilon_o$. At time $t=0$, a plane wave pulse is considered incident on the cube. The problem is to find the scattered fields and the induced current on the surfaces of the cube at subsequent time t, $(t>0)$.

4.1 Formulation

Maxwell's equations are once again used as the point of departure and are solved by the finite-difference technique. Equations (2.1) and (2.2) are expressed in rectangular component form assuming there is no charge or current sources in the medium. The set of first order partial differential equations to be solved is:

$$\varepsilon_o \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}$$

$$
\begin{aligned}
\varepsilon_o \frac{\partial E_x}{\partial t} &= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\
\mu_o \frac{\partial H_z}{\partial t} &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \\
\mu_o \frac{\partial H_y}{\partial t} &= \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\mu_o \frac{\partial H_x}{\partial t} &= \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}
\end{aligned}
\tag{4.1}
$$

A convenient difference scheme has been suggested by YEE [2]. To set up the difference equations a suitable set of grid points are shown in Figure 14a.

With the difference-scheme as shown, (4.1) is then replaced by a set of difference equations as follow:

$$
\begin{aligned}
E_z^{n+1}(I,J,K+\tfrac{1}{2}) &= E_z^{n}(I,J,K+\tfrac{1}{2}) + \frac{\Delta t}{\varepsilon_o \Delta x}\left[H_y^n(I+\tfrac{1}{2},J,K+\tfrac{1}{2}) - H_y^n(I-\tfrac{1}{2},J,K+\tfrac{1}{2})\right] \\
&\quad - \frac{\Delta t}{\varepsilon_o \Delta y}\left[H_x^n(I,J+\tfrac{1}{2},K+\tfrac{1}{2}) - H_x^n(I,J-\tfrac{1}{2},K+\tfrac{1}{2})\right] \\
E_y^{n+1}(I,J+\tfrac{1}{2},K) &= E_y^{n}(I,J+\tfrac{1}{2},K) + \frac{\Delta t}{\varepsilon_o \Delta z}\left[H_x^n(I,J+\tfrac{1}{2},k+\tfrac{1}{2}) - H_x^n(I,J+\tfrac{1}{2},K-\tfrac{1}{2})\right] \\
&\quad - \frac{\Delta t}{\varepsilon_o \Delta x}\left[H_z^n(I+\tfrac{1}{2},J+\tfrac{1}{2},K) - H_z^n(I-\tfrac{1}{2},J+\tfrac{1}{2},K)\right] \\
E_x^{n+1}(I+\tfrac{1}{2},J,K) &= E_x^{n}(I+\tfrac{1}{2},J,K) + \frac{\Delta t}{\varepsilon_o \Delta y}\left[H_z^n(I+\tfrac{1}{2},J+\tfrac{1}{2},K) - H_z^n(I+\tfrac{1}{2},J-\tfrac{1}{2},K)\right] \\
&\quad - \frac{\Delta t}{\varepsilon_o \Delta z}\left[H_y^n(I+\tfrac{1}{2},J,K+\tfrac{1}{2}) - H_y^n(I+\tfrac{1}{2},J,K-\tfrac{1}{2})\right] \\
H_z^{n+1}(I+\tfrac{1}{2},J+\tfrac{1}{2},K) &= H_z^{n}(I+\tfrac{1}{2},J+\tfrac{1}{2},K) + \frac{\Delta t}{\mu_o \Delta y}\left[E_x^{n+1}(I+\tfrac{1}{2},J+1,K) - E_x^{n+1}(I+\tfrac{1}{2},J,K)\right] \\
&\quad - \frac{\Delta t}{\mu_o \Delta x}\left[E_y^{n+1}(I+1,J+\tfrac{1}{2},K) - E_y^{n+1}(I,J+\tfrac{1}{2},K)\right] \\
H_y^{n+1}(I+\tfrac{1}{2},J,K+\tfrac{1}{2}) &= H_y^{n}(I+\tfrac{1}{2},J,K+\tfrac{1}{2}) + \frac{\Delta t}{\mu_o \Delta x}\left[E_z^{n+1}(I+1,J,K+\tfrac{1}{2}) - E_z^{n+1}(I,J,K+\tfrac{1}{2})\right] \\
&\quad - \frac{\Delta t}{\mu_o \Delta z}\left[E_x^{n+1}(I+\tfrac{1}{2},J,K+1) - E_x^{n+1}(I+\tfrac{1}{2},J,K)\right] \\
H_x^{n+1}(I,J+\tfrac{1}{2},K+\tfrac{1}{2}) &= H_x^{n}(I,J+\tfrac{1}{2},K+\tfrac{1}{2}) + \frac{\Delta t}{\mu_o \Delta z}\left[E_y^{n+1}(I,J+\tfrac{1}{2},K+1) - E_y^{n+1}(I,J+\tfrac{1}{2},K)\right] \\
&\quad - \frac{\Delta t}{\mu_o \Delta y}\left[E_z^{n+1}(I,J+1,K+\tfrac{1}{2}) - E_z^{n+1}(I,J,K+\tfrac{1}{2})\right]
\end{aligned}
\tag{4.2}
$$

where the following notation is used :

$$E_z^{n+1}(I,J,K+\tfrac{1}{2}) = E_z(I\Delta x, J\Delta y, (K+\tfrac{1}{2})\Delta z, (n+1)\Delta t)$$

$$H_z^{n}(I+\tfrac{1}{2},J+\tfrac{1}{2},K) = H_z((I+\tfrac{1}{2})\Delta x, (J+\tfrac{1}{2})\Delta y, K\Delta z, n\Delta t)$$

and so forth for the other field components.

4.2 Amplification Matrix and Stability Criteria

To develop the stability criteria, let the initial field components be, in general, the form of an exponential function, i.e.

$$E_z^n(I,J,K) = E_z^n e^{jk_1 I\Delta x} e^{jk_2 J\Delta y} e^{jk_3 K\Delta z}$$

$$E_z^n(I,J,K+\tfrac{1}{2}) = E_z^n(I,J,K) e^{jk_3\Delta z/2}$$

where k_1, k_2, k_3 are components of the propagation constant along the x, y, and z direction respectively, and $j = \sqrt{-1}$. Using these notations, (4.2) can be written:

$$E_z^{n+1}(I,J,K) = E_z^n(I,J,K) + \frac{\Delta t}{\varepsilon_o \Delta x}(e^{jk_1\Delta x/2} - e^{-jk_1\Delta x/2})H_y^n(I,J,K) - \frac{\Delta t}{\varepsilon_o \Delta y}(e^{jk_2\Delta y/2} - e^{-jk_2\Delta y/2})H_x^n(I,J,K)$$

$$E_y^{n+1}(I,J,K) = E_y^n(I,J,K) + \frac{\Delta t}{\varepsilon_o \Delta z}(e^{jk_3\Delta z/2} - e^{-jk_3\Delta z/2})H_x^n(I,J,K) - \frac{\Delta t}{\varepsilon_o \Delta x}(e^{jk_1\Delta x/2} - e^{-jk_1\Delta x/2})H_z^n(I,J,K)$$

$$E_x^{n+1}(I,J,K) = E_x^n(I,J,K) + \frac{\Delta t}{\varepsilon_o \Delta y}(e^{jk_2\Delta y/2} - e^{-jk_2\Delta y/2})H_z^n(I,J,K)$$
$$- \frac{\Delta t}{\varepsilon_o \Delta z}(e^{jk_3\Delta z/2} - e^{-jk_3\Delta z/2})H_y^n(I,J,K)$$
$$H_z^{n+1}(I,J,K) = H_z^n(I,J,K) + \frac{\Delta t}{\mu_o \Delta y}(e^{jk_2\Delta y/2} - e^{-jk_2\Delta y/2})E_x^{n+1}(I,J,K)$$
$$- \frac{\Delta t}{\mu_o \Delta x}(e^{jk_1\Delta x/2} - e^{-jk_1\Delta x/2})E_y^{n+1}(I,J,K)$$
$$H_y^{n+1}(I,J,K) = H_y^n(I,J,K) + \frac{\Delta t}{\mu_o \Delta x}(e^{jk_1\Delta x/2} - e^{-jk_1\Delta x/2})E_z^{n+1}(I,J,K)$$
$$- \frac{\Delta t}{\mu_o \Delta z}(e^{jk_3\Delta z/2} - e^{-jk_3\Delta z/2})E_x^{n+1}(I,J,K)$$
$$H_x^{n+1}(I,J,K) = H_x^n(I,J,K) + \frac{\Delta t}{\mu_o \Delta z}(e^{jk_3\Delta z/2} - e^{-jk_3\Delta z/2})E_y^{n+1}(I,J,K)$$
$$- \frac{\Delta t}{\mu_o \Delta y}(e^{jk_2\Delta y/2} - e^{-jk_2\Delta y/2})E_z^{n+1}(I,J,K)$$

where $\Delta\tau = c\Delta t$

$$Z = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$a = 2\frac{\Delta\tau}{\Delta x}\sin(k_1\Delta x/2)$$

$$b = 2\frac{\Delta\tau}{\Delta y}\sin(k_2\Delta y/2)$$

$$d = 2\frac{\Delta\tau}{\Delta z}\sin(k_3\Delta z/2)$$

And noting that $(e^{jk_1\Delta x/2} - e^{-jk_1\Delta x/2}) = j2\sin(k_1\Delta x/2)$. The set of equations (4.3) can be written in a matrix notation:

$$\begin{bmatrix} E_z^{n+1}(I,J,K) \\ E_y^{n+1}(I,J,K) \\ E_x^{n+1}(I,J,K) \\ ZH_z^{n+1}(I,J,K) \\ ZH_y^{n+1}(I,J,K) \\ ZH_x^{n+1}(I,J,K) \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} E_z^{n}(I,J,K) \\ E_y^{n}(I,J,K) \\ E_x^{n}(I,J,K) \\ ZH_z^{n}(I,J,K) \\ ZH_y^{n}(I,J,K) \\ ZH_x^{n}(I,J,K) \end{bmatrix}$$

where $[M]$ is the amplification matrix and is given by:

$$[M] = \begin{bmatrix} 1 & 0 & 0 & 0 & ja & -jb \\ 0 & 1 & 0 & -ja & 0 & jd \\ 0 & 0 & 1 & jb & -jd & 0 \\ 0 & -ja & jb & (1-b^2-a^2) & bd & ad \\ ja & 0 & -jd & bd & (1-a^2-d^2) & ab \\ -jb & jd & 0 & ad & ab & (1-d^2-b^2) \end{bmatrix}$$

Von Neumann's condition for the difference scheme to converge is that the eigenvalues of the amplification matrix $M(\xi)$ not exceed one in absolute value for all real values of ξ.

To obatin the eigenvalues of (4.4), it is necessary to solve for the roots of the following secular equation:

$$\begin{vmatrix} 1-\lambda & 0 & 0 & 0 & ja & -jb \\ 0 & 1-\lambda & 0 & -ja & 0 & jd \\ 0 & 0 & 1-\lambda & jb & -jd & 0 \\ 0 & -ja & jb & (1-\lambda-a^2-b^2) & bd & ad \\ ja & 0 & -jd & bd & (1-\lambda-a^2-d^2) & ab \\ -jb & jd & 0 & ad & ab & (1-\lambda-b^2-d^2) \end{vmatrix} = 0 \qquad (4.5)$$

where λ is the eigenvalue of (4.5). The calculation of (4.5) is quite straightforward; however it is tedious. Fortunately, a very simple result is obtained. It is

$$(1-\lambda)^2\left[\lambda^2-(2-a^2-b^2-d^2)\lambda+1\right]^2 = 0$$

The eigenvalues λ which satisfy the above equation will not exceed one in absolute value if and only if

$$(a^2+b^2+d^2) \leqslant 4$$

Substituting in the values for a, b, and d yields the condition on the grid size for convergence.

$$\left[\frac{\Delta\tau}{\Delta x}\right]^2 + \left[\frac{\Delta\tau}{\Delta y}\right]^2 + \left[\frac{\Delta\tau}{\Delta z}\right]^2 \leqslant 1 \qquad (4.6)$$

4.3 Some Aspects of Calculations

The system of difference equations of (4.2) is assured of having a solution provided the initial conditions and boundary conditions are specified. Since a cube is present in the medium, the boundary conditions on the cube must be also included.

BOUNDARY CONDITIONS

In order to require that the tangential electric field vanish on the perfectly conducting cube surfaces, they are conveniently represented by a collection of surfaces of smaller cubes, the sides of which are parallel to the coordinate axes. Plane surfaces perpendicular to the x-axis will be chosen so as to contain grid points where E_y and E_z are defined. Similarly, plane surfaces perpendicular to the other axes are chosen. This procedure is required since the sides of the cube do not necessarily fall on grid points where the field components appropriate for the boundary conditions are defined. (Figure 14a).

In order to limit the extent of our calculations region, artificial boundaries are placed a few grid spaces from the cube, the values of the field components on these boundaries are computed by linear extrapolation.

INITIAL CONDITIONS

The initial field is considered to be a uniform plane wave. Since the field components are required to be continuous functions it is acceptable to assume they have a half-sine wave profile. Moreover, they are also assumed to be propagating in the negative x-direction. Hence only two components of the pulse exist, they are:

$$E_z(x,y,z,t) = \sin\left[A(x-14\Delta x+ct)\right] \quad 0 \leq A(x-14\Delta x+ct) \leq \pi$$
$$= 0 \quad \text{elsewhere} \qquad (4.7)$$
$$H_y(x,y,z,t) = E_z(x,y,z,t)/Z$$

where A is a real constant.

The computation procedure is nearly the same as that discussed in previous chapter, however, we now have a three dimensional grid space. The whole space is divided into grid points where the field components are evaluated, and the system of equations (4.2) is then used to compute another grid space at each successive time-increment Δt. Appropriate boundary conditions for the scatterer are then applied to the corresponding grid points on each surface of the scatterer.

4.4 Numerical Results

Numerical results are presented for the incident pulse as given by (4.7) where

$$A = \pi(8\Delta x)^{-1}$$

i.e. the pulse width is considered to be $8\Delta x$ units. For computational stability, a convenient choice which satisfies (4.6) is as follows:

$$\Delta x = \Delta y = \Delta z$$

$$\Delta\tau = c\Delta t$$

$$\frac{\Delta\tau}{\Delta x} = \frac{1}{2}$$

The physical length of one side of the cube is considered to be $L = 10\Delta x$ units. The region which is used in computation has dimensions of $24\Delta x \times 18\Delta y \times 18\Delta x$; (Figure 14b.). Ideally, we should consider a computational region that is much larger than the dimensions of the obstacle, so that its boundaries would not affect significantly the computation of field components near the obstacle. Since an IBM 360 Model 40 was used to obtain the numerical data, sufficient memory storage to fulfill the aforementioned requirements was not available. However, linear extrapolation may be used to simulate those conditions. For a limited duration of time, the results which we obtained should approximate those of the original problem.

The program which was developed (See Appendix II.) was used to compute the induced current on various surfaces of the cube. Figure 15 and Figure 16 show the distribution of the induced surface charge density and induced surface current on the front surface of the cube. Since it is inconvenient to present a two-dimensional pattern of the distributions, a line through the center of the cube face, K = 9, is chosen to show the distribution of charges and currents as a function of $J\Delta y$. Figure 17 and Figure 18 show these distributions on the upper surface of the cube and Figure 19 and Figure 20 show those distributions on the side surface. Note that the terminology of "front surface", "upper surface" and "side surface" are defined explicitly in Figure 14b.

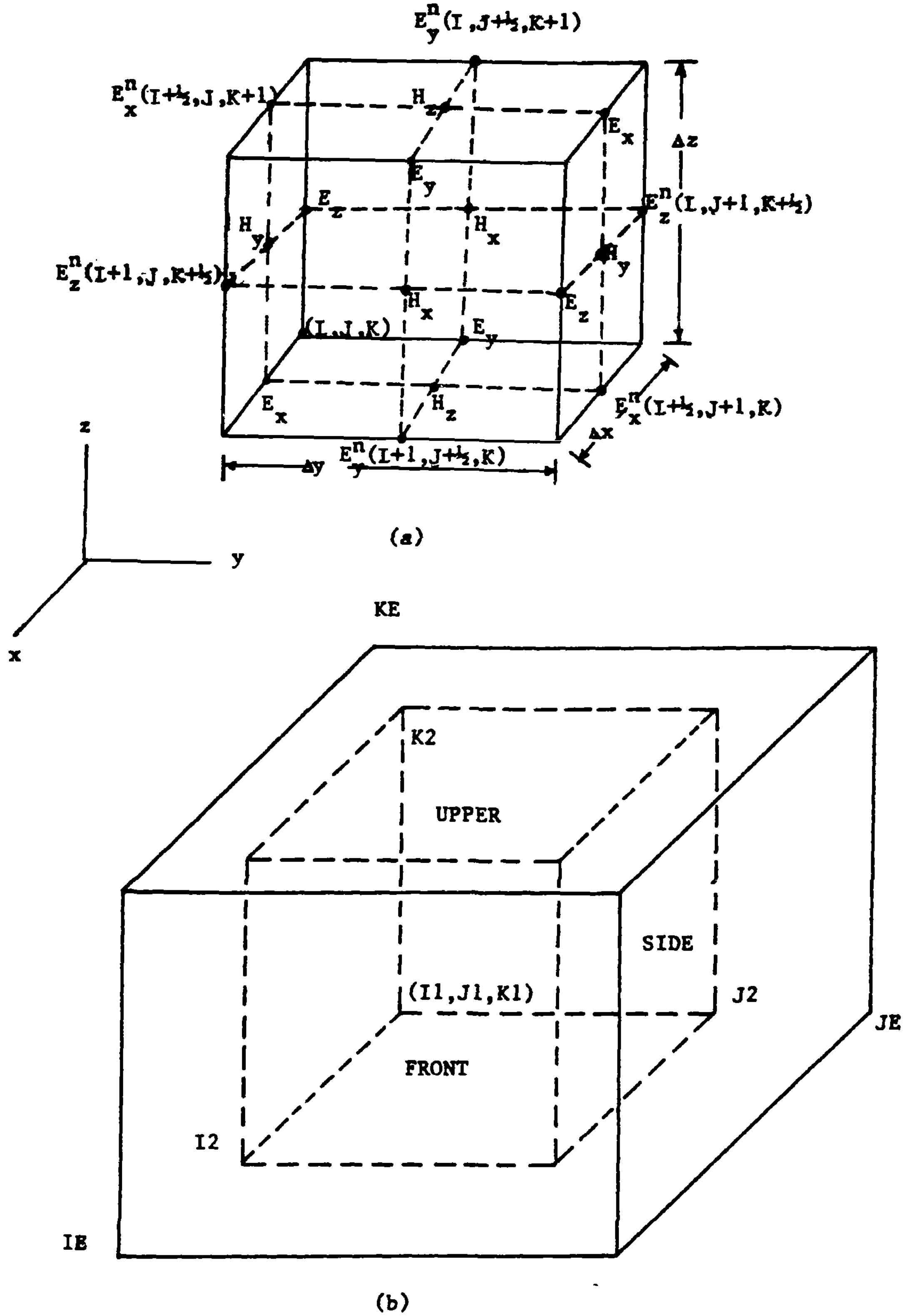

Figure 14: Positions of various field components.

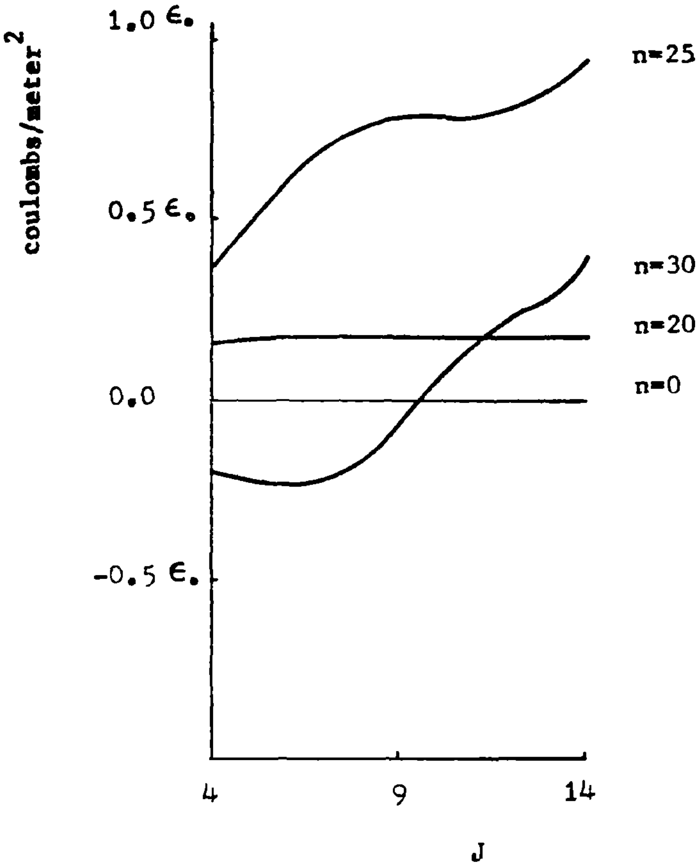

Figure 15: Surface charge distribution induced on the front surface (I2=14) of the cube at K=9. Region of the cube is $4 \leq I \leq 14$, $4 \leq J \leq 14$ and $4 \leq K \leq 14$. $\epsilon_0 = 8.85 \times 10^{-12}$ farad/m.

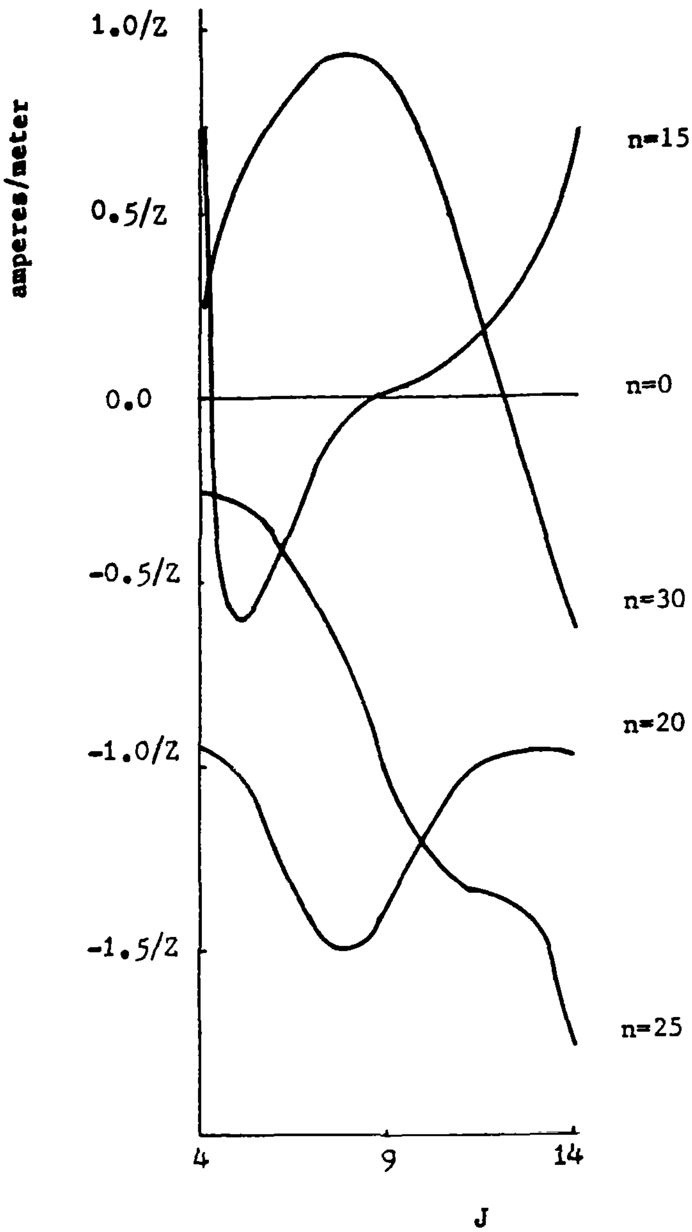

Figure 16: Current distribution induced on the front surface (I2=14) of the cube at K=9. Note that this is the component in y-direction. Z=376.7 ohms.

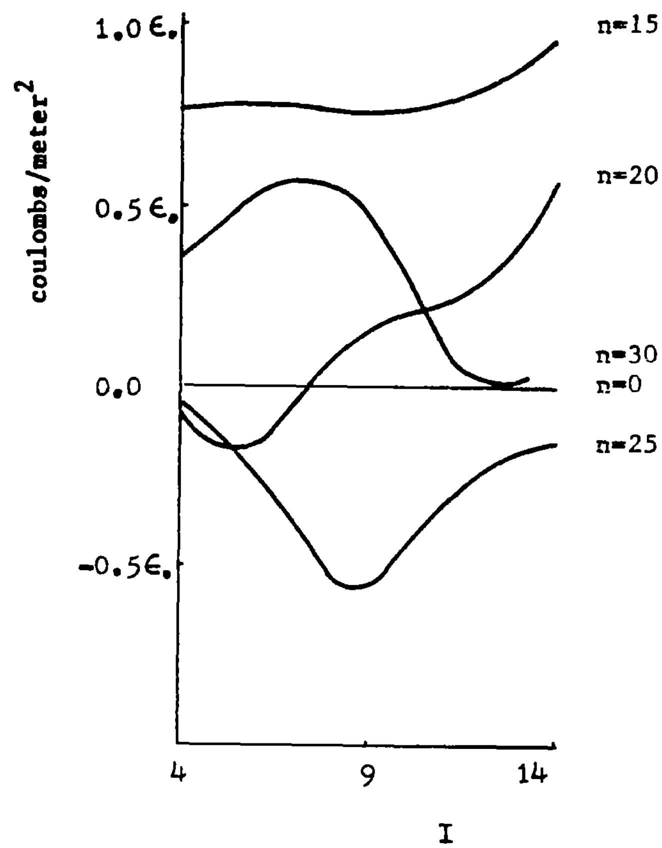

Figure 17: Surface charge distribution induced on the upper surface (K2=14) of the cube at J=9. ϵ_0=8.85 x 10^{-12} farad/m.

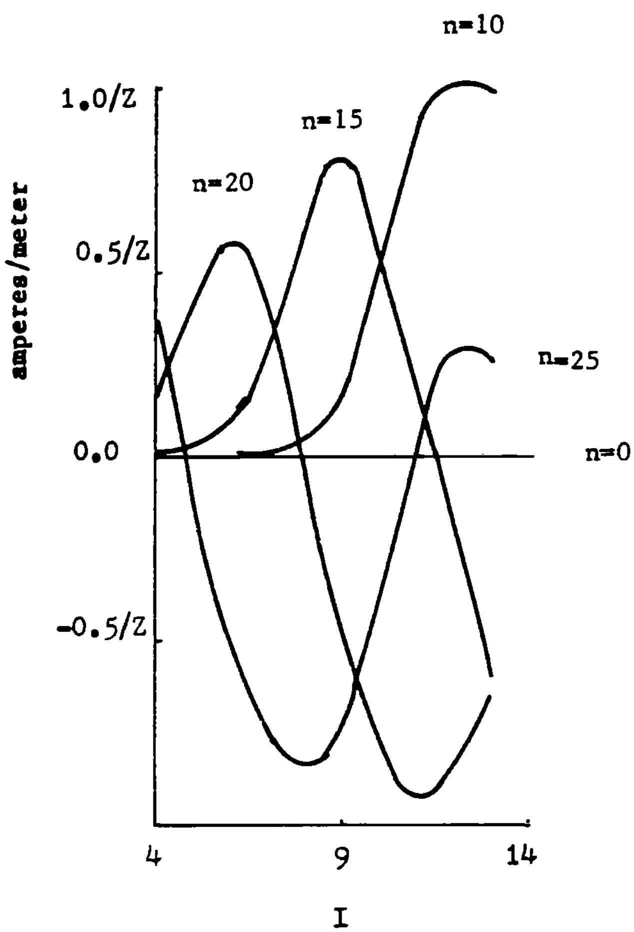

Figure 18: Current distribution induced on the upper surface (K2=14) of the cube at J=9. Note that this is the component in x-direction. Z=376.7 ohms.

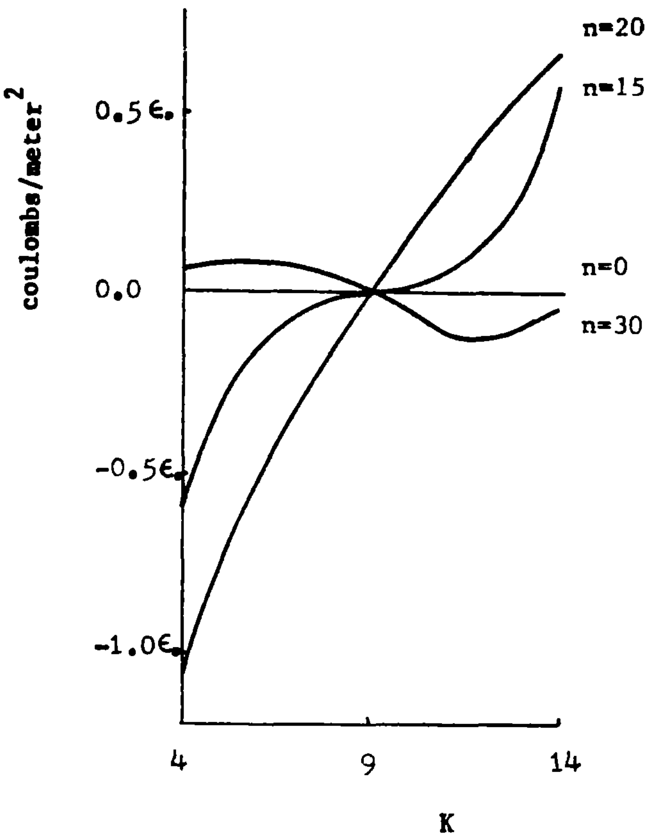

Figure 19: Surface charge distribution induced on the side surface (J2=14) of the cube at J=9. $\epsilon_0 = 8.85 \times 10^{-12}$ farad/m.

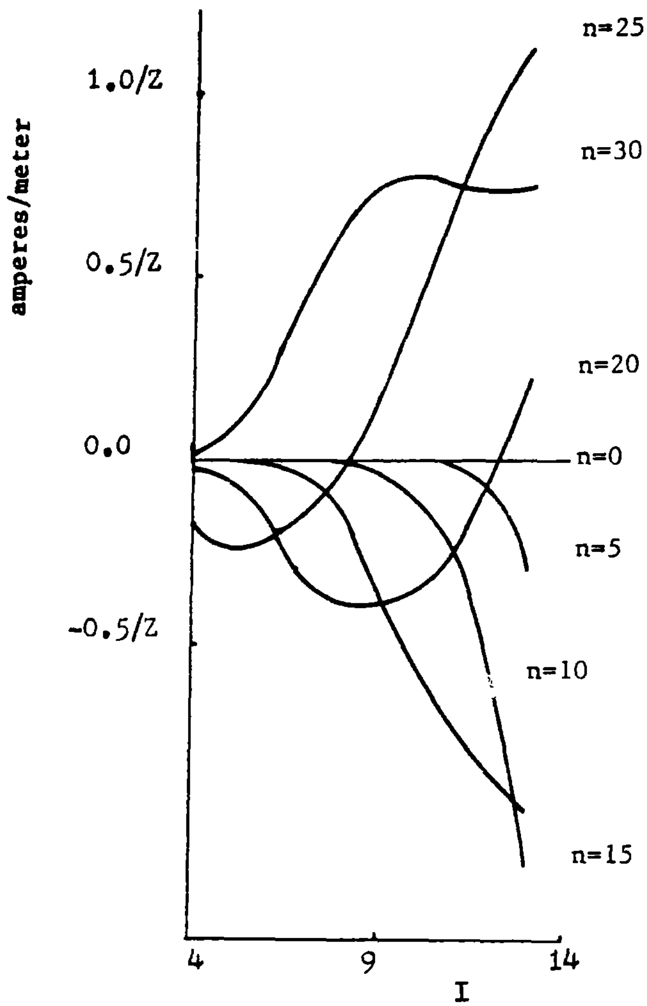

Figure 20: Current distribution induced on the side surface (J2=14) of the cube at K=9. Note that this is the component in x-direction. Z=376.7 ohms.

CHAPTER V

SUMMARY AND CONCLUSIONS

The object of this thesis was to present the finite-difference method for treating electromagnetic scattering problems. Since such scattering effects are governed by Maxwell's equations, the problem thus reduced to the numerical solution of a set of first order partial differential equations.

A general approach to the numerical solution by using the finite-difference technique has been discussed in Chapter II. Whether such a solution is equivalent to the true solution can be, in general, considered in terms of the stability of the difference scheme chosen. A suitable choice of the difference scheme requires that Von Neumann's condition be satisfied. This will guarantee a meaningful solution.

The problem chosen for illustrating the technique in Chapter III is two dimensional scattering in a time-varying, inhomogeneous medium. A perfectly conducting cylinder was the scatterer. The induced current distribution on the surface of the cylinder was obtained as well as the field components. The attenuation of the pulse in the lossy medium is clearly demonstrated. In Chapter IV, a three dimensional scattering problem has been considered. Although the technique used is limited to three dimensional scattering problems by the memory capacity of present computers, it does provide a new method for the solution of electromagnetic scattering problems. Numerical results are presented for the induced charges and the induced current on the surfaces of the cube that is illuminated by a uniform plane wave pulse.

No attempt was made in this thesis to discuss the existence and uniqueness theorems for the numerical solutions of partial differential equations with variable coefficients such as are obtained in the study of wave propagation in time-varying, inhomogeneous media. The existing mathematical theory in this area is generally inadequate, and we have only a recourse to a combination of intuition and experimental evidence (5).

It was the experience of the author that the proper arrangement of the initial field was very important to the convergence of the solution. This, of course, would also affect the choice of the difference scheme. Another thing noted was that the numerical solution was generally cocerned with a finite time duration, we may therefore construct some artificial boundaries so as to limit the region of calculation. Of course, ideally we require that these boundaries not affect the results in the region of interest. If we have sufficient computer memory capacity, this can always be done.

The subject discussed in this thesis has provided a new approach to the solution of the electromagnetic scattering problems. The idea of approximating all space with grids leads to the treatment of more general problems, for example, scattering from obstacles of arbitrary configuration. In fact, this solution technique is very powerful, limited only by the capability of the digital computer that is used.

APPENDIX I

The fortran program which is used to obtain the data in Chapter III is presented here to facilitate further numerical investigations. The program is written in the universal FORTRAN IV programmer's language (12). Comment statements included in the program should give enough information in order to make use of the program without having to be intimately familiar with the theory behind the program.

```
C     FINITE DIFFERENCE METHODS FOR ELECTROMAGNETIC PULSE
C     SCATTERING FROM A FINITE LENGTH CYLINDER IN TIME-
C     VARYING, INHOMOGENEOUS MEDIUM
C         *     *     *     *     *     *     *     *     *     *     *
C     DESCRIPTION OF VARIABLES
C     I  = COORDINATE OF THE GRID POINT IN Z-DIRECTION
C     J  = COORDINATE OF THE GRID POINT IN R-DIRECTION
C     IN = OUTER BOUNDARY OF THE Z-DIRECTION
C     JM = OUTER BOUNDARY OF THE R-DIRECTION
C     Z  = INTRINSIC IMPEDANCE
C     PI = 3.141592
C     ISL,ISE = SPECIFY THE LOCATION OF THE CYLINDER IN
C     Z-DIRECTION
C     JSC,JSE = SPECIFY THE LOCATION OF THE CYLINDER IN
C     R-DIRECTION
C     IPULSE = LEADING END OF THE INPUT PULSE
C     PWIDTH = PULSE WIDTH
C     DELCON = INCREMENT OF CONDUCTIVITY OF THE MEDIUM AT
C     EVERY TIME STEP
C     PERMTY = PERMITTIVITY OF THE MEDIUM
C     AMPLTU = AMPLITUDE OF THE INPUT PULSE
C     BETA    = 2.405/R,  WHERE  R  IS THE RADIUS OF THE
C                          WAVEGUIDE
C     K2,K3 = DUMMY VARIABLES FOR CONTROLLING THE OUTPUT
C            STATEMENTS
C     A = (DEL T)/(DEL Z) = 0.5,  GRID RATIO AS DEFINED
C     B = (DEL T)/(DEL R) = 0.5,  GRID RATIO AS DEFINED
C     ******************************************************
      DIMENSION EZO(100,41),ERO(100,41),HPHIO(100,41),
     1FACTR2(100),VSIN1(100),VSIN2(100),VSIN11(100),
     2FACTR1(100),VSIN22(100)
      INTEGER CASE
      READ(1,3) IN,JM,ISL,ISE,JSC,JSO,IPULSE,PWIDTH,DELCON,
     1PERMTY,A,B,BETA
    3 FORMAT(7I3,6F6.3)
      WRITE(3,71) IN,JM,ISL,ISE,JSC,JSO,IPULSE,PWIDTH,DELCON,
     1PERMTY,A,B,BETA
   71 FORMAT(7I5,6F8.3//)
      WRITE(3,1)
    1 FORMAT(40X,'NUMERICAL SOLUTION OF MAXWELL S EQUATIONS')
      WRITE(3,2)
    2 FORMAT(21X,'WAVE SCATTERING FROM A FINITE CYLINDER IN '
     1,'TIME-VARYING AND INHOMOGENEOUS MEDIUM'//)
      AMPLTU=1.0
      PI=3.141592
      Z=376.7
      DELZ=0.025
      K2=16
      K3=31
      ISE1=ISE-1
```

```
      JSO1=JSO-1
      IN1=IN-1
      JM1=JM-1
      F=PI/(DELZ*PWIDTH)
      AML=Z*B*DELZ/PERMTY*0.5
      IPUEND=IPULSE+PWIDTH-1
  134 READ(1,135) CASE
  135 FORMAT(I1)
      GO TO (4,6,8,999),CASE
    4 WRITE(3,5)
    5 FORMAT(15X,'CASE ONE   WAVE SCATTERING IN '
     1,'NON-CONDUCTING MEDIUM'/)
      GO TO 10
    6 WRITE(3,7)
    7 FORMAT(15X,'CASE TWO   WAVE SCATTERING IN '
     1,'TIME-VARYING CONDUCTIVITY MEDIUM'/)
      GO TO 10
    8 WRITE(3,9)
    9 FORMAT(15X,'CASE THREE   WAVE SCATTERING IN '
     1'INHOMOGENEOUS AND TIME-VARYING MEDIUM'/)
   10 CONTINUE
      DO 11 J=1,JM
      DO 11 I=1,IN
      HPHIO(I,J)=0.0
      ERO(I,J)=0.0
   11 EZO(I,J)=0.0
      GO TO (13,13,12),CASE
   12 IPULS1=IPULSE+9
      GO TO 141
   13 IPULS1=IPULSE
C
C       INCIDENT WAVE CALCULATION AS THE INPUT DATA
  141 DO 14 I=IPULS1,IPUEND
      DO 14 J=1,JM
      T=J+0.5-1
      X=BETA*T*DELZ
      BESSI=X/2.-(X**3)/16.0+(X**5)/384.0-(X**7)/18432.0
     1+(X**9)/147456.0
      VSIN=SIN((I+0.5-IPULSE)*PI/PWIDTH)
      VCOS=COS((I+0.5-IPULSE)*PI/PWIDTH)
   14 HPHIO(I,J)=-2.0*F*BESSI*VSIN*VCOS/(PERMTY*BETA*Z)*AMPLTU
      IPULS2=IPULS1+1
      IPUEN2=IPUEND+1
      DO 15 J=1,JM
      DO 15 I=IPULS2,IPUEN2
      T=J+0.5-1
      X=BETA*T*DELZ
      BESSI=X/2.-(X**3)/16.0+(X**5)/384.0-(X**7)/18432.0
     1+(X**9)/147456.0
      VSIN=SIN((I-.25-IPULSE)*PI/PWIDTH)
```

```
      VCOS=COS((I-.25-IPULSE)*PI/PWIDTH)
   15 ERO(I,J)=-2.0*F*BESSI*VSIN*VCOS/BETA*AMPLTU
      DO 16 J=1,JM
      DC 16 I=IPULS1,IPUENC
      T=J-1
      X=BETA*T*DELZ
      BESSO=1.0-(X**2)/4.0+(X**4)/64.C-(X**6)/2304.0
     1+(X**8)/147456.0
      VSIN=SIN((I+.25-IPULSE)*PI/PWIDTH)**2
   16 EZO(I,J)=BESSO*VSIN*AMPLTU
      GO TO(19,19,17),CASE
   17 DO 18 J=1,JM
      DC 18 I=1,IPULS1
   18 EZO(I,J)=EZO(IPULS1,J)
      CO 28 I=IPULSE,IPULS2
      VSIN1(I)=SIN((I+.25-IPULS2   )*PI/PWICTH)**2
   28 VSIN2(I)=SIN((I-IPULS2)*PI/PWIDTH)**2
      DC 29 I=1,IPULSE
      VSIN2(I)=VSIN2(IPULSE)
   29 VSIN1(I)=VSIN1(IPULSE)
      DC 30 I=IPULS2,IN
      VSIN2(I)=C.0
   30 VSIN1(I)=C.0
   19 DO 20 I=1,IN
      FACTR1(I)=C.0
   20 FACTR2(I)=0.0
      NSTEP=0
      COND=C.0
      JX=0
   21 CCNTINUE
      WRITE(3,22) NSTEP,FACTR1(10),FACTR1(25)
   22 FCRMAT(15X,'N=',I3,10X,'FACTR1(10)=',F8.5,4X,
     1'FACTR1(25)=',F8.5/)
      WRITE(3,311) (VSIN2(I),I=1,IN)
  311 FORMAT(1X,'VSIN2(I)',10F11.4/)
      WRITE(3,38)
C
C     OUTPUT STATEMANTS  GENERATING THE INDUCED CURRENT ON
C     THE SURFACE CF THE CYLINCER
      K1=4
   33 WRITE(3,34) K1
   34 FORMAT(2X,'HPHI',5X,'J=',I3)
C
C     OUTPUT STATEMENTS GENERATING THE ELECTRIC FIELCS
      WRITE(3,35) ((HPHIC(I,J),I=ISL,ISE),J=K1,K1,1)
   35 FORMAT(1X,10E13.4)
      WRITE(3,36)
   36 FORMAT(2X,'ERC')
      WRITE(3,35) ((ERO(I,J),I=1,IN),J=K1,K1,1)
      WRITE(3,37)
```

```
   37 FORMAT(2X,'EZO')
      WRITE(3,35) ((EZO(I,J),I=1,IN),J=K1,K1,1)
      WRITE(3,38)
   38 FORMAT(//)
      K1=K1+K2
      IF(K1-K3) 33,39,39
   39 CONTINUE
      GO TO (32,23,25),CASE
   23 COND=COND+DELCON
      DO 26 I=1,IN
      FACTR1(I)=COND*AML
   26 FACTR2(I)=FACTR1(I)
      GO TO 32
   25 CONTINUE
      IF(NSTEP-50) 251,251,252
  251 COND=COND+DELCON
  252 CONTINUE
      DO 31 I=1,IN
      FACTR1(I)=COND*VSIN1(I)*AML
   31 FACTR2(I)=COND*VSIN2(I)*AML
   32 CONTINUE
C
C     FIELD COMPONENTS ARE EVALUATED THROUGH THE DIFFERENCE
C     EQUATIONS. BOUNDARY CONDITIONS ARE THEN APPLIED.
C     EXTRAPOLATION OF THE END POINTS MUST BE DONE AT THIS
C     TIME.
      DO 40 J=1,JM1
      DO 40 I=1,IN1
      HPHIO(I,J)=HPHIO(I,J)+A*(EZO(I,J+1)-EZO(I,J))/Z
     1-B*(ERO(I+1,J)-ERO(I,J))/Z
   40 CONTINUE
C     EXTRAPOLATION OF THE OUTER BOUNDARY
      DO 41 J=1,JM
   41 HPHIO(IN,J)=2.0*HPHIO(IN1,J)-HPHIO(IN-2,J)
      DO 42 I=1,IN1
   42 HPHIO(I,JM)=2.0*HPHIO(I,JM1)-HPHIO(I,JM-2)
      DO 50 J=1,JM1
      DO 50 I=1,IN1
      ERO(I+1,J)=(ERO(I+1,J)*(1.0-FACTR1(I+1))-Z*B*(HPHIO(I+
     11,J)-HPHIO(I,J)))/(1.0+FACTR1(I+1))
   50 CONTINUE
C     EXTRAPOLATION OF THE OUTER BOUNDARY
      DO 51 J=1,JM
      ERO(1,J)=2.0*ERO(2,J)-ERO(3,J)
   51 ERO(IN,J)=2.0*ERO(IN1,J)-ERO(IN1-1,J)
C     SCATTERER BOUNDARY CONDITIONS
      DO 52 J=JSO,JSO
      DO 52 I=ISL,ISE1
   52 ERO(I,J)=0.0
      DO 60 J=1,JM1
```

```
      DO 60 I=1,IN1
      EZC(I,J+1)=(EZO(I,J+1)*(1.0-FACTR2(I))+Z*A*(HPHIO(I,J+
     11)-HPHIO(I,J))+Z*A*(HPHIC(I,J+1)+HPHIC(I,J))/(2.0*(J)))
     2/(1.0+FACTR2(I))
   60 CONTINUE
C     EXTRAPOLATION OF THE OUTER BOUNDARY
      DO 61 I=1,IN
      EZC(I,1)=2.0*EZC(I,2)-EZC(I,3)
   61 EZO(I,JM)=0.0
C     SCATTERER BOUNDARY CONDITIONS
      DO 62 J=JSC,JSC1
      DO 62 I=ISL,ISE
   62 EZC(I,J)=0.0
C     INCREASE TIME STEP AND COMPUTE EACH FIELD COMPONENT
C     AGAIN
      NSTEP=NSTEP+1
      GO TO (172,172,171),CASE
  171 IF(JX-1) 63,64,63
   64 JX=0
      DO 69 I=2,IN
      VSIN11(I)=VSIN1(I-1)
   69 VSIN22(I)=VSIN2(I-1)
      DO 691 I=2,IN
      VSIN1(I)=VSIN11(I)
  691 VSIN2(I)=VSIN22(I)
      GO TO 172
   63 JX=1
  172 CONTINUE
      IF(NSTEP-10) 39,21,200
  200 IF(NSTEP-20) 39,21,201
  201 IF(NSTEP-30) 39,21,202
  202 IF(NSTEP-40) 39,21,134
  999 STOP
      END
100 41 40 70  1  4 22 18.0  0.002  1.0   0.5  0.5   0.001
1
2
3
4
```

APPENDIX II

The fortran program which is used to obtain the data in Chapter IV is presented here to facilitate further numerical investigations. The program is written in the universal FORTRAN IV programmers' language (12). Comment statements included in the program should give enough information in order to make use of the program without having to be intimately familiar with the theory behind the program.

```
C      ELECTROMAGNETIC PULSE SCATTERING FROM A CUBE BY USING
C      FINITE DIFFERENCE METHOD
C              *                  *                  *
C      DESCRIPTION OF VARIABLES
C      I = COORDINATE OF THE GRID POINT IN X-DIRECTION
C      J = COORDINATE OF THE GRID POINT IN Y-DIRECTION
C      K = COORDINATE OF THE GRID POINT IN Z-DIRECTION
C      I1,J1,K1,I2,J2,K2 = SPECIFY THE OUTER REGION OF THE
C      ARTIFICIAL BOUNDARIES
C      IPULSE = LEADING END OF THE INPUT PULSE
C      PWIDTH = PULSE WIDTH
C      NSTEP = TIME STEP
C
C      *****************************************************
C
       DIMENSION EX(24,18,18),EY(24,18,18),EZ(24,18,18),
      1          HX(24,18,18),HY(24,18,18),HZ(24,18,18)
       READ(1,88) IE,JE,KE,I1,J1,K1,I2,J2,K2,IPULSE,PWIDTH
    88 FORMAT(10I3,F6.2)
       WRITE(3,88)IE,JE,KE,I1,J1,K1,I2,J2,K2,  IPULSE,PWIDTH
       IE1=IE-1
       IE2=IE-2
       JE1=JE-1
       JE2=JE-2
       KE1=KE-1
       KE2=KE-2
       I21=I2-I1
       J21=J2-J1
       K21=K2-K1
       I221=I2-1
       J221=J2-1
       K221=K2-1
       PI=3.141592
       IWIDTH=IPULSE+PWIDTH
       DO 1 I=1,IE
       DO 1 J=1,JE
       DO 1 K=1,KE
       EZ(I,J,K)=0.0
       EX(I,J,K)=0.0
       EY(I,J,K)=0.0
       HZ(I,J,K)=0.0
       HX(I,J,K)=0.0
     1 HY(I,J,K)=0.0
C
C      INPUT FIELD CALCULATION
       DO 2 I=IPULSE,IWIDTH
       EZ(I,1,1)=SIN((I-IPULSE)*PI/PWIDTH)
     2 HY(I,1,1)=SIN((I+0.5-IPULSE)*PI/PWIDTH)
       DO 3 I=IPULSE,IWIDTH
       DO 3 J=1,JE
```

```
      DO 3 K=1,KE
      EZ(I,J,K)=EZ(I,1,1)
    3 HY(I,J,K)=HY(I,1,1)
      NSTEP=0
   80 WRITE(3,91) NSTEP
   91 FORMAT(20X,'N=',I3)
      WRITE(3,112)
  112 FORMAT(/)
C
C     OUTPUT STATEMENTS WHICH GENERATE THE INCUCEC CHARGES
C     AND INDUCED CURRENT ON VARIOUS SURFACE CF THE CUBE
      WRITE(3,92)
   92 FORMAT(4X,'EZ')
      WRITE(3,93) (EZ(I,16,16),I=1,IE)
   93 FORMAT(2X,10E13.4)
      WRITE(3,112)
      WRITE(3,95)
   95 FORMAT(4X,'EX PATTERN CN THE FRCNT SURFACE CF THE '
     1'CUBE'
      DC 97 K=K1,K2
      WRITE(3,93) (EX(I2,J,K),J=J1,J2)
   97 CONTINUE
      WRITE(3,112)
      WRITE(3,958)
  958 FORMAT(4X,'EX PATTERN ON THE  BACK SURFACE OF THE '
     1'CUBE'
      DC 978 K=K1,K2
      WRITE(3,93) (EX(I1,J,K),J=J1,J2)
  978 CONTINUE
      WRITE(3,112)
      WRITE(3,951)
  951 FORMAT(4X,'EZ PATTERN ON THE LPPER SURFACE CF THE '
     1'CUBE'
      DO 971 I=I1,I2
      WRITE(3,93) (EZ(I,J,K2),J=J1,J2)
  971 CCNTINUE
      WRITE(3,112)
      WRITE(3,94)
   94 FORMAT(4X,'EY PATTERN CN THE  SICE SURFACE CF THE '
     1'CUBE'
      DC 96 K=K1,K2
      WRITE(3,93) (EY(I,J2,K) ,I=I1,I2)
   96 CCNTINUE
      WRITE(3,112)
      WRITE(3,955)
  955 FORMAT(4X,'HY PATTERN ON THE FRONT SURFACE OF THE '
     1'CUBE'
      DO 975 K=K1,K221
      WRITE(3,93) (HY(I2,J,K),J=J1,J2)
  975 CCNTINUE
```

```
      WRITE(3,112)
      WRITE(3,959)
  959 FORMAT(4X,'HY PATTERN ON THE  BACK SURFACE OF THE '
     1'CUBE'
      DO 979 K=K1,K221
      WRITE(3,93) (HY(I1,J,K),J=J1,J2)
  979 CONTINUE
      WRITE(3,112)
      WRITE(3,952)
  952 FORMAT(4X,'HZ PATTERN ON THE FRONT SURFACE OF THE '
     1'CUBE'
      DO 972 K=K1,K2
      WRITE(3,93) (HZ(I2,J,K),J=J1,J221)
  972 CONTINUE
      WRITE(3,112)
      WRITE(3,941)
  941 FORMAT(4X,'HZ PATTERN ON THE  BACK SURFACE OF THE '
     1'CUBE'
      DO 961 K=K1,K2
      WRITE(3,93) (HZ(I1,J,K),J=J1,J221)
  961 CONTINUE
      WRITE(3,112)
      WRITE(3,953)
  953 FORMAT(4X,'HX PATTERN ON THE UPPER SURFACE OF THE '
     1'CUBE'
      DO 973 J=J1,J221
      WRITE(3,93) (HX(I,J,K2),I=I1,I2)
  973 CONTINUE
      WRITE(3,112)
      WRITE(3,956)
  956 FORMAT(4X,'HY PATTERN ON THE UPPER SURFACE OF THE '
     1'CUBE'
      DO 976 J=J1,J2
      WRITE(3,93) (HY(I,J,K2),I=I1,I221)
  976 CONTINUE
      WRITE(3,112)
      WRITE(3,954)
  954 FORMAT(4X,'HX PATTERN ON THE SIDE  SURFACE OF THE '
     1'CUBE'
      DO 974 K=K1,K221
      WRITE(3,93) (HX(I,J2,K),I=I1,I221)
  974 CONTINUE
      WRITE(3,112)
      WRITE(3,957)
  957 FORMAT(4X,'HZ PATTERN ON THE SIDE  SURFACE OF THE '
     1'CUBE'
      DO 977 K=K1,K2
      WRITE(3,93) (HZ(I,J2,K),I=I1,I221)
  977 CONTINUE
      WRITE(3,111)
```

```
  111 FORMAT(///)
C
C       EVALUATE EACH FIELC COMPONENT THROUGH THE CIFFERENCE
C       EQUATIONS. BCUNCARY CCNCITICNS ARE THEN APPLIED
C       IMMEDIATELY AFTER THE FIELD WHICH HAS BEEN CCMPLTEC.
C       EXTRAPOLATION OF THE BOUNCARIES MUST BE DONE AT THIS
C       TIME.
   81 DO 10 I=2,IE
      DC 10 J=2,JE
      CO 10 K=1,KE
   10 EZ(I,J,K)=EZ(I,J,K)+(HY(I,J,K)-HY(I-1,J,K))/2.C
     1-(HX(I,J,K)-HX(I,J-1,K))/2.C
C       EXTRAPOLATE ENC CONDITIONS
      DC 11 J=2,JE
      DO 11 K=1,KE
   11 EZ(1,J,K)=(2.0*EZ(2,J,K)-EZ(3,J,K))/2.0
      DC 12 I=2,IE
      DO 12 K=1,KE
   12 EZ(I,1,K)=(2.C* EZ(I,2,K)-EZ(I,3,K))/2.C
      DC 13 K=1,KE
   13 EZ(1,1,K)=((2.C*EZ(2,1,K)-EZ(3,1,K))/2.C+(2.0*EZ(1,2,K
     1)-EZ(1,3,K))/2.C)/2.C
C       SCATTER BCUNCARY CCNCITICNS
      DO 16 J=J1,J2,J21
      DC 16 K=K1,K221
      DC 16 I=I1,I2
   16 EZ(I,J,K)=0.0
      DC 15 I=I1,I2,I21
      DC 15 K=K1,K221
      DC 15 J=J1,J2
   15 EZ(I,J,K)=0.0
      DC 20 I=2,IE
      DC 20 J=1,JE
      DC 20 K=2,KE
   20 EY(I,J,K)=EY(I,J,K)+(HX(I,J,K)-HX(I,J,K-1))/2.0
     1-(HZ(I,J,K)-HZ(I-1,J,K))/2.C
C       EXTRAPOLATE CUTER BCUNDARY
      DO 21 J=1,JE
      DC 21 K=2,KE
   21 EY(1,J,K)=(2.0*EY(2,J,K)-EY(3,J,K))/2.0
      CO 22 J=1,JE
      DC 22 I=2,IE
   22 EY(I,J,1)=(2.C*EY(I,J,2)-EY(I,J,3))/2.0
      DC 23 J=1,JE
   23 EY(1,J,1)=((2.0*EY(2,J,1)-EY(3,J,1))/2.0+(2.0*EY(1,J,2
     1)-EY(1,J,3))/2.C)/2.C
C       SCATTER BCUNCARY CONCITIONS
      DC 24 K=K1,K2,K21
      DO 24 J=J1,J221
      DO 24 I=I1,I2
```

```
   24 EY(I,J,K)=0.0
      DC 25 I=I1,I2,I21
      DO 25 J=J1,J221
      DO 25 K=K1,K2
   25 EY(I,J,K)=C.0
      DO 30 I=1,IE
      DC 30 J=2,JE
      DO 30 K=2,KE
   30 EX(I,J,K)=EX(I,J,K)+(HZ(I,J,K)-HZ(I,J-1,K))/2.0
     1-(HY(I,J,K)-HY(I,J,K-1))/2.0
C     EXTRAPOLATE CLTER BOUNDARY
      DO 31 I=1,IE
      DC 31 K=2,KE
   31 EX(I,1,K)=(2.0*EX(I,2,K)-EX(I,3,K))/2.0
      DO 32 I=1,IE
      DC 32 J=2,JE
   32 EX(I,J,1)=(2.0*EX(I,J,2)-EX(I,J,3))/2.0
      DC 33 I=1,IE
   33 EX(I,1,1)=((2.C*EX(I,2,1)-EX(I,3,1))/2.0+(2.0*EX(I,1,2
     1)-EX(I,1,3))/2.0)/2.0
C     SCATTER BOUNCARY CCNDITICNS
      DO 34 K=K1,K2,K21
      CO 34 J=J1,J2
      DC 34 I=I1,I221
   34 EX(I,J,K)=C.0
      DO 35 J=J1,J2,J21
      DC 35 I=I1,I221
      DC 35 K=K1,K2
   35 EX(I,J,K)=0.0
      DC 40 I=1,IE1
      DO 40 J=1,JE1
      DO 40 K=1,KE
   40 HZ(I,J,K)=HZ(I,J,K)+(EX(I,J+1,K)-EX(I,J,K))/2.0
     1-(EY(I+1,J,K)-EY(I,J,K))/2.0
C     EXTRAPOLATE CUTER BCUNCARY
      DO 41 K=1,KE
      DO 41 J=1,JE1
   41 HZ(IE,J,K)=(2.0*HZ(IE1,J,K)-HZ (IE2,J,K))/2.0
      DO 42 K=1,KE
      DC 42 I=1,IE1
   42 HZ(I,JE,K)=(2.0*HZ(I,JE1,K)-HZ(I,JE2,K))/2.C
      DO 43 K=1,KE
   43 HZ(IE,JE,K)=((2.0*HZ(IE1,JE,K)-HZ(IE2,JE,K))/2.C+(2.C*
     1HZ(IE,JE1,K)-HZ(IE,JE2,K))/2.0)/2.0
      CO 5C I=1,IE1
      DC 50 J=1,JE
      DC 5C K=1,KE1
   50 HY(I,J,K)=HY(I,J,K)+(EZ(I+1,J,K)-EZ(I,J,K))/2.0
     1-(EX(I,J,K+1)-EX(I,J,K))/2.0
C     EXTRAPOLATION CF CUTER BCUNDARY
```

```
      DO 74 J=1,JE
      DO 74 K=1,KE1
   74 HY(IE,J,K)=(2.0*HY(IE1,J,K)-HY(IE2,J,K))/2.0
      DO 75 J=1,JF
      CC 75 I=1,IE1
   75 HY(I,J,KE)=(2.0*HY(I,J,KE1)-HY(I,J,KE2))/2.0
      DO 76 J=1,JE
   76 HY(IE,J,JE)=((2.0*HY(IE1,J,JE)-HY(IE2,J,JE))/2.0+(2.C*
     1HY(IE,J,KE1)-HY(IE,J,KE2))/2.0)/2.0
      CO 60 I=1,IE
      DC 60 J=1,JE1
      DO 60 K=1,KE1
   60 HX(I,J,K)=HX(I,J,K)+(EY(I,J,K+1)-EY(I,J,K))/2.0
     1-(EZ(I,J+1,K)-EZ(I,J,K))/2.0
C     EXTRAPOLATICN CF THE OUTER BCUNDARY
      CC 77 I=1,IE
      DC 77 K=1,KE1
   77 HX(I,JF,K)=(2.C*HX(I,JE1,K)-HX(I,JE2,K))/2.0
      DO 78 I=1,IE
      CC 78 J=1,JE1
   78 HX(I,J,KE)=(2.C*HX(I,J,KE1)-HX(I,J,KE2))/2.C
      CC 79 I=1,IE
   79 HX(I,JE,KE)=((2.C*HX(I,JE1,KE)-HX(I,JE2,KE))/2.0+(2.0*
     1HX(I,JE,KE1)-HX(I,JE,KE2))/2.0)/2.0
C
C     INCREASE TIME STEP AND EVALUATE EACH FIELC CCMPCNENT
C     AGAIN
      NSTEP=NSTEP+1
      IF(NSTEP- 5) 81,80,200
  200 IF(NSTEP-10) 81,80,2C1
  201 IF(NSTEP-15) 81,80,202
  202 IF(NSTEP-2C) 81,80,2C3
  203 IF(NSTEP-25) 81,8C,2C4
  204 IF(NSTEP-30) 81,80,82
   82 STOP
      END
 24 18 18  4  4  4 14 14 14 14   8.0
```

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