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RECEIVING CHARACTERISTICS OF IMPEDANCE LOADED SLOT CONFIGURATIONS

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ABSTRACT

A general theory is developed for determining the current in the load impedance of any slot configuration in terms of the current in the load impedance of the complementary strip (wire) circuit. It is assumed that these duals are illuminated by plane wave electromagnetic fields. Following establishment of the procedures and techniques for analysis of strip and slot circuits, the practical situation presented by a bomb in the open bomb bay of an aircraft with load impedance connected between one end of the weapon and fuselage is treated. The load current is found approximately when the incident plane wave electric field is polarized in both transverse and longitudinal directions with respect to a strip that is equivalent electrically to a specific right circular cylinder that represents the weapon.

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Professor Ronold W. P. King, of Harvard University, discussed the subject of complementarity with the writer a number of times during the preparation of this paper. The weapon in the open bomb bay problem was suggested by Mr. G. E. Morgan of Autonetics.

RECEIVING CHARACTERISTICS OF IMPEDANCE LOADED SLOT CONFIGURATIONS

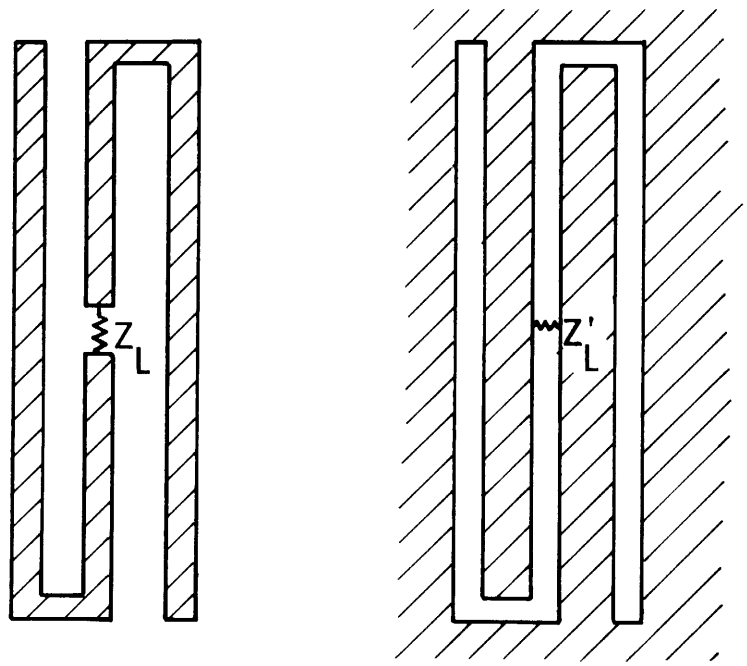
Introduction

In the field of electromagnetic compatibility, specifically in the area of EMP and EMR hazards to ordnance, many unusual circuits that respond to incident electromagnetic waves are encountered. In actuality these circuits are receiving antennas and must be analyzed as such, using applicable prinicples of electromagnetics. Radio frequency fields may enter a bomb or missile by means of cables or slots. If the interior fields are sufficiently intense, the resulting current flow in some key components may cause malfunction of the weapon.

The purpose of the present paper is to present a procedure for determining the receiving characteristics of an impedance loaded slot configuration in terms of the receiving properties of its dual, or complementary, structure. Actually, slot and strip receiving antennas may be analyzed independently of each other; however, the antenna engineer, especially the writer, generally feels more comfortable analyzing wire (strip) circuits than slot configurations. Accordingly, it is helpful to be in a position to analyze slot configurations in terms of strip circuits. This can be made a very straightforward task.

If the strip circuit consists of more than one conductor, it will normally exhibit both antenna and transmission line characteristics. Hence, all such configurations will be described in the present paper as antenna, transmission lines. It is well known that a two-wire transmission line terminated at its ends in load impedances is susceptible to RF pickup. The response is usually small. But a two-wire transmission line approximately one-half wavelength long, short-circuited at each end, and containing a load impedance in series with one conductor at its center makes an excellent receiving antenna! This circuit is the well-known conventional two-conductor folded dipole receiving antenna. It exhibits antenna and transmission line properties simultaneously.

A typical impedance loaded strip antenna, transmission line and its complement—an impedance loaded slot antenna, transmission line—are illustrated by Figure 1. For the desired polarization of the incident electric field one solves the strip circuit for the current I_L in the impedance Z_L and then determines I_L' in Z_L' following one of the methods set forth in this paper. It should be mentioned that the strip circuit need not consist of strips of the same width. Analytical methods are available for finding I_L in Z_L with this modification in the structure. Note that strip and slot structures are duals when the strips exactly fit into the slots.



(a) Strip Antenna, Transmission Line (b) Dual of (a)

Figure 1. Impedance Loaded Strip and Slot Receiving Antenna, Transmission Lines That are Complementary

Some ten years ago the writer became interested in slot receiving antennas as related to radio frequency hazards to ordnance. In an unpublished Sandia Laboratories report[1] a slot receiving antenna is analyzed. As illustrated by Figure 2, it consists of a slot 2h meters in length and W meters in width cut in an infinite perfectly conducting plane of vanishing thickness. Load impedance Z_L' is connected to the slot terminals, i.e., across the slot at its midpoint. An expression is obtained for the load current in terms of the amplitude of an incident plane wave electric field directed parallel to the load impedance, i.e., transverse to the slot. This problem is solved several different ways in the present paper to lay the foundation for the analysis of more advanced slot receiving antenna, transmission lines.

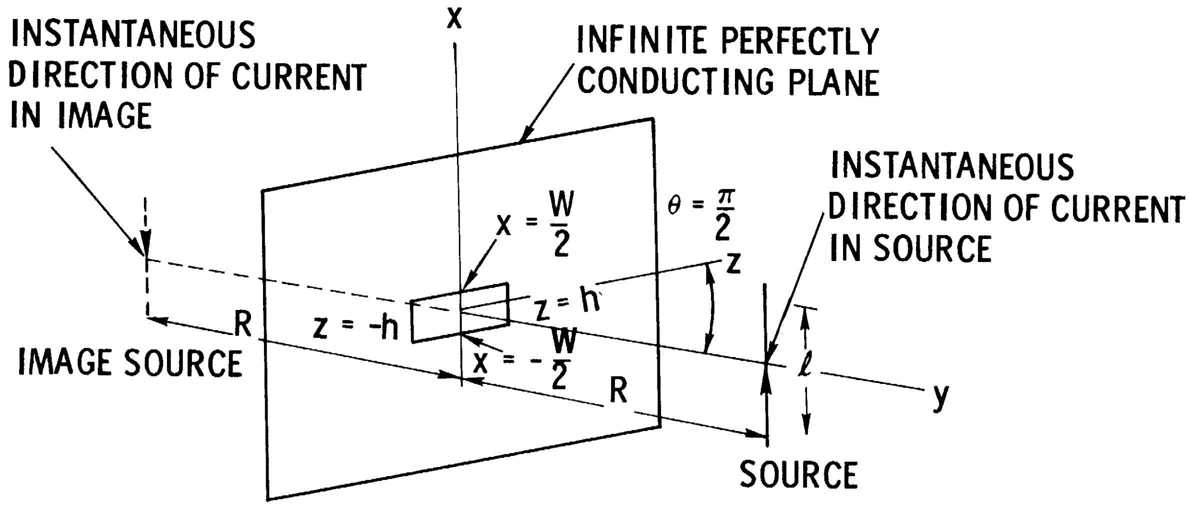


Figure 2(a). Slot Cut in an Infinite Perfectly Conducting Plane of Infinitesimal Thickness

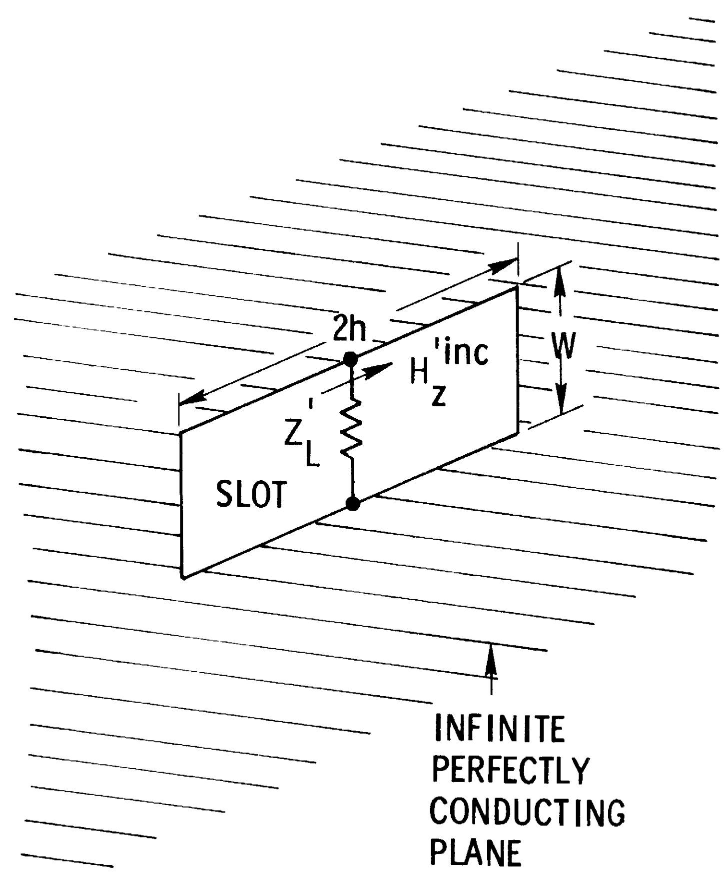


Figure 2(b). Receiving Slot Antenna With Load Z_L'

The subject of complementarity is an involved one. For a detailed treatment the reader is referred to the literature.[2] In this paper only a sufficient number of elementary details relating to complementarity are introduced to permit solution of practical problems.

From what has been said, it should be evident to the reader that he must be prepared to calculate the current in the load impedance of the strip configuration. This can be complicated, but the writer has been privileged to present the ground work for this undertaking in several issues of the IEEE Transactions on Electromagnetic Compatibility.[3-7]

Maxwell's Equations in Source-Free Regions

The field equations in free space are

$$\left.\begin{aligned} \epsilon_0 \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{E} &= -j\omega\mu_0 \vec{H} \\ \nabla \times \vec{H} &= j\omega\epsilon_0 \vec{E} \\ \mu_0 \nabla \cdot \vec{H} &= 0 \end{aligned}\right\} . \tag{1}$$

for an assumed but suppressed time dependence $\exp(j\omega t)$, where $\omega = 2\pi f$ is the radian frequency. In the rationalized mks system of units $\epsilon_0 = 8.85 \times 10^{-12}$ farads/m and $\mu_0 = 4\pi \times 10^{-7}$ henries/m. The electric and magnetic fields at an observation point are $\vec{E}$ and $\vec{H}$, respectively. The units of $\vec{E}$ are volts/m and $\vec{H}$ are amperes/m. Important combinations of the constants, ϵ_0 and μ_0, are

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \simeq 3 \times 10^8 \text{ m/sec} \tag{2}$$

and

$$\zeta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi \simeq 377 \text{ ohms.} \tag{3}$$

In theoretical discussions of impedance loaded strip and slot antenna, transmission lines it is convenient to employ the principle of complementarity. Complementarity imposes the requirement that the strip circuit just fill the slot circuit, and the electric and magnetic fields are interchanged according to

$$\vec{E}' = \zeta_0 \vec{H} \tag{4a}$$

$$\vec{H}' = -\vec{E}/\zeta_0 . \tag{4b}$$

Equally satisfactory are the interchanges $\vec{E}' = -\zeta_0 \vec{H}$ and $\vec{H}' = \vec{E}/\zeta_0$. Equations (4a) and (4b) written in Cartesian coordinates are

$$\left.\begin{aligned} E'_x &= \zeta_0 H_x \\ E'_y &= \zeta_0 H_y \\ E'_z &= \zeta_0 H_z \\ H'_x &= -E_x/\zeta_0 \\ H'_y &= -E_y/\zeta_0 \\ H'_z &= -E_z/\zeta_0 \end{aligned}\right\} . \tag{5}$$

This transformation is not to be confused with relations such as $H_\phi = E_\theta/\zeta_0$, which have no validity except in the far zone of a radiating system. Note that the subscripts in the foregoing expression differ, but in (5) the subscripts are the same for any given equation. One may select $\vec{E}$ and $\vec{H}$ to represent the fields of the strip or slot configuration. The writer has selected $\vec{E}$ and $\vec{H}$ to represent the fields of the strip circuit. Accordingly, all primed quantities apply to the slot configuration. Also, the transformation given by (4) is the one consistently applied in this paper.

If one substitutes (4) into (1), the result is

$$\left.\begin{aligned} \epsilon_0 \nabla\cdot\vec{E}' &= 0 \\ \nabla\times\vec{E}' &= -j\omega\mu_0\vec{H}' \\ \nabla\times\vec{H}' &= j\omega\epsilon_0\vec{E}' \\ \mu_0\nabla\cdot\vec{H}' &= 0 \end{aligned}\right\} . \tag{6}$$

The field equations are thus invariant to the transformation delineated by (4). The normalization constant ζ_0, defined by (3), is fixed once it has been decided to use the rationalized mks system of units. Attention is invited to the fact that a minus sign is involved in (4a) or (4b); otherwise, Maxwell's equations are violated. Some writers in this field appear to have ignored this fact.[9, 10] Another obvious but very important point is that the direction of wave propagation specified by $\vec{E} \times \vec{H}$ is the same as for $\vec{E}' \times \vec{H}'$.

The Famous Booker Formula Relating the Input Impedance and Input Admittance of Unloaded Complementary Structures

In 1946 Booker published his now-famous formula relating the input impedance of a strip dipole to the input admittance of its dual, the slot antenna.[8-10] It is

$$\frac{Z_{in}}{Y'_{in}} = \frac{\zeta_0^2}{4} \tag{7}$$

where $Y'_{in} = 1/Z'_{in}$. In this paper the primed impedances apply to the slot configuration; the unprimed, to the complementary structure (i.e., the strip circuit).

Generalization of the Booker Formula

The generalization of the Booker formula can best be effected by considering a specific circuit. Figure 3(a) portrays a strip transmission line with load Z_s. The length of the line is s and the strips are of width W, spaced a distance b apart measured center to center. The characteristic impedance of the line is

$$Z_c = \frac{\zeta_0}{\pi} \ln\left(\frac{b}{a}\right) = \frac{\zeta_0}{\pi} \ln\left(\frac{4b}{W}\right) \tag{8}$$

where $a = W/4$.[11] The input impedance is

$$Z_{in} = Z_c \left\{\frac{Z_s + j Z_c \tan \beta s}{Z_c + j Z_s \tan \beta s}\right\} . \tag{9}$$

Here, $\beta = \omega/c$ is the propagation constant. The line is considered to be dissipationless.

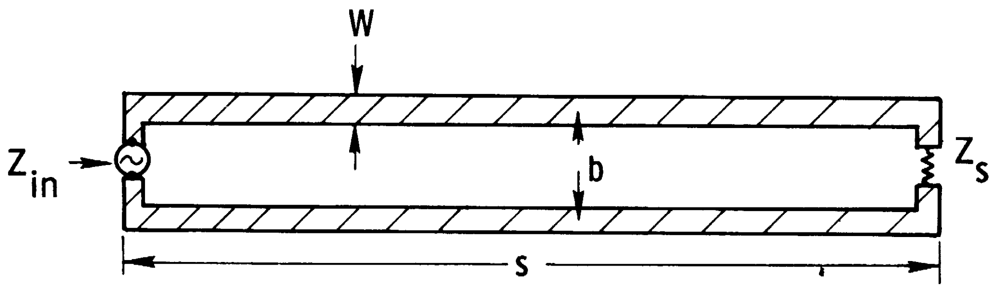


Figure 3(a). Impedance Loaded Strip Antenna, Transmission Line

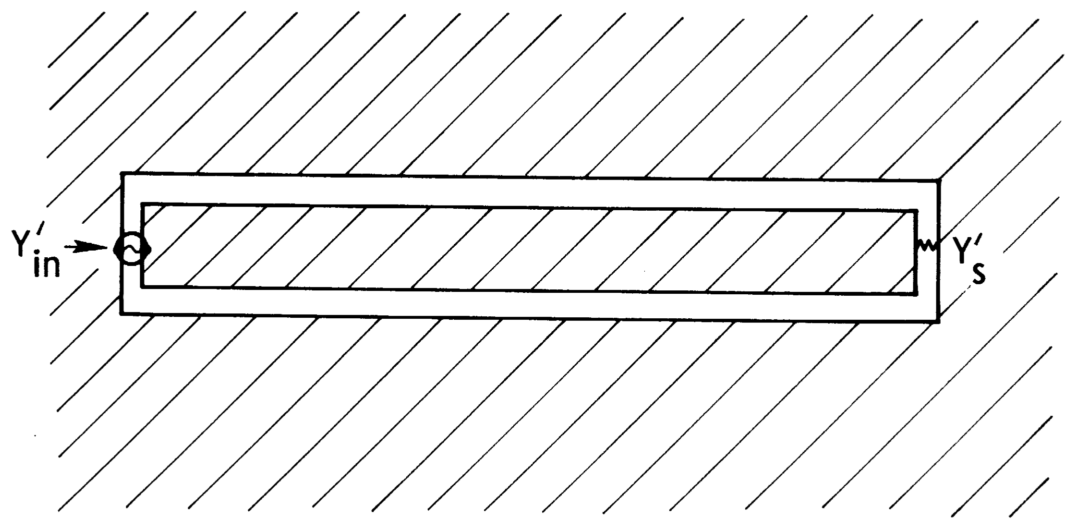


Figure 3(b). Complement of Circuit 3(a)

Assume now that the input admittance of the dual (Figure 3(b))can be obtained directly from (9) by replacing the unprimed impedances by primed admittances. Then

$$Y'_{in} = Y'_c \left\{ \frac{Y'_s + j Y'_c \tan \beta s}{Y'_c + j Y'_s \tan \beta s} \right\} . \tag{10}$$

Write

$$Y'_{in} = \frac{4Z_{in}}{\zeta_0^2} \tag{11}$$

$$Y'_c = \frac{4Z_c}{\zeta_0^2} \tag{12}$$

$$Y'_s = \frac{4Z_s}{\zeta_0^2} \tag{13}$$

paralleling (7).

Substitute (11) through (13) into (10). Voilà - (9) is obtained!

The above may appear to be a frivolous or insipid exercise, but it is a result of great importance. Suppose that one is required to analyze the slot configuration that is the dual of Figure 1, reference 6. The slot loading admittances Y'_{s1}, Y'_{s2}, etc., are specified. Then (13) may be employed to find Z_{s1}, Z_{s2}, etc., needed in analyzing the strip configuration. Equations (11) through (13) are to be regarded as scaling relations.

Elementary Slot Receiving Antenna

Consider the slot receiving antenna pictured in Figure 2. The incident plane wave electric field is directed along the positive z axis and is thus parallel to the strip. The usual practice is followed here by measuring θ from the major axis of the strip as illustrated. In the present instance $\theta = \pi/2$. The open-circuit voltage of the strip V_{oc} and the short-circuit current of the slot I'_{sc} are given by the relations

$$V_{oc} = 4h_e\left(\frac{\pi}{2}\right)E_z^{inc} \tag{14}$$

and

$$I'_{sc} = -4h_e\left(\frac{\pi}{2}\right)H'^{inc}_z , \tag{15}$$

respectively. Equations (14) and (15) are complementary relations and define the effective half-length of the strip $h_e(\pi/2)$. The quantity $h_e(\pi/2)$ is dimensionally a length, and is measured in meters. Values of $h_e(\pi/2)$ may be found in the literature for isolated cylindrical antennas.[12, 13] As mentioned earlier, the equivalent radius of the strip is $a = W/4$.[11]

It should be noted that the source and image source excite the slot because the infinite perfectly conducting plane containing the slot is an almost perfect mirror. In order for strip configurations to be true complements of the slot configurations, it is necessary to multiply the incident field exciting the strip circuit by a factor of 2. This accounts for the multiplying factor of 4 in (14), instead of the usual factor of 2. This subject is discussed more fully later in the paper.

From circuit (a) (Figure 4) it is evident that

$$I_L = \frac{4h_e\left(\frac{\pi}{2}\right)E_z^{inc}}{Z_{in} + Z_L} \tag{16}$$

where use has been made of (14). Remember that (16) applies when the strip is illuminated by two identical fields of the same amplitude and phase. From circuits (b) and (c) (Figure 4)

$$I'_{sc} = \frac{V'_{oc}}{Z'_{in}} = \frac{I'_L\left(Z'_{in} + Z'_L\right)}{Z'_{in}} . \tag{17}$$

Hence,

$$I'_L = \frac{I'_{sc}\ Z'_{in}}{Z'_{in} + Z'_L} . \tag{18}$$

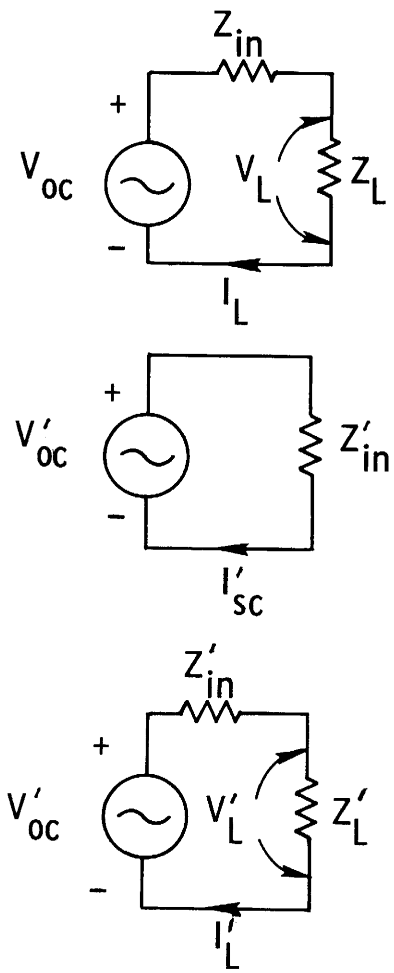
But

$$V'_L = I'_L Z'_L = \frac{I'_{sc} Z'_{in} Z'_L}{Z'_{in} + Z'_L} = \frac{I'_{sc}}{Y'_{in} + Y'_L} \tag{19}$$

or, upon inserting the value of I'_{sc} from (15),

$$V'_L = -\frac{4h_e\left(\frac{\pi}{2}\right)H'^{inc}_z}{Y'_{in} + Y'_L} . \tag{20}$$

Equations (16) and (20) are complementary. Once (16) is written, (20) follows by parallelism.



(a) Strip Circuit

(b) Slot Circuit

(c) Slot Circuit

Figure 4. Some Equivalent Circuits of Strip and Slot Receiving Antennas

Now

$$I'_L = \frac{V'_L}{Z'_L} = -\frac{4h_e\left(\frac{\pi}{2}\right)H_z'^{\,inc}}{Z'_L\left(Y'_L + Y'_{in}\right)} \,. \tag{21}$$

By using Booker's formula, $Y'_{in} = 4\,Z_{in}/\zeta_0^2$, (21) may be written

$$I'_L = \frac{h_e\left(\frac{\pi}{2}\right)H_z'^{\,inc}\,\zeta_0^2}{\frac{\zeta_0^2}{4} + Z_{in}\,Z'_L} \,. \tag{22}$$

This formula for the load current I'_L first appeared in reference 1. Inasmuch as the incident magnetic field is in the z direction, it follows that the incident electric field is parallel to Z'_L, i.e., transverse to the slot. The input impedance of the strip is Z_{in}. Formulas for this quantity appear in the literature.[14, 15]

There are various ways to solve the elementary slot receiving antenna problem, but all methods must give the same result. One may eliminate $4h_e(\pi/2)$ from (16) and (21). This yields

$$I_L H_z'^{inc}\left(Z_{in} + Z_L\right) = - I'_L Z'_L \left(Y'_{in} + Y'_L\right) E_z^{inc} , \qquad (23)$$

or

$$I'_L = - \frac{I_L H_z'^{inc}\left(Z_{in} + Z_L\right)}{Z'_L\left(Y'_{in} + Y'_L\right)E_z^{inc}} = \frac{I_L\left(Z_{in} + Z_L\right)}{Z'_L\left(Y'_{in} + Y'_L\right)\zeta_0} , \qquad (24)$$

where use has been made of (5), i.e., $H_z'^{inc} = - E_z^{inc}/\zeta_0$. From Booker's formula it is known that $Y'_{in} = 4Z_{in}/\zeta_0^2$. Also let the lumped load impedances be transformed according to

$$Y'_L = \frac{1}{Z'_L} = \frac{4Z_L}{\zeta_0^2} . \qquad (25)$$

Using these relations in (24) yields

$$I'_L = I_L \frac{\zeta_0}{4Z'_L} . \qquad (26)$$

In deriving (26), the transformations $H_z'^{inc} = - E_z^{inc}/\zeta_0$ and $Z_L Z'_L = \zeta_0^2/4$ are employed. If Z'_L is specified, one calculates Z_L for use in the complementary circuit from the latter relation. The current I_L is then determined. In calculating I_L, two sources are assumed to illuminate the strip circuit.

The Open Bomb Bay Problem When the Incident Electric Field is Parallel to the Axis of the Weapon

To solve the problem outlined in the title of this section, it is necessary to first solve the strip line problem, when illuminated by two sources, for the current I_L. The strip configuration is illustrated by Figure 5(a); its complement, by Figure 5(b).

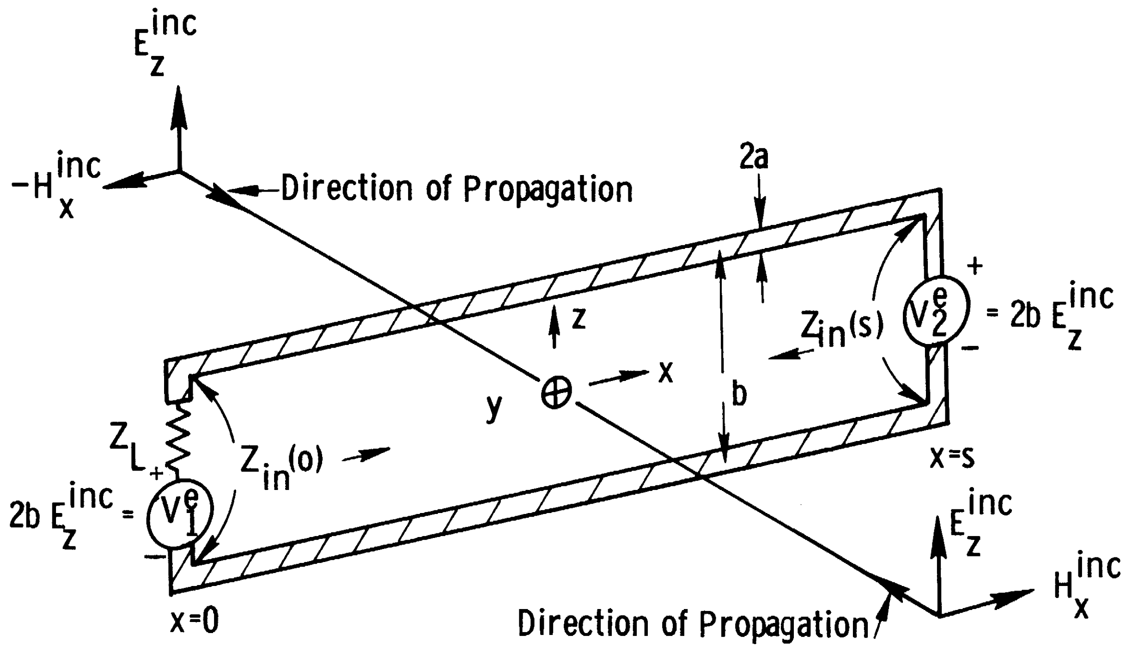


Figure 5(a). Two-Conductor Strip Line With Transverse Electric Field Excitation

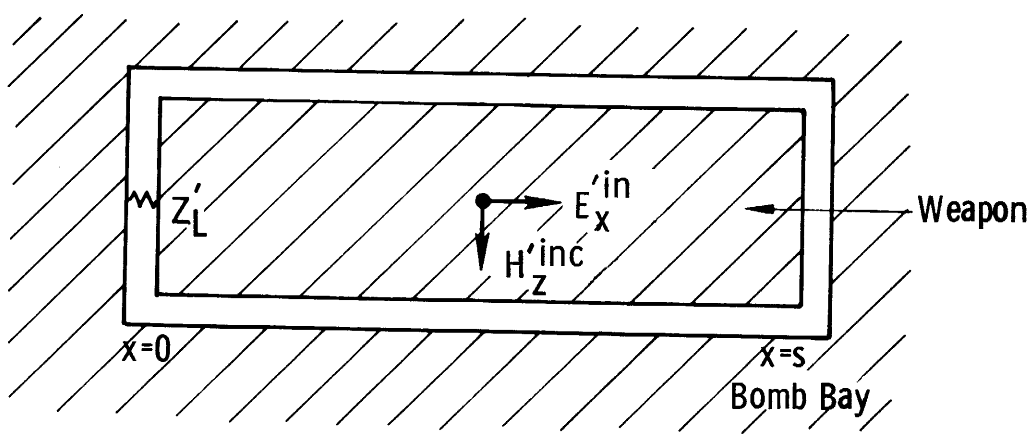


Figure 5(b). Complement of Figure 5(a)

It is clear that by the use of two sources the total magnetic field in the xoz plane of Figure 5(a) is zero. Then by (5) it follows that the tangential electric field on the infinite sheet will be zero (Figure 5(b)). This is a boundary condition that must be satisfied. Note that in Figure 5(b) the incident electric field is directed along the axis of the weapon.

The voltages V_1^e and V_2^e are equal (see Figure 5(a)) because the fields arrive at both terminations simultaneously. That is,

$$V_1^e = V_2^e = 2b\,E_z^{inc} . \tag{27}$$

Now the load current I_L is given by

$$I_L = I_{x1}(o) - I_{x2}(o) \tag{28}$$

where

$$I_{x1}(o) = \frac{V_1^e}{Z_L + Z_{in}(o)} = \frac{2b\,E_z^{inc}}{Z_L + Z_{in}(o)} \tag{29}$$

and[16]

$$I_{x2}(o) = \frac{I_{x2}(s)\,Z_c}{Z_c \cos\beta s + jZ_L \sin\beta s} . \tag{30}$$

Also,

$$I_{x2}(s) = \frac{V_2^e}{Z_{in}(s)} = \frac{2b\,E_z^{inc}}{Z_{in}(s)} . \tag{31}$$

In (29) and (31),

$$Z_{in}(o) = jZ_c \tan\beta s \tag{32}$$

and

$$Z_{in}(s) = Z_c \left\{ \frac{Z_L + jZ_c \tan\beta s}{Z_c + jZ_L \tan\beta s} \right\} . \tag{33}$$

In (30), (32), and (33), Z_c is given by (8), $\beta = \omega/c$, and the length of the strip line is s.

The foregoing equations yield

$$I_L = \frac{2b\,E_z^{inc}\left(\cos\beta s - 1\right)}{Z_L \cos\beta s + jZ_c \sin\beta s} . \tag{34}$$

One may now write the complement of (34) by referring to (16) and (20). It is

$$V'_L = \frac{2b\, H'^{inc}_z \left(1 - \cos\beta s\right)}{Y'_L \cos\beta s + jY'_c \sin\beta s} . \tag{35}$$

But

$$I'_L = V'_L Y'_L = \frac{2b\, H'^{inc}_z \left(1 - \cos\beta s\right)}{\cos\beta s + Z'_L Y'_c \sin\beta s} . \tag{36}$$

The characteristic admittance Y'_c of the slot circuit may be determined in terms of Z_c of the strip circuit by the use of (12).

Alternatively, one may substitute $Z_L = \zeta_0^2 Y'_L/4$, $Z_c = \zeta_0^2 Y'_c/4$, and $E_z^{inc} = -\zeta_0 H'^{inc}_z$ in (34) and employ (26). Thus,

$$I'_L = \frac{2b\, H'^{inc}_z \left(1 - \cos\beta s\right)\zeta_0^2}{4Z'_L \left(\frac{\zeta_0^2 Y'_L}{4} \cos\beta s + j\,\frac{\zeta_0^2 Y'_c}{4} \sin\beta s\right)} = \frac{2b\, H'^{inc}_z \left(1 - \cos\beta s\right)}{\cos\beta s + jZ'_L Y'_c \sin\beta s} , \tag{37}$$

the same result as that obtained before. This is the final formula for the current in the impedance Z'_L of Figure 5(b) when the incident electric field is directed parallel to the axis of the weapon. Note that if $\beta s = 2n\pi$ and $n = 1, 2, 3 \cdots$, then $I'_L = 0$.

The Open Bomb Bay Problem When a Component of the Incident Electric Field is Transverse to the Axis of the Weapon

Figure 6 represents the dual of the desired slot configuration. The incident electric field is now assumed to be directed parallel to the strips of length s rather than parallel to the terminations, as in Figure 5(a). However, the solution of any problem is independent of the coordinates selected. A slight embellishment of a published theory[17] gives the current I_L in Z_L of Figure 6. For two sources the result is*

$$I_L = -j \frac{4E_z^{inc} \sin\left(\frac{\beta b}{2} \sin\phi\right) \sin\beta s}{\beta\left(Z_L \cos\beta s + jZ_c \sin\beta s\right)}. \tag{38}$$

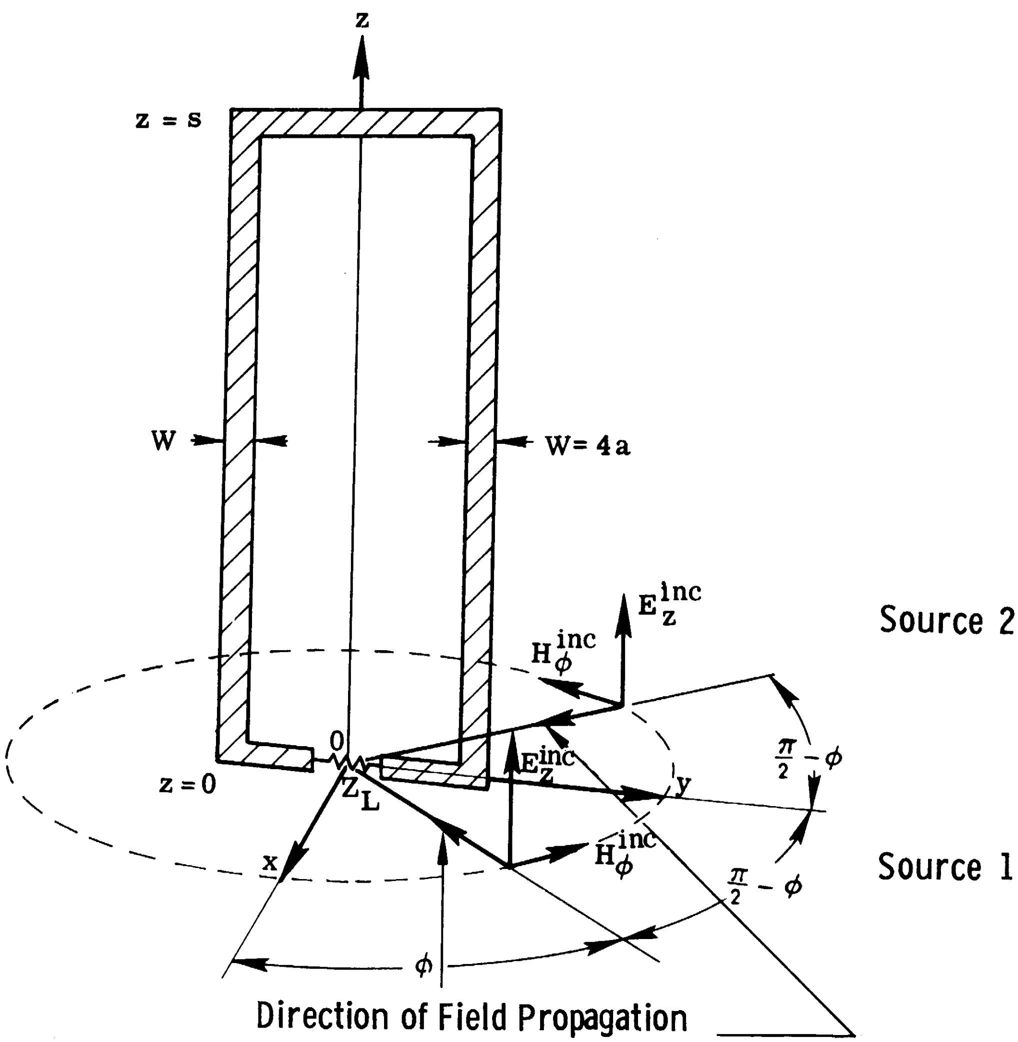


Figure 6. Impedance Loaded Strip Line With Two Field Sources

*When $\phi = 3\pi/4$ and there is one source,

$$I_L = j \frac{2E_z^{inc} \sin\left(\frac{\beta b}{2}\right) \sin\beta s}{\beta\left(Z_L \cos\beta s + jZ_c \sin\beta s\right)}.$$

This is precisely the result obtained from Eqs (28) and (30) of reference 17 for $Z_s = 0$.

Employing (26) together with (38) and the relations $Z_L = \zeta_0^2 Y_L'/4$, $Z_c = \zeta_0^2 Y_c'/4$, and $E_z^{inc} = -\zeta_0 H_z'^{inc}$, as before, yields

$$I_L' = j \frac{4H_z'^{inc} \sin\left(\frac{\beta b}{2} \sin\phi\right) \sin\beta s}{\beta\left(\cos\beta s + jZ_L' Y_c' \sin\beta s\right)}. \tag{39}$$

Note that when $\beta s = n\pi$ and $n = 1, 2, 3 \cdots$, the current $I_L' = 0$. Now $E_z^{inc} = -\zeta_0 H_z'$ and $H_\phi^{inc} = E_\phi'^{inc}/\zeta_0$. Accordingly, $E_\phi'^{inc}$ is precisely transverse to the slot circuit (the dual of Figure 6) only when $\phi = 0$ or π radians. In these cases (39) shows that $I_L' = 0$. This is not a suprising result. When $\phi = 0$ or π in (38), $I_L = 0$. When the load current is zero in the strip circuit, it is certainly zero in the complementary antenna, transmission line.

Measured and Computed Slot Impedances

The receiving slot configurations discussed in this paper are cut in an infinite perfectly conducting plane of vanishing thickness. Cavity-backed slots are not considered. Unfortunately, there is no available analytical expression for the impedance of a slot of finite dimensions cut in a hollow cylinder of finite length and radius. To circumvent this difficulty, impedance measurements were made at the Physical Sciences Laboratory, New Mexico State University, on (1) a slot 12 inches in length by 1.45 inches in width cut in the middle of a 3- by 9-foot ground plane, and (2) the same slot configuration with the ground plane rolled up into an open-ended cylinder 11.46 inches in diameter and 108 inches in length. The results of these measurements are presented in Tables I and II, respectively. Readers are reminded that accurate measurements in the area of electromagnetic fields and waves are exceedingly difficult to make and should be carried out only by experienced people.

In Table III the computed impedance of a center-driven slot 12 inches in length by 1.45 inches in width cut in an infinite perfectly conducting plane of vanishing thickness is presented. The formula used is $Z_{in}' = \zeta_0^2/4Z_{in}$, where Z_{in} is the input impedance of the strip dipole. It has been pointed out that the equivalent cylindrical radius a of a strip of width W is $a = W/4$.

It should be mentioned that it is common practice to compute the input impedance of a cavity-backed slot by using the formula $Z_{in}Z'_{in} = \zeta_0/4$. For details the reader is referred to the literature.[8]

Conclusions

The writer has presented a generalized theory for impedance loaded slot antenna, transmission lines in terms of the complementary strip configurations. It is to be emphasized that the strip circuit problem and its dual may be solved independently. Some researchers may not wish to introduce the strip configuration at all; however, inasmuch as the writer has had considerable experience solving wire circuit problems, he feels more confident of the results obtained for slot configurations by first introducing their complements.

TABLE I

Measured Impedance of a Center-Driven Slot Free to Radiate in Each Half Space Cut in a 3- by 9-Foot Flat Plate. (The long dimension of the slot is at right angles to the long dimension of the flat sheet.) Slot Dimensions: 12 Inches in Length by 1.45 Inches in Width.

βh	Z' (ohms)	|Z'| (ohms)
0.3192	5.0 + j67.5	67.69
0.3830	8.5 + j80.5	80.95
0.4469	12 + j91.0	91.79
0.5107	18 + j116	116.9
0.5745	21 + j103	105.1
0.6384	28 + j133	135.9
0.7022	39 + j148	153.0
0.7660	53 + j150	158.4
0.8300	65 + j174	185.7
0.8937	125 + j228	260.0
0.9576	228 + j240	331.0
1.021	320 + j215	385.5
1.085	388 + j125	407.6
1.117	398 + j28	399.0
1.149	388 + j15	388.2
1.213	350 + j38	352.0
1.277	302 + j65	308.9
1.341	282 + j48	286.0
1.404	245 - j13	245.3
1.468	216 - j31	218.3
1.532	202 - j31	204.4
1.596	181 - j49	187.5
1.756	105 - j61	121.3
1.915	112 - j22	117.6
2.075	111 + j25	113.8
2.234	108 + j39	114.8
2.394	136 + j0.0	136.0
2.553	160 + j36	164.0
2.713	136 + j64	150.3
2.905	178 + j78	194.3

TABLE II

Measured Impedance of a Center-Driven Slot Cut Circumferentially in the Middle of a Cylinder 11.46 Inches in Diameter and 108 Inches in Length
Slot Dimensions: 12 Inches in Length by 1.45 Inches in Width

βh	Z' (ohms)	|Z'| (ohms)
0.3192	5.5 + j64	64.24
0.3830	8.0 + j72	72.44
0.4469	11 + j81	81.74
0.5107	14 + j96	96.95
0.5745	16 + j101	101.7
0.6384	20 + j112	113.8
0.7022	25 + j97	100.2
0.7660	29 + j110	113.8
0.7980	32 + j123	127.1
0.8300	40 + j137	142.7
0.8937	62 + j161	172.5
0.9576	101 + j191	216.1
1.021	145 + j203	249.5
1.085	194 + j219	292.5
1.149	183 + j190	263.8
1.213	184 + j175	253.9
1.277	160 + j156	223.5
1.341	150 + j156	216.4
1.404	147 + j140	203.0
1.468	176 + j130	218.8
1.532	185 + j94	207.4
1.596	202 + j58	210.0
1.660	191 + j56	199.1
1.756	160 − j20	161.2
1.915	116 − j4	116.1
2.075	80 + j30	85.44
2.234	59 + j41	71.55
2.394	65 + j62	89.63
2.553	118 + j64	134.1
2.713	67 + j79	103.2
2.873	52 + j77	92.28

TABLE III

Computed Impedance of a Center-Driven Slot Free to Radiate in Each Half Space. Slot Dimensions: 12 Inches in Length by 1.45 Inches in Width

βh	Z' (ohms)	|Z'| (ohms)
0.100	0.0012 + j15.13	15.13
0.125	0.0030 + j18.91	18.91
0.150	0.0062 + j22.69	22.69
0.175	0.0115 + j26.47	26.47
0.200	0.0196 + j30.25	30.25
0.225	0.0315 + j34.04	34.04
0.250	0.0479 + j37.82	37.82
0.275	0.0702 + j41.60	41.60
0.300	0.0994 + j45.38	45.38
0.325	0.1369 + j49.16	49.16
0.350	0.1842 + j52.94	52.94
0.375	0.2427 + j56.73	56.73
0.400	0.3142 + j60.51	60.51
0.425	0.4004 + j64.29	64.29
0.450	0.5033 + j68.07	68.07
0.475	0.6248 + j71.85	71.85
0.500	1.255 + j94.58	94.60
0.700	6.531 + j149.4	149.5
0.900	28.79 + j232.4	234.1
1.100	134.5 + j366.8	390.6
1.200	293.6 + j419.1	511.7
1.300	518.9 + j307.9	603.4
1.400	551.8 + j39.47	553.2
1.500	414.9 + j101.1	427.1
1.600	297.2 − j124.1	322.1
1.700	222.3 − j108.5	247.3
1.800	175.8 − j83.65	194.7
1.900	145.9 − j58.05	157.1
2.000	125.7 − j33.34	130.1
2.100	111.6 − j9.808	112.0
2.200	101.4 + j13.19	102.2
2.300	93.67 + j36.00	100.4
2.400	87.70 + j58.97	105.7
2.500	82.86 + j82.55	117.0
2.600	78.79 + j107.2	133.0
2.700	75.28 + j132.8	152.6
2.800	72.19 + j159.9	175.4
2.900	69.69 + j188.4	200.9
3.000	68.06 + j218.7	229.1
3.100	67.95 + j250.8	259.8
3.200	69.90 + j284.4	292.9
3.300	75.48 + j320.6	329.4
3.400	86.01 + j359.0	369.1
3.500	103.3 + j399.6	327.6
3.600	130.9 + j442.8	461.7
3.700	173.5 + j488.3	518.2
3.800	238.9 + j533.1	584.2
3.900	338.5 + j566.6	660.1
4.000	482.3 + j562.7	741.1

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9. John D. Kraus, "Antennas," McGraw-Hill Book Company, Inc., 1950, pp 367-371, Section 13-5, "The Impedance of Slot Antennas."
It appears that Kraus develops the famous Booker formula for the impedance of a slot antenna in terms of the impedance of the complementary dipole by means of scalar quantities throughout, and thus avoids the introduction of a sign convention. The writer has not explored the possibility of doing the problem in this manner. The justification for (13-29) and (13-30) is not apparent. If these equations are intended to be statements of complementarity, they violate Maxwell's equations, as is demonstrated in the text of the present paper.

10. Edward C. Jordon, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., 1950, pp 587-593, Section 15.10, "Slot Antennas."
Jordon states in (15-68) and (15-69) that $\vec{E}' = k_1\vec{H}$ and $\vec{H}' = k_2\vec{E}$. From (4) of the present paper, $\vec{E}' = \zeta_0\vec{H}$ and $\vec{H}' = -\vec{E}/\zeta_0$. Evidently, $k_1 = \zeta_0$ and $k_2 = -1/\zeta_0$ so that $k_1/k_2 = -\zeta_0^2$. A compensating negative sign has to be introduced somewhere else in the theory if both Z_s and Z_d appearing in (15-72) are to have positive real parts. (See also the note under reference 9.)

11. Ronold W. P. King, "The Theory of Linear Antennas," Harvard University Press, 1956, Chapter 1, p 20, Eq 45.

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13. Ronold W. P. King, Harry R. Mimno, and Alexander H. Wing, "Transmission Lines Antennas and Wave Guides," McGraw-Hill Book Company, Inc., 1945, p 165, Eq 31.5 (provided that $\beta h < \pi/2$).

14. Reference 11, Chapter 2.

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16. Reference 13, p 19, Eq 18.22.

17. Reference 3, p 6, Eqs (28) and (30) with $Z_s = 0$.