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THE INTERIOR ELECTROSTATIC FIELD OF A SEMI-INFINITE CYLINDRICAL CAVITY IN A GROUNDED, CONDUCTING HALF-SPACE

by

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ABSTRACT

The electrostatic field inside a semi-infinite cylindrical cavity present in a grounded, conducting half-space is determined using Green's theorem. An integral equation is derived for the aperture potential by requiring continuity of the normal component of the electric field in the opening. This integral equation is solved formally by expanding the unknown aperture potential in a convenient series form. Only the leading series coefficient is required to approximate the electrostatic field in the cavity away from the opening. Approximate numerical values are given for this leading coefficient.

FOREWORD

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CONTENTS

1. INTRODUCTION

In the absence of the cylindrical cavity shown in figure 1(a), an ambient electrostatic field, E_0, is established along the z-axis. The presence of the cylindrical cavity alters the ambient charge distribution on the surface of the conducting half-space giving rise to a potential distribution in the opening. The reactive field in the region where $z > 0$ [region II in fig. 1(b)] and the field in the cavity [region I in fig. 1(b)] can be calculated from knowledge of the potential distribution in the opening at $z = 0$. The essence of the boundary-value problem, then, is the determination of the electrostatic potential in the opening.

An electrostatic field, $\overline{E}(\overline{r})$, can be represented as the gradient of a scalar potential function, $\Phi(\overline{r})$:

$$\overline{E}(\overline{r}) = \overline{\nabla}\,\Phi(\overline{r}) \tag{1}$$

where the scalar potential function satisfies Laplace's equation in the absence of free charge. In a charge-free region bounded by the surface S, the scalar potential function within the region can be obtained from Green's theorem:

$$\Phi(\overline{r}) = -\oint_S \Phi(\overline{r}')[\overline{\nabla}'\,G(r,r')]\cdot\hat{n}'\,d\overline{r}' \tag{2}$$

where $G(\overline{r},\overline{r}')$ is Green's function, the integration is over the closed surface S, and $\hat{n}'$ is an outward unit normal to S. The Green's function is the potential function due to a point charge at $\overline{r} = \overline{r}'$ within S, i.e.:

$$\nabla^2\,G(\overline{r},\overline{r}') = -\delta(\overline{r} - \overline{r}') \tag{3}$$

subject to the boundary condition that

$$G(\overline{r},\overline{r}') = 0 \text{ for } \overline{r} \text{ on S.} \tag{4}$$

The procedure for solving this electrostatic boundary-value problem is analogous to that followed by Latham and Lee[1] in their treatment of the corresponding magnetostatic problem. First, the appropriate Green's functions must be derived for regions I and II. Next, an integral equation is derived for the electrostatic potential in the opening by equating the z-components of the electrostatic field from regions I and II at the opening. Lastly, the resulting integral equation is solved by assuming a convenient series form for the unknown potential in the opening.

2. GREEN'S FUNCTIONS

Referring to the cylindrical coordinate system in figure 1(c), the Green's function in region I satisfies

$$\nabla^2\,G_I(\overline{r},\overline{r}') = -1/\rho\;\delta(z - z')\;\delta(\rho - \rho')\;\delta(\phi - \phi') \tag{5}$$

where

$$G_I(\overline{r},\overline{r}')\Big|_{\substack{\rho=a\\ z<0}} = G_I(\overline{r},\overline{r}')\Big|_{\substack{\rho<a\\ z=0}} = 0 \tag{6}$$

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[1] Latham, R.W., and Lee, K.S.H., "A Study of Some Factors Affecting the Interior Fields of a Semi-Infinite Pipe Exposed to a Low-Frequency Magnetic Field," EMP Interaction Note X, Air Force Weapons Laboratory, Kirtland AFB, New Mexico, August 1967.

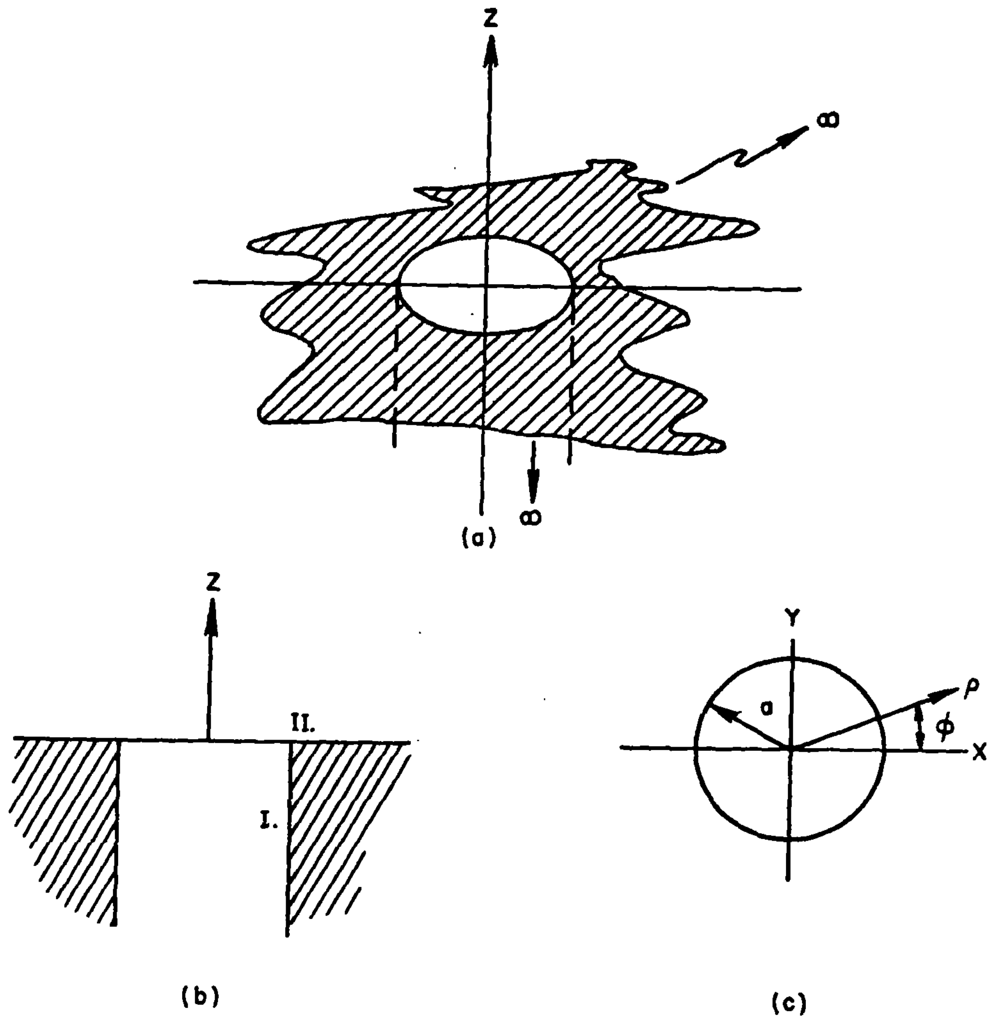

Figure 1. Semi-infinite cylindrical cavity in a grounded conducting half-space.

A Fourier-Bessel series can be constructed to represent the product of radial and azimuthal Dirac delta functions appearing on the right-hand side of equation (5):

$$-\frac{1}{\rho}\,\delta(\rho-\rho')\,\delta(\phi-\phi') = -\sum_{\substack{r=1\\n=0}}^{\infty} A_{nr}\,J_n(\mu_{nr}\rho)\cos n(\phi-\phi')$$

where $\mu_{nr}a$ is the argument which yields the r-*th* zero of the n-*th* order Bessel function. Multiplying the last result by $\rho J_m(\mu_{mr}\rho)\cos m\phi$, integrating over ϕ from 0 to 2π and over ρ from 0 to a, yields

$$A_{nr} = \frac{\epsilon_n J_n(\mu_{nr}\rho')}{\pi a^2[J_n'(\mu_{nr}a)]^2} \tag{7}$$

where the prime on the Bessel function in the denominator indicates differentiation with respect to the argument and

$$\epsilon_n = \begin{cases}1, & n=0\\ 2, & n=1,2,3\ldots\end{cases}$$

Consequently, equation (5) can be replaced by

$$\nabla^2\,G_I(\overline{r},\overline{r}') = -\delta(z-z')\sum_{\substack{r=1\\n=0}}^{\infty} A_{nr}J_n(\mu_{nr}\rho)\cos n(\phi-\phi') \tag{8}$$

where A_{nr} is given in equation (7). Continuing, the Green's function can also be expanded in a Fourier-Bessel series; for example

$$G_I(\overline{r},\overline{r}') = \sum_{\substack{r=1\\n=0}}^{\infty} J_n(\mu_{nr}\rho)\cos n(\phi-\phi')\,f_{nr}(z) \tag{9}$$

where $f_{nr}(z)$ is an unknown function to be determined. Employing equation (9) in equation (8) leads to

$$\frac{d^2 f_{nr}(z)}{dz^2} - \mu_{nr}^2 f_{nr}(z) = -A_{nr}\,\delta(z-z') \tag{10}$$

which is solvable using conventional methods of ordinary differential equations.

From the boundary conditions that

$$\lim_{z\to 0} f_{nr}(z) \to 0$$

and

$$\lim_{z\to\infty} f_{nr}(z) \to 0$$

the solution of equation (10) becomes

$$f_{nr}(z) = -\frac{A_{nr}}{\mu_{nr}}\sinh\mu_{nr}z\int_{-\infty}^{z}\exp(\mu_{nr}s)\,\delta(s-z')ds$$

$$-\frac{A_{nr}}{\mu_{nr}}\exp(\mu_{nr}z)\int_{z}^{0}\sinh\mu_{nr}s\,\delta(s-z')ds \tag{11}$$

which, for the situation of interest, reduces to

$$z < z' \leq 0, \; f_{nr}(z) = -(A_{nr}/\mu_{nr}) \exp(\mu_{nr}z) \sinh \mu_{nr}z' \qquad (12)$$

Therefore, Green's function in region I is

$$G_I(\overline{r},\overline{r}') = -\sum_{\substack{r=1\\n=0}}^{\infty} \frac{\varepsilon_n \cos n(\phi - \phi')J_n(\mu_{nr}\rho)J_n(\mu_{nr}\rho') \exp(\mu_{nr}z)\sinh \mu_{nr}z'}{a^2\pi\mu_{nr}[J_n'(\mu_{nr}a)]^2} \qquad (13)$$

The Green's function in region II is a solution of

$$\nabla^2 G_{II}(\overline{r},\overline{r}') = -\delta(x - x')\delta(y - y')\delta(z - z') \qquad (14)$$

subject to the boundary condition

$$G_{II}(\overline{r},\overline{r}')\Big|_{z=0} = 0 \qquad (15)$$

It is not necessary to solve equation (14) directly since the potential of a point charge above a grounded conducting plane is well known. From image theory, for example, Green's function in region II is

$$G_{II}(\overline{r},\overline{r}') = \frac{1}{4\pi}\left\{\left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{-1/2} - \left[(x - x')^2 + (y - y')^2 + (z + z')^2\right]^{-1/2}\right\} \qquad (16)$$

3. THE INTEGRAL EQUATION FOR THE APERTURE POTENTIAL

The potential in region I follows directly from equation (2):

$$\Phi_I(\overline{r}) = -\oint_{S_I} \Phi_I(\overline{r}')\,[\overline{\nabla}'G_I(\overline{r},\overline{r}')]\cdot\hat{n}'\,d\overline{r}'$$

The surface S_I is the surface of the semi-infinite cylindrical cavity but since the lateral portion is a grounded conductor, the surface integral has a nonzero value only over the opening. Noting the azimuthal symmetry of the problem, then, leads to

$$\Phi_I(\rho,z) = -\int_0^a \int_0^{2\pi} \Phi_I(\rho',0)\, \frac{\partial G_I(\overline{r},\overline{r}')}{\partial z'}\Big|_{z'=0} \rho'd\rho'd\phi' \qquad (17)$$

The potential in region II is made up of an ambient potential, $\Phi_0(\overline{r})$, and a reactive potential due to the presence of the cavity:

$$\Phi_{II}(\overline{r}) = \Phi_0(\overline{r}) - \oint_{S_{II}} \Phi_{II}(\overline{r}')\,[\overline{\nabla}'G_{II}(\overline{r},\overline{r}')]\cdot\hat{n}'d\overline{r}'$$

where

$$\Phi_0(\overline{r}) = E_0 z \qquad (18)$$

The surface S_{II} is the grounded conducting plane at $z = 0$ such that the integral has a nonzero value, again, only over the opening:

$$\Phi_{II}(\rho,z) = E_0 z + \int_0^a \int_0^{2\pi} \Phi_{II}(\rho',0)\, \frac{\partial G_{II}(\overline{r},\overline{r}')}{\partial z'}\Big|_{z'=0} \rho'd\rho'd\phi' \qquad (19)$$

Since the potential in the aperture must vanish at $\rho' = a$, $\Phi_I(\rho',0)$ and $\Phi_{II}(\rho',0)$ must be equal and cannot differ by some constant value. So, for convenience, we define

$$\Phi_I(\rho',0) = \Phi_{II}(\rho',0) = E_0 f(\rho') \qquad (20)$$

Enforcing the requirement that the z-component of the electric field be continuous in the aperture yields

$$\int_0^{2\pi}\int_0^{a} f(\rho') \sum_{\substack{r=1\\n=0}}^{\infty} \frac{\varepsilon_n \mu_{nr} \cos n(\phi - \phi') J_n(\mu_{nr}\rho) J_n(\mu_{nr}\rho')}{a^2\pi[J_n'(\mu_{nr}a)]^2} \rho' d\rho' d\phi' \tag{21}$$

$$= 1 + \int_0^{2\pi}\int_0^{a} f(\rho') \frac{1}{2\pi[\rho'^2 + \rho^2 - 2\rho\rho' \cos(\phi - \phi')]^{3/2}} \rho' d\rho' d\phi'$$

for $\rho \leq a$ which is an integral equation for the aperture potential. This integral equation can be put in a more convenient form by using

$$\rho = ax, \text{ and } \rho' = ax' \tag{22}$$

$$\zeta_{nr} = \mu_{nr} a \tag{23}$$

and

$$\theta = \phi' - \phi \tag{24}$$

Equation (21) now becomes

$$\int_{-\phi}^{2\pi-\phi}\int_0^{1} f(x') \sum_{\substack{r=1\\n=0}}^{\infty} \frac{\varepsilon_n \zeta_{nr} \cos n\theta \; J_n(\zeta_{nr}x) J_n(\zeta_{nr}x')}{a\pi[J_n'(\zeta_{nr})]^2} x'dx'd\theta \tag{25}$$

$$= 1 + \int_{-\phi}^{2\pi-\phi}\int_0^{1} f(x') \frac{1}{2\pi a(x'^2 + x^2 - 2xx' \cos\theta)^{3/2}} x'dx'd\theta \text{ for } 0 \leq x \leq 1$$

4. FORMAL SOLUTION OF THE INTEGRAL EQUATION

The first step in solving equation (25) for the aperture potential is to perform the integrations over θ. The integration over θ appearing on the left-hand side of equation (25) is trivial:

$$\int_{-\phi}^{2\pi-\phi} \cos n\theta \, d\theta = 2\pi \cos n\phi \; \delta_{n0}$$

where δ_{n0} is a Kroneker delta which is given by

$$\delta_{n0} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

The integration over θ on the right-hand side of equation (25) can be achieved by noting[2]

$$\frac{1}{R} = (P^2 + Z^2)^{-\frac{1}{2}} = \int_0^{\infty} e^{\pm kZ} J_0(kP)dk$$

where the plus (minus) sign in the exponent is for $Z < 0$ ($Z > 0$). The last result can be differentiated twice with respect to Z and evaluated at Z =0 in order to obtain

$$\frac{1}{P^3} = -\int_0^{\infty} k^2 J_0(kP)dk$$

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[2] Panofsky, W.K.H., and Phillips, M., Classical Electricity and Magnetism, Addison-Wesley, Reading, Massachusetts, 1962, p. 89.

where we choose

$$P = (x^2 + x'^2 - 2xx' \cos\theta)^{1/2}$$

Hence, expanding $J_0(kP)$ via the addition theorem for Bessel functions and integrating over θ provides

$$\frac{1}{2\pi a}\int_{-\phi}^{2\pi-\phi}\frac{d\theta}{(x^2 + x'^2 - 2xx'\cos\theta)^{3/2}} = -\frac{1}{a}\int_0^\infty k^2 J_0(kx)J_0(kx')dk$$

The integral equation for the aperture potential, equation (25), becomes

$$\frac{2}{a}\int_0^1 f(x')\sum_{r=1}^{\infty}\frac{\zeta_r J_0(\zeta_r x)J_0(\zeta_r x')}{[J_0'(\zeta_r)]^2}x'dx'$$

$$= 1 - \frac{1}{a}\int_0^1 f(x')\left[\int_0^\infty k^2 J_0(kx)J_0(kx')dk\right]x'dx' \quad (26)$$

for $0 \leq x \leq 1$ wherein

$$\zeta_r = \zeta_{0r} \quad (27)$$

The following form is assumed for the aperture potential:

$$f(x') = \sum_{s=1}^{\infty} C_s J_0(\zeta_s x'),\; 0 \leq x' \leq 1 \quad (28)$$

where C_s must be determined from the integral equation. The potential in the opening given in equation (28) does indeed vanish at the edge where $x' = 1$. Next, we employ equation (28) in equation (26), multiply by $xJ_0(\zeta_t x)$ and integrate over x from 0 to 1 to obtain

$$\frac{2}{a}\int_0^1\int_0^1 xx'\sum_{s=1}^{\infty}C_s J_0(\zeta_s x')J_0(\zeta_t x)\sum_{r=1}^{\infty}\frac{\zeta_r J_0(\zeta_r x)J_0(\zeta_r x')}{[J_0'(\zeta_r)]^2}dx'dx$$

$$= \int_0^1 xJ_0(\zeta_t x)\,dx$$

$$-\frac{1}{a}\int_0^1 xJ_0(\zeta_t x)\left\{\int_0^1 x'\sum_{s=1}^{\infty}C_s J_0(\zeta_s x')\left[\int_0^\infty k^2 J_0(kx)J_0(kx')dk\right]dx'\right\}dx$$

which can be written as

$$\frac{2}{a}\sum_{\substack{r=1\\s=1}}^{\infty}\frac{C_s\zeta_r}{[J_0'(\zeta_r)]^2}\left[\int_0^1 x'J_0(\zeta_s x')J_0(\zeta_r x')dx'\right]\left[\int_0^1 xJ_0(\zeta_r x)J_0(\zeta_t x)dx\right]$$

$$= \int_0^1 xJ_0(\zeta_t x)dx$$

$$-\frac{1}{a}\sum_{s=1}^{\infty}C_s\int_0^\infty\frac{1}{k^2}\left[\int_0^k y'J_0(u_s y')J_0(y')dy'\right]\left[\int_0^k yJ_0(u_t y)J_0(y)dy\right]dk \quad (29)$$

where

$$y = kx \text{ and } y' = kx' \tag{30}$$

with

$$u_x = \zeta_s/k \text{ and } u_t = \zeta_t/k \tag{31}$$

Several of the required integrations in equation (29) are given in tables of integrals:[3]

$$\int_0^1 xJ_0(\zeta_m x)J_0(\zeta_n x)dx = \frac{1}{2}\,[J_0'(\zeta_n)]^2\ \delta_{mn}$$

$$\int_0^1 xJ_0(\zeta_m x)dx = \frac{J_1(\zeta_m)}{\zeta_m}$$

$$\int_0^k vJ_0(u_m v)J_0(v)dv = \frac{k}{u_m^2 - 1}\,u_m J_1(u_m k)J_0(k)$$

where m and n are integers and all other parameters are as defined previously. Using these results for the integrals appearing in equation (29) and performing some algebraic manipulation yields

$$\frac{\zeta_t}{2a}\,[J_0'(\zeta_t)]^4\,C_t = \frac{J_1(\zeta_t)}{\zeta_t} - \frac{C_t}{a}\,\zeta_t^2\,[J_1(\zeta_t)]^2 \int_0^\infty \frac{k^2[J_0(k)]^2}{(\zeta_t^2 - k^2)^2}\,dk$$
$$- \sum_{\substack{s=1\\ s\neq t}}^{\infty} \frac{C_s}{a}\,\zeta_s\,\zeta_t\,J_1(\zeta_s)J_1(\zeta_t) \int_0^\infty \frac{k^2[J_0(k)]^2}{(\zeta_t^2 - k^2)(\zeta_s^2 - k^2)}\,dk \tag{32}$$

Finally, equation (32) can be expressed in a more succinct form by introducing

$$D_s = C_s\zeta_s J_1(\zeta_s)/a \tag{33}$$

and

$$M_{ts} = \int_0^\infty \frac{k^2[J_0(k)]^2}{(\zeta_t^2 - k^2)(\zeta_s^2 - k^2)}\,dk + \frac{[J_0'(\zeta_s)]^2}{2\zeta_s}\,\delta_{st} \tag{34}$$

where δ_{st} is a Kroneker delta, so that

$$\sum_{s=1}^{\infty} M_{ts}D_s = \frac{1}{\zeta_t^2},\quad t = 1,2,3,\ldots \tag{35}$$

This infinite system of algebraic equations can be formally stated in matrix form as

$$\underline{M}\ \underline{D} = \underline{R} \tag{36}$$

where

$$\underline{M} = (M_{ts}) \qquad t,s = 1,2,3,\ldots \tag{37}$$

$$\underline{D} = (D_s) \qquad s = 1,2,3,\ldots \tag{38}$$

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[3] Abramowitz, M., and Stegun, I., Handbook of Mathematical Functions, Dover, New York, 1965, pp. 484-485.

and

$$\underline{R} = (1/\zeta_t^2) \qquad t = 1,2,3,\ldots \tag{39}$$

If the determinant of the matrix $\underline{M}$ is nonzero, the formal solution of the matrix equation, equation (36), is

$$\underline{D} = \underline{M}^{-1}\,\underline{R} \tag{40}$$

where $\underline{M}^{-1}$ is the inverse of the matrix $\underline{M}$.

5. SUCCESSIVE APPROXIMATIONS FOR THE FIELD IN THE CAVITY

Only very special matrices of infinite order can be inverted rigorously. A practical approach that is commonly employed is to truncate arbitrarily a matrix of infinite order and invert the truncated matrix numerically or analytically using standard techniques. Higher-order approximations are achieved by inverting truntcated matrices of larger orders. Following this approach, the algebraic system in equation (35) can be approximated by the following finite system of algebraic equations:

$$\sum_{s=1}^{L} M_{ts}\, D_s^{(L)} = \frac{1}{\zeta_t^2},\ t = 1,2,\ldots,L \tag{41}$$

where $D_s^{(L)}$ is an approximation to D_s for $s = 1,2,\ldots,L$, L being some integer greater than or equal to one. Furthermore, the implicit assumption is made that if

$$|D_s^{(L+1)} - D_s^{(L)}| << |D_s^{(L+1)}|$$

then

$$|D_s - D_s^{(L+1)}| << |D_s|$$

The potential in region I can be specified by using the expansion for the aperture potential, equation (28), in equation (17) and performing the necessary integrations. The potential in region I is, thus,

$$\Phi_I(\rho,z) = E_0 \sum_{r=1}^{\infty} C_r\, J_0(\zeta_r\rho/a)\,\exp(\zeta_r z/a) \tag{42}$$

The potential away from the opening can be approximated as

$$\Phi_I(\rho,z)\Big|_{|z|>a} \rightarrow E_0C_1J_0(\zeta_1\rho/a)\,\exp(\zeta_1 z/a) \tag{43}$$

such that

$$\mathbf{E}^{z<0}(\rho,z)\Big|_{|z|>a} \rightarrow \hat{e}_\rho[-E_0(C_1/a)\zeta_1J_1(\zeta_1\rho/a)\,\exp(\zeta_1 z/a)]$$

$$+ \hat{e}_z[E_0(C_1/a)\zeta_1\, J_0(\zeta_1\rho/a)\,\exp(\zeta_1 z/a)] \tag{44}$$

Consequently, only the leading expansion coefficient, C_1, is necessary to adequately approximate the electrostatic field in the cavity away from the opening.

In order to obtain approximate numerical values for C_1, the integral that appears in the definition of M_{ts}, equation (34), must be evaluated. This integral is treated in the appendix of this memorandum. The following numerical values are readily obtained:

$$C_1^{(1)} = 0.5872a$$
$$C_1^{(2)} = 0.6024a \tag{45}$$
$$C_1^{(3)} = 0.6070a$$

where

$$D_s^{(L)} = C_s^{(L)} \zeta_s J_1(\zeta_s)/a \tag{46}$$

6. CONCLUSION

The electrostatic field in the cavity can be calculated more accurately at any position in the cavity by solving large systems of algebraic equations. Presumably, any desired degree of accuracy could be achieved with the aid of a high-speed digital computer. Also, the electrostatic field in region II can be readily specified from equation (19). Lastly, the findings of this study should be valid for time-varying electric fields when the wavelengths of the ambient field are much greater than the cavity diameter--i.e., in the so-called quasistatic approximation.

APPENDIX

In this appendix the integral appearing in equation (34) is evaluated for several values of the parameters that appear in its integrand. This integral is denoted, here, as

$$i(\zeta_s,\zeta_t) = \int_0^\infty \frac{k^2[J_0(k)]^2}{(\zeta_t^2 - k^2)(\zeta_s^2 - k^2)}\,dk \qquad \text{(A-1)}$$

Latham and Lee have evaluated the integral denoted by

$$I(m,n,i\zeta_t) = \int_0^\infty \frac{x^2 J_m(x) J_n(x)}{(x^2 - \zeta_t^2)^2}\,dx \qquad \text{(A-2)}$$

and found that

$$I(m,n,i\zeta_t) = \frac{\pi}{4}[-1]^{(m+n)/2}\left\{\sum_{p=0}^{\infty} \frac{(i\zeta_t/2)^{m+n+2p-1}[p+(m+n+1)/2]\ \Gamma(2p+m+n+1)}{\Gamma(p+1)\ \Gamma(p+m+1)\ \Gamma(p+n+1)\ \Gamma(p+m+n+1)}\right\}$$

$$-\frac{\pi}{4}[-1]^{(m+n)/2}\left\{\sum_{q=\frac{-(m+n)}{2}}^{\infty} \frac{(i\zeta_t/2)^{m+n+2q}[q+(m+n)/2+1]\ \Gamma(2q+m+n+2)}{\Gamma(q+3/2)\ \Gamma(q+m+3/2)\ \Gamma(q+n+3/2)\ \Gamma(q+m+n+3/2)}\right\} \qquad \text{(A-3)}$$

Now, there is a relationship between the integrals defined in equations (A-1) and (A-2), viz.

$$i(\zeta_t,\zeta_t) = I(0,0,i\zeta_t)$$

such that

$$i(\zeta_t,\zeta_t) = \frac{\pi}{4}\,\frac{2}{i\zeta_t}\sum_{p=0}^{\infty} \frac{(i\zeta_t/2)^{2p}\ (p + 1/2)\ \Gamma(2p + 1)}{[\Gamma(p + 1)]^4}$$

$$-\frac{\pi}{4}\sum_{q=0}^{\infty} (-1)^q\,\frac{(\zeta_t/2)^{2q}\ (q + 1)\ \Gamma(2q + 1)}{[\Gamma(q + 3/2)]^4}$$

The last result can be simplified by using the identity

$$[J_0(z)]^2 = \sum_{p=0}^{\infty} \frac{(-1)^p\ \Gamma(2p + 1)}{[\Gamma(p + 1)]^4}\left(\frac{z}{2}\right)^{2p}$$

and its derivative with respect to z, i.e.

$$-zJ_0(z)J_1(z) = \sum_{p=0}^{\infty} \frac{(-1)^p\ \Gamma(2p + 1)p}{[\Gamma(p + 1)]^4}\left(\frac{z}{2}\right)^{2p}$$

such that

$$-\zeta_t J_0(\zeta_t)J_1(\zeta_t) + \frac{[J_0(\zeta_t)]^2}{2} = \sum_{p=0}^{\infty} \frac{(i\zeta_t/2)^{2p}(p + 1/2)\ \Gamma(2p + 1)}{[\Gamma(p + 1)]^4} = 0$$

since $J_0(\zeta_t) = 0$. Therefore

$$i(\zeta_t,\zeta_t) = \sum_{p=0}^{\infty} B_p\ \zeta_t^{2p} \qquad \text{(A-4)}$$

where

$$B_p = \frac{(-1)^{p+1} 2^{6p-2} (p + 1)(p!)^4 (2p + 1)}{\pi (p + 1/2)^4 [(2p)!]^3} \quad \text{(A-5)}$$

Numerical evaluation of equation (A-4) on a digital computer provides:

t	ζ_t	$i(\zeta_t, \zeta_t)$
1	2.4048	0.1798
2	5.5201	0.0853
3	8.6537	0.0556
4	11.7915	0.0356

When ζ_t and ζ_s are distinct, equation (A-1) can be written as

$$i(\zeta_s, \zeta_t) = \frac{1}{(\zeta_t^2 - \zeta_s^2)} [F(\zeta_s) - F(\zeta_t)] \quad \text{(A-6)}$$

where

$$F(\zeta_n) = \int_0^\infty \frac{\zeta_n^2 [J_0(k)]^2}{\zeta_n^2 - k^2} \, dk \quad \text{(A-7)}$$

The function defined in equation (A-7) can be expressed, however, as

$$F(\zeta_n) = -2 \int_0^{\xi_n} \zeta_n' i(\zeta_n', \zeta_n') d\zeta_n' \quad \text{(A-8)}$$

and, therefore

$$F(\zeta_n) = -2 \sum_{p=0}^{\infty} B_p \frac{(\zeta_n)^{2p+2}}{2p + 2} \quad \text{(A-9)}$$

Hence, numerical evaluation of equation (A-6) on a digital computer provides

s	t	$i(\zeta_s, \zeta_t)$
1	2	-0.0109
1	3	-0.0060
2	3	-0.0032

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY U.S. Army Safeguard System Command Huntsville, Alabama 35807

13. ABSTRACT

The electrostatic field inside a semi-infinite cylindrical cavity present in a grounded, conducting half-space is determined using Green's theorem. An integral equation is derived for the aperture potential by requiring continuity of the normal component of the electric field in the opening. This integral equation is solved formally by expanding the unknown aperture potential in a convenient series form. Only the leading series coefficient is required to approximate the electrostatic field in the cavity away from the opening. Approximate numerical values are given for this leading coefficient.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Electrostatic Green's functions	8	3				
Integral equations	8	3				
Semi-infinite cylindrical cavity	8	3				