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COMPUTER MODELS FOR ANTENNAS*

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ABSTRACT

An overview of wire-antenna computer modeling is given, with emphasis on the interface problem. The formulation and numerical-solution methods are summarized, and applications are demonstrated with numerous examples.

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1. INTRODUCTION

Numerical methods based on integral-equation formulations are receiving increasing acceptance for application to real-life electromagnetic radiation and scattering problems. Computer codes have been developed and validated for both surface and wire geometries in both the frequency and time domains for modeling infinite, homogeneous medium problems. Some of these basic procedures have also been extended to the analysis of structures located near a planar interface. In this presentation we will discuss the general topic of computer models for wire antennas from a frequency-domain viewpoint with emphasis directed to antennas located near the ground-air interface. Some preliminary considerations are discussed in Section 2, followed by a brief summary of a specific formulation and numerical treatment in Section 3, with sample numerical results given in Section 4.

2. PRELIMINARY CONSIDERATIONS

The derivation of an integral equation for a wire structure can be accomplished in many ways. What is basically involved is the writing of Maxwell's equations in integral form so that the scattered or secondary fields are given in terms of integrals over induced source distributions. By expressing the secondary field over loci of points where the behavior of the total field (incident or primary plus secondary) is known via boundary or continuity conditions, an integral equation for the induced source is obtained in terms of the primary field. Two broad general classes of integral equations are obtained, depending upon whether the forcing function (primary field) is electric or magnetic. An electric forcing function gives rise to a Fredholm integral equation of the first kind, so called because the unknown appears only under the integral. A magnetic forcing field gives rise to a Fredholm integral equation of the second kind, in which the unknown also appears outside the integral. While derivatives of the unknown may occur as well, these equations are commonly called integral equations, rather than integro-differential equations as would be strictly correct.

Generally speaking, it has been found that the magnetic-field type of integral equation is better suited for smooth, closed surfaces than it is for thin-plate or shell geometries and wires (Poggio and Miller, 1973). The converse is generally true of the electric-field type of integral equation. It is the latter then that is most commonly employed for treating wire structures. Also involved in developing wire integral equations are the approximations that (1) the circumferential current is negligible, (2) the circumferential variation of the longitudinal current can be ignored; and (3) the thin-wire or reduced kernel can be used in place of the actual surface integration.

Many analytically equivalent integral equations for wires based upon the electric field can be derived. Three of the most commonly employed are the Hallen or vector potential type (Mei, 1965), the scalar-vector potential version (Harrington, 1968), and the Pocklington integral equation (Richmond, 1965). All are solved within the framework of the moment (or matrix) method but each exhibits distinctive characteristics which must be taken into account in its numerical treatment. The Hallen equation, for example, can produce results using a pulse-current basis of accuracy comparable to those obtained from the Pocklington equation solved with a three-term (constant, sine, and cosine) basis for simple structures (Miller and Deadrick, 1973a). The Hallen equation is not, however, readily extendable to the complex geometries that the Pocklington equation can handle (Butler, 1972).

Although pulse-current (Richmond, 1965; Curtis, 1972) and linear-current (Chao and Strait, 1970) bases have been quite widely used, and can under suitable circumstances be essentially equivalent, they are not as efficient for modeling traveling wave structures, regardless of the integral equation employed, as are sinusoidal bases which possess non-constant derivatives and which can closely resemble the actual current solution. Sinusoidal bases have appeared in sub-sectional or subdomain form in both the three-term expansion mentioned above and in the piecewise sinusoidal (Richmond, 1969) or two-term form. Fourier series have also been studied as complete-domain sinusoidal bases, but have not been widely adopted because they require more integration effort than subsectional bases and can lead to ill-conditioned matrices (Richmond, 1965).

The weight or test functions most often used have been delta functions, although Galerkin's method with both linear (two-term) (Chao and Strait, 1970) and sinusoidal (two-term) (Richmond, 1969) functions has also been quite widely applied. The term "point matching" refers to the use of delta-function weights. A comparison of numerical convergence rates for several common methods applied to a straight-wire scatterer is shown in Fig. 1 (Miller et al., 1974).

In addition to the problem of choosing basis and weight functions, there are other special aspects of the numerical development which must be considered when selecting a code for computer modeling. Three of these aspects are discussed below.

2.1 JUNCTION TREATMENT

Any subsectional approach which employs finite-difference operators in the integral equation or other than a pulse-current basis necessitates special consideration of both simple (two wires) and multiple (three or more wires) junctions. What is essentially required is a way to relate in some physically and mathematically reasonable way the current basis of each subsection (segment) to those of its neighbors. When pulse bases are used in the scalar-vector potential integral equation, the finite-difference operator spans two segments and thus leads to a charge which involves the two corresponding pulse-current samples (Harrington, 1968). For two- or three-term bases, the condition of current amplitude and slope continuity at each simple junction leads to equations which permit all the constants in the current expansion to be given in terms of current samples at the segment junctions or centers (Miller and Deadrick, 1973a). A slightly different handling of the three-term basis was developed by Yeh

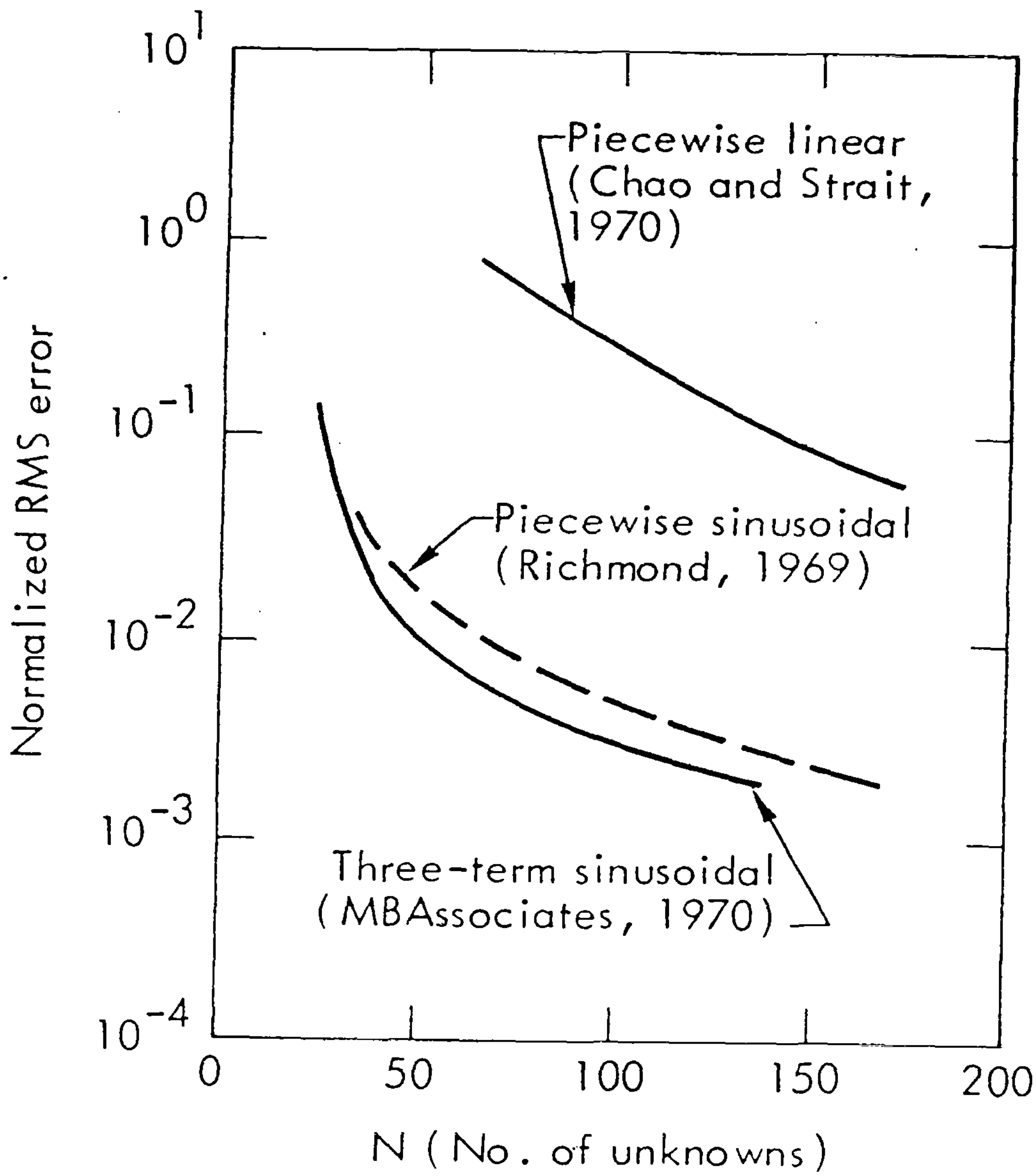

Figure 1. Convergence rate for several solution methods (after Miller et al., 1973).

and Mei (1967), in which the current is extrapolated from a given segment to the adjacent segment centers, but which is otherwise basically the same.

When a multiple junction is concerned, the treatment can get considerably more involved. The pulse-basis approach mentioned above has been extended to the multiple junction (Curtis, 1972) by dividing the total junction charge between the junction segments according to the ratio of their individual areas to the total area. This approach has been found useful for the three-term expansion as well (Miller and Deadrick, 1973a). The two-term expansions have been applied to multiple junctions by overlapping M - 1 of the bases a pair at a time at an M-segment junction (Chao and Strait, 1970). Application of the three-term expansion to the multiple junction has been accomplished by MBAssociates using the Yeh and Mei simple-junction procedure by incorporating a composite segment having the averaged length and total current of the M - 1 connected segments (Gee et al., 1971). A more elaborate multiple-junction approach has been developed for the three-term expansion by Andreason and Harris (1968). Their procedure apparently is the only one in which the junction geometry plays an explicit role in establishing the current relationships at the junction. Although all of these approaches evidently can produce satisfactory results, there is little or no direct evidence of their validity in terms of the junction current and charge distributions. It should be noted that the numerical results have been found in some cases to be quite sensitive to the junction treatment (Miller and Deadrick, 1973a). Further, the above list by no means exhausts all possible alternatives for the junction treatment.

2.2 SOURCE MODELS

Determination of quantities such as absolute gain, efficiency, radiated power, input power, etc. requires not only the antenna current distribution but also the input characteristics, particularly the feed-point impedance (or admittance). The feedpoint admittance can be found in various ways, but when using the integral-equation approach one usually defines it in terms of source-region current per unit of terminal voltage. In order to calculate this quantity, a realistic source model is needed that not only provides an appropriate means for numerically exciting the antenna but also permits ready evaluation or specification of the effective terminal voltage. Thus if, as in a point-matching procedure, the excitation arises as a tangential field on the source segment of length Δ, the driving voltage might be assumed to be $-E^I\Delta$ if E^I is constant on the source segment and zero elsewhere. This assumption may be invalid, however, with the result that the actual voltage can only be obtained by integrating the tangential field in the vicinity of the source segment (Miller and Deadrick, 1973a). Somewhat less ambiguity should arise from Galerkin-type approaches where the boundary conditions are integrated, so that the classical delta-function source might be numerically approximated. An alternative source model for point matching is provided as a current slope discontinuity, which also approximates a delta-function source field. The current bases, junction treatment, and weight functions can all influence the usefulness of these alternative source models. In case of uncertainty, once the current distribution has been found, the impendance can be computed from the classical EMF method, although at the expense of the additional integration which this entails.

2.3 INTEGRATION

Integration is understandably an essential part of the moment method, being involved in applying the integral operator to the current bases and, in a Galerkin method, evaluating the inner product of this result with the weight functions. For most wire programs, these operations, which lead to the generalized impedance matrix, dominate the total solution time for numbers of unknowns less than ~ 200. It is thus important that the integration time be minimized consistent with the overall accuracy requirements.

One way to approach this goal is to choose appropriate bases and weight functions. The two-term sinusoidal current basis, for example, requires no numerical integration when the Pocklington integral equation is used together with point matching. This particular combination is not very accurate, however, (Miller et al., 1974). By adding the constant term, much better results are obtained, with the slight additional expense of the numerical integration required to find the longitudinal field of this current term; the radial component can be analytically expressed. Alternatively, use of a sinusoidal weight function (Richmond, 1969) also gives much improved results and surprisingly requires numerical integration, at most, of sine and cosine integrals. The piecewise linear basis used with the scalar-vector potential equation cannot be analytically integrated, but good results are obtained with four-point rectangular integration of both the operator and inner-product integrals. In addition, instead of applying a numerical integration to the self-term, a series expansion which gives a closed-form expression is used (Harrington, 1967). When numerical integration is resorted to, various adaptive routines and special techniques are available to improve efficiency (Miller and Burke, 1969).

3. WIRE ANTENNA ANALYSIS

It can be appreciated that there are many options available to the analyst concerning the integral equation to be selected and its numerical treatment in developing a computer model for application to wire antennas. In order to limit this discussion to a reasonable length, our attention will be primarily directed to an approach based on the Pocklington integral equation solved using a three-term subsectional basis (constant, sine, and cosine) and point matching. Unless otherwise indicated, antenna sources are introduced as tangential electric fields, with the Yeh and Mei (1967) form of current extrapolation used for simple junctions and the MBAssociates extension of this extrapolation method to multiple junctions. Both the source model and junction (simple and multiple) treatment used in this code may exhibit deficiencies, but when applied with care (e.g., equal segment lengths near sources and at multiple junctions) the code has proved to be valid and reliable. A brief overview of the relevant equations and numerical treatment used for free space and various interface theories and some special topics is given in this section. Numerical results follow in Section 4.

3.1 INFINITE, HOMOGENEOUS, ISOTROPIC MEDIA

The Pocklington-type integral equation for a wire structure of contour $C(\bar{r})$ can be expressed in the form

$$\hat{s}\cdot\bar{E}^I(s) = \frac{i\omega\mu}{4\pi} \int_{C(\bar{r})} I(s')G_0(s,s')ds'; \quad s\varepsilon C(\bar{r}), \qquad (1)$$

where

$$G_0(s,s') = \left[\hat{s}\cdot\hat{s}' + \frac{1}{k^2}(\hat{s}\cdot\nabla)(\hat{s}'\cdot\nabla)\right] \cdot g_0(\bar{r},\bar{r}'),$$

$$g_0\ (r,r') = \frac{e^{-ikR}}{R},$$

$$R = |\bar{r} - \bar{r}'| \geq a(\bar{r}),$$

$$\hat{s} = \frac{\nabla C(\bar{r}')}{|\nabla C(\bar{r})|},$$

and

$$\hat{s}' = \frac{\nabla C(r')}{|\nabla C(r')|}.$$

where as usual k is the infinite-medium wave member, the permeability and permittivity are denoted by μ and ε, $a(\bar{r})$ is the wire radius at $\bar{r}$, and $\bar{E}^I$ is the incident electric field.

Reduction of this equation to matrix form involves these seven steps:

(1) Approximating $C(\bar{r})$ as a piecewise linear sequence of N segments of length Δ_i, $i = 1, \ldots, N$, so that

$$C(\bar{r}) \simeq \sum_{i=1}^{N} \Delta_i\ \hat{s}_i,$$

with $\hat{s}_i$ the unit tangent vector to $C(\bar{r})$ at $\bar{r} = \bar{r}_i$ (use of straight segments is not mandatory, but it is very convenient in simplifying the current integration);

(2) Introducing the subsectional bases

$$I_i(s') = A_i + B_i \sin [k(s' - s_i)] + C_i \cos [k(s' - s_i)]$$

to represent the unknown current (the final unknowns will be the N-sampled current values $I_i = A_i + C_i$, $i = 1, \ldots, N$, at the center of each of the N segments);

(3) A current interpolation procedure whereby the individual A_i, B_i, and C_i constants are expressed in terms of the sampled current values;

(4) Use of the N delta-function weights $\delta(s - s_j)$, $j = 1, \ldots, N$, to obtain an Nth-order impedance matrix of N independent field equations (note that the weight functions sample the field at the segment centers, and are thus "collocated" with the current sample locations);

(5) Specification of the N incident or primary field vector components $E_i = E^I(s_j) \cdot \hat{s}_j$, $j = 1, \ldots, N$, which are the tangential fields at the N segment centers;

(6) Matrix manipulation to obtain an admittance equivalent of the impedance matrix; and

(7) Computation of the current distribution and whatever field components, if any, are desired.

The total computer solution time is well approximated by $AN^2 + BN^3$, where the "A" term corresponds to step (4) and the "B" term to step (6). For the code under the consideration here and for a CDC-7600 computer, $A \approx 4 \times 10^{-4}$ sec and $B \approx 2 \times 10^{-6}$ sec.

3.2 PERFECTLY CONDUCTING HALF-SPACE

Equation (1) as written applies to wire structures excited as antennas or scatterers and located in infinite, isotropic, homogeneous media of arbitrary (possibly lossy) permittivity and permeability. It can easily be extended to permit the modeling of magnetic or electric image planes. For example, the perfectly conducting ground analog of Eq. (1) is, for an antenna elevated above a ground plane at z = 0,

$$s \cdot \bar{E}^I(s) = \frac{i\omega\mu}{4} \int_{C(r)} I(s')[G_0(s,s') + G_I(s,s'^*)]\, ds', \qquad (2)$$

where

$$g_I = \frac{e^{-ikR^*}}{R^*},$$

$$R^* = |\bar{r} - \bar{r}'^*|,$$

$$\bar{r}'^*(x,y,z) = \bar{r}'(x,y,-z),$$

$$s'^* = \frac{\nabla C(\bar{r}'^*)}{|\nabla C(\bar{r}'^*)|}.$$

Similar forms can be written for a magnetic interface and for an interior right-angle corner. If the corner angle is otherwise arbitrary but related to π as an integer multiple, a discrete spectrum of angular images is obtained, but the essence of the integral equation form is preserved. Precisely the same line of approach can also be used for interior problems where the wire structure is located between two parallel magnetic or electric planes (Taylor, 1970).

3.3 IMPERFECTLY CONDUCTING HALF-SPACE

A problem which is not so computationally simple to handle, but which is of perhaps greater practical interest, is that of an antenna located (buried or elevated) near the ground-air interface. This is a topic of considerable longevity in electromagnetics; a formal solution was worked out for this problem in 1909 by Sommerfeld (Sommerfeld, 1909). The numerical complexity of evaluating the Sommerfeld integrals (which appear in the integral equation kernel) for arbitrary source and observation-point locations and ground parameters, however, has prevented the Sommerfeld theory from being routinely used for such problems. Consequently, while

Consequently, while some progress has been made in applying the Sommerfeld theory, alternative approaches to the antenna-ground problem have also been pursued. These various methods are briefly discussed below.

3.3.1 The Sommerfeld Theory

Details of the steps in deriving the Sommerfeld integrals may be found elsewhere (Sommerfeld, 1964). Here we will simply write one version of Eq. (1) which accounts for the interface reflected field via the Sommerfeld theory; alternative forms are also available, differing essentially in how the perfect-ground image terms are handled. It is

$$\hat{s}\cdot\bar{E}^I(s) = \frac{i\omega\mu}{4\pi} \int_{C(\bar{r})} I(s')\, ds' \bullet \Big\{ G_0(s,s') + G_I(s,s'^*)$$

$$+ \left(\cos\beta + \frac{1}{k^2}\frac{\partial^2}{\partial s\,\partial z}\right) \sin\beta'\, g_{Hz} - \cos\beta'\, g_{Vz}$$

$$+ \sin\beta' \left[\sin\beta \cos(\alpha - \alpha') + \frac{1}{k^2}\frac{\partial^2}{\partial s\,\partial t'}\right] g_{Ht} \Big\}, \qquad (3)$$

where $\alpha = \alpha(\bar{r})$ and $\beta = \beta(\bar{r})$ are the direction angles of the wire at $\bar{r}$, $\hat{t}'$ is the horizontal projection of $\hat{s}'$, J_n is the Bessel function of order n, and

$$g_{Ht} = 2 \int_0^\infty \frac{\lambda}{\mu + \mu_E} J_0(\lambda\rho)\, e^{-\gamma(z+z')}\, d\lambda,$$

$$g_{Ht} = \frac{-\cos(\phi - \alpha')}{k^2} \int_0^\infty \frac{\mu - \mu_E}{\varepsilon_E\mu + \mu_E} \bullet J_1(\lambda\rho)\, e^{-\gamma(z+z')}\lambda^2\, d\lambda,$$

$$g_{Vz} = 2 \int_0^\infty \frac{\mu_E}{\varepsilon_E\mu + \mu_E} \bullet J_0(\lambda\rho) e^{-\gamma(z+z')} \frac{\lambda}{\mu}\, d\lambda,$$

$$\rho = \sqrt{(x - x')^2 + (y - y')^2 + a^2},$$

$$\phi = \tan^{-1} [(y - y')/(x - x')],$$

$$\gamma = \sqrt{\lambda^2 - k^2},$$

$$\gamma_E = \sqrt{\lambda^2 - \varepsilon_E k^2},$$

with ε_E the lower half-space permittivity relative to the upper.

The presence of the double integral in Eq. (3), particularly the Sommerfeld portion, makes it quite time-consuming and sensitive to evaluate. In spite of that, the basic moment method can be used to solve it, but, in addition to the usual constraints imposed on current sampling, it is necessary to take into account the source distance from the interface.

3.3.2 Modified Image Theory

In many cases, although they may not be always easy to identify a priori, the rigor represented by Eq. (3) is unnecessary; various approximations will be found adequate. The accuracy actually required of the computer model may be debatable, but it is probably reasonable to seek something on the order of experimental error. One approach which has been found, for simple antennas, to agree within 10-15% of the Sommerfeld results for input impedance, and so which appears useful in view of the above observation, is the reflection coefficient approximation (Miller et al., 1972a, 1972b). It involves representing the interface-reflected fields in terms of their perfect-ground images multiplied by the Fresnel plane-wave reflection coefficients for the TE and TM field components evaluated at the specular reflection point. This approximation leads to the integral equation given below:

$$\hat{s}\cdot\bar{E}^I(s) = \frac{i\omega\mu}{4\pi} \int_{C(\bar{r})} I(s') \bullet [G_0(s,s') + R_M G_I(s,s'^*)$$

$$+ (R_E - R_M) \sin\beta \sin\beta' \quad \sin(\phi - \alpha) \sin(\phi - \alpha') \, g_I(\bar{r},\bar{r}'^*)] \, ds', \quad (4)$$

where R_E and R_M are the TE and TM reflection coefficients given by

$$R_E = \frac{\sqrt{\varepsilon_E - \sin^2\theta} - \cos\theta}{\sqrt{\varepsilon_E - \sin^2\theta} + \cos\theta},$$

$$R_M = \frac{\varepsilon_E \cos\theta - \sqrt{\varepsilon_E - \sin^2\theta}}{\varepsilon_E \cos\theta + \sqrt{\varepsilon_E - \sin^2\theta}},$$

with θ the angle of incidence with respect to vertical. (Although Eqs. (3) and (4) are written expressly for the reflected field, similar expressions also hold for the field transmitted across the interface.)

Since the reflection-coefficient integral equation (4) differs only trivially from that for the perfect-ground case given by (2), it may be appreciated that its numerical solution is obtained with almost equal efficiency, in marked contrast to the situation which holds for the rigorous theory. The reflection coefficient approximation is, in addition, applicable to a laterally inhomogeneous ground with little further complication. Layered grounds can also be handled using this approach.

3.3.3 Interface Source Distribution

The Sommerfeld theory is not the only rigorous formulation which can be derived for the antenna-ground problem. Some variations of that approach which still involve integral-type elementary source solutions, are summarized by Baños (1966). A completely different method of treatment which results in integration in real space (the interface) rather than wave-space integration (the Sommerfeld or λ integral) can be postulated. One way is to treat the interface tangential fields as unknowns in addition to the antenna current distribution itself. One can then solve for these surface sources together with the antenna current by applying the moment method to the coupled integral equations which result. An obvious disadvantage of this approach is that many more unknowns require consideration, an infinite number in principle, but finite in practice since only the region near the antenna need be modeled. Advantages are that the computer-time penalty imposed by the interface-related calculation is relatively independent of antenna size, and the Sommerfeld integrals are entirely circumvented, with no nested numerical integrals being encountered.

The integral equation which results from this treatment has the form

$$\hat{s}\cdot\overline{E}^{I}(s) = \frac{i\omega\mu}{4\pi}\int_{C(\overline{r})} I(s')\, G_0(s,s')\, ds' - \hat{s}\cdot\int_A \Big\{ i\omega\mu\, [z \times \overline{H}(\overline{r}')]\, g_0(\overline{r},\overline{r}') - [\hat{z}\times\overline{E}(\overline{r}')] \times \nabla' g_0(\overline{r},\overline{r}') - [\hat{z}\cdot\overline{E}(\overline{r}')]\nabla' g_0(\overline{r},\overline{r}')\, da' \Big\} ;\ s\varepsilon C(\overline{r}) \qquad (5)$$

$$0 = \hat{z}\times\frac{i\omega\mu}{4\pi}\int_{C(r)} I(s')\overline{G}_0(\overline{r},s')ds' - \frac{1}{4\pi}\hat{z}\times\int_A\Big\{ i\omega\mu[\hat{z}\times\overline{H}(\overline{r}')]g_0(\overline{r},\overline{r}') - [\hat{z}\cdot\overline{E}(\overline{r}')]\nabla' g_0(\overline{r},\overline{r}')\Big\}\, da' ;\ \overline{r}\varepsilon A$$

$$0 = \hat{z}\times\nabla\times\frac{1}{4\pi}\int_{C(\overline{r})} I(s')\overline{G}_0(\overline{r},s')ds' + \frac{1}{4\pi}\hat{z}\times\int_A\Big\{ i\omega\varepsilon[\hat{z}\times\overline{E}(\overline{r}')]g_0(\overline{r},\overline{r}') - [\hat{z}\cdot\overline{H}(\overline{r}')]\nabla' g_0(\overline{r},\overline{r}')\Big\}\, da' ;\ \overline{r}\varepsilon A$$

where A is an area on the z = 0 ground plane under the antenna, $\bar{E}(\bar{r}')$ and $\bar{H}(\bar{r}')$ are the ground-plane source distributions, and

$$\bar{G}_0(\bar{r},s') = \left[\hat{s}' + \frac{1}{k^2}\nabla(\hat{s}'\cdot\nabla)\right] g_0(\bar{r},\bar{r}').$$

Note that the surface integrals in the latter two of the above equations must be evaluated in a principal-value sense.

This approach has not yet been implemented.

3.3.4 Surface Source Approximations

In the same way that the reflection-coefficient approximation follows in a straightforward way from the rigorous Sommerfeld theory, approximations to the interface source distribution analysis discussed above naturally suggest themselves. Two we consider here are the surface-impedance and physical-optics approximations.

3.3.5 Surface-Impedance Approximation

Under the condition that the surface impedance concept is valid $[(\sin^2\theta)/\varepsilon_E \ll 1]$ (King, 1969a), the tangential components of the electric and magnetic fields at the surface are to a good approximation related as

$$E_{tan} = -H_{tan} Z_{surf},$$

where

$$Z_{surf} = \eta/\sqrt{\varepsilon_E},$$

with η the upper medium wave impedance.

Upon employing this relationship in (5) the surface integral equations decouple and the number of surface unknowns is decreased by half. A corresponding reduction in the order of the overall linear system is thus achieved, with a potential significant saving in both the computer storage and solution time. Either of the two surface equations can be retained for use together with the structure-related integral equation. For convenience we might select the electric-field equation since then all required interaction coefficients involve the electric field only.

3.3.6 Physical-Optics Approximation

Even further simplification of (5) can be achieved by invoking a physical-optics type of approximation for the surface fields. But in contrast to the usual physical-optics magnetic field, which is given by $H_{tan} = 2H_{tan}^{inc}$, we instead use

$$H_{tan} = (1 + R)H_{tan}^{inc},$$

with R the reflection coefficient together with the surface impedance approximation for E_{tan} to allow for a finite ground conductivity. This permits the total-surface field distribution to be expressed in terms of the currents on the wire structure and leads to the same number of unknowns as for the Sommerfeld theory, but with the advantage of a much simpler integral-equation kernel. A perfect-ground type of integral equation could be derived by decomposing the surface source contribution to the fields on the structure into a part due to the perfect ground (due to $2H_{tan}^{inc}$), which can be analytically integrated to give the usual perfect ground image, and a perturbation term $(R - 1)H_{tan}^{inc}$, which will require numerical integration. While in general we might use a Fresnel reflection coefficient for each incremental source and surface path, it would be simpler, and not inconsistent with approximations already employed, to everywhere approximate R by its normal incidence form. Note that by appropriate pairing of source and observation points on the wire structure and use of a single reflection coefficient evaluated at their specular point, we obtain the reflection coefficient approximation already discussed.

3.3.7 The Compensation Theorem

Application of the compensation theorem to ground-plane problems has received considerable attention (Monteath, 1951; Mittra, 1961; King, 1969b). It has been used to determine the input impedance of vertical monopoles located over various ground configurations, including determining the effect of ground-screen size. However, more general antenna problems have evidently not been attempted with this theory. The reason for this lies, apparently, not in limitations inherent in the theory itself, but in its numerical implementation. A ground plane integral is involved, which, for all but the simplest situations, requires numerical evaluation.

The compensation theorem "is essentially an exact perturbation technique in which the fields in the unperturbed state are known" (King, 1969b). If the unperturbed state is the case of a perfectly conducting ground plane and the perturbed state is the actual ground problem of interest, then we obtain for the antenna input impedance

$$Z' = Z + \frac{1}{I^2} \int_A \bar{H} \cdot \hat{z} \times \bar{E}' \, da,$$

where the primes denote perturbed quantities and I is the feedpoint current. Since the perfect-ground magnetic-field distribution can be accurately solved for, evaluation of Z' hinges on finding $\bar{E}'$. This is usually accomplished by using the surface impedance approximation, i.e., $E'_{tan} = -H'_{tan} \cdot Z_{surf}$, and then assuming $H'_{tan} \simeq H_{tan}$. These steps facilitate the calculation and permit use of the perfect-ground result as a sort of canonical solution to find the antenna impedance for the finitely conducting ground.

3.3.8 Ground-Wave Propagation

Another important aspect of the antenna-ground problem is that of determining the propagation characteristics of waves launched along the interface.

Such surface waves are an important mechanism for short- and intermediate-distance communication. A common problem which arises in assessing the path loss of a surface-wave mode link is that presented by an inhomogeneous path due to surface impedance changes or profile variations from the ideal smooth, curved earth.

A general integral-equation computer code for analyzing this problem has been developed by Ott (1971). The rigor of Ott's approach, while permitting the accurate modeling of quite complex paths, is not needed for many of the so-called mixed-path problems, where surface impedance changes only are of interest. For these situations, which typically involve propagation across a coastline, an approximate procedure derived by Eckersley (David and Vogue, 1969) and Millington (1949; Millington and Isted, 1950) is quite adequate. It involves use of the standard Norton attenuation functions and leads to the following generalized expression for the vertical surface-wave field E^V over an N-part mixed path as

$$E^V = \frac{i\omega\mu}{4\pi}\left[\left(\sum_{i=1}^{N_B} F_{Li}^B F_{zi} + \sum_{i=N_B+1}^{N_B+N_E} F_{Li}^E\right) I_i \Delta_i f_i\right] F_P f_R, \tag{6}$$

where superscripts B and E denote buried and elevated antenna segments, respectively, there being a total of $N_B + N_E$, and I_i is the current at the center of segment i of length Δ_i, coordinates x_i, y_i, z_i and direction angles α_i and β_i with respect to the x-y (ground) plane and the x axis, respectively. Further,

$$F_{Li}^B = \frac{\sin \alpha_i}{n^2} + \sqrt{n^2 - \sin^2\theta_i}\, \cos(\phi - \beta_i) \cos \alpha_i$$

and

$$F^E_{Li} = \sin \alpha_i \sin \theta + \sqrt{n^2 - \sin^2 \theta_i} \cos (\phi - \beta_i) \cos \alpha_i$$

are the surface-wave launching efficiencies of the buried and elevated segments, where n is the refractive index of the lower half-space at the antenna location, and

$$F_{zi} = \exp [-ik_0 z_i (\sqrt{n^2 - 1} + 1)],$$

$$f_i = \exp \left\{ -ik_0 [\sin \theta_i (x_i \cos \phi + y_i \sin \phi) - z_i \cos \theta_i] \right\},$$

$$f_r = \frac{e^{-ik_0 R_T}}{R_T},$$

$$\theta_i = \tan^{-1} \left(\frac{R_T}{z + z_i} \right),$$

$$\phi = \cos^{-1} \left(\frac{R_T \cdot \hat{x}}{R_T} \right),$$

$$R_T = \sqrt{z^2 + r^2} \sim r ,$$

with ϕ the observation angle with respect to the x axis, z the observation height, and r the radial distance to the observation point from the origin. Finally, F_{pi} is the mixed-path propagation factor, and has the form

$$F_{pi} = 2 \left(\frac{\prod_{i=1}^{N} F_{i,R_i} \prod_{i=1}^{N} F_{i,R-R_{i-1}}}{\prod_{i=1}^{N-1} F_{i+1,R_i} \prod_{i=1}^{N-1} F_{i,R-R_i}} \right),$$

where $F_{i,R}$ is the TM-mode Norton attenuation function (King, 1956) calculated for a path of length R and having electrical parameters ε_i and σ_i, and with R_i given by

$$R_i = \sum_{j=1}^{i} r_j ,$$

where r_j is the path length across part j of the N-part path. The above expression is written for a flat earth and assumes source and observation points close enough to the interface that the Fresnel plane-wave reflection coefficients are -1. Implicit in its derivation is the fact that the attenuation rate of a surface wave is primarily determined by the local wavefront curvature and local surface impedance.

3.4 SPECIAL TOPICS

In addition to the above topics, there are other problem areas concerning wire-antenna computer modeling that deserve attention. Some of them are summarized here.

3.4.1 Ground Screens

The compensation theorem has been employed in various ways to analyze ground-screen effects as mentioned above. The reflection-coefficient approximation has also been used for this purpose. It offers an easily implemented procedure for analyzing a broad class of ground-screen configurations with greater efficiency than available in general from the compensation theorem.

What is essentially required in order to include the ground-screen influence in the reflection-coefficient calculation is a modified reflection coefficient which takes into account the reflecting properties of the screen-ground combination. This is possible if the surface impedance of the combination is known. For ground screens whose wires are in good electrical contact with the soil, the effective surface impedance Z'_{surf} may be taken to be (Wait, 1969)

$$Z'_{surf} \approx \frac{Z_{surf}\, Z_{screen}}{Z_{surf} + Z_{screen}} ,$$

where Z_{screen} is the screen impedance. For a radial screen having N wires of radius a, the screen impedance at distance ρ from the center is given by (Wait, 1969)

$$Z_{screen} \approx \frac{i\mu\omega\rho}{N} \ln (\rho/Na).$$

A corresponding formula for a parallel grid of wires, whose center spacing is d, is

$$Z_{screen} \approx \frac{i\mu\omega d}{2\pi} \ln (d/2\pi a).$$

Meshes consisting of locally orthogonal wires having different spacings might be treated as anisotropically conducting planes whose principal-direction impedances are obtained from the parallel-wire formula using their corresponding spacings. From Z'_{surf} we infer an effective ground permittivity for use in computing the Fresnel reflection coefficients, and are thus able to include the screen in the integral-equation calculation. The anisotropic case requires decomposition of the TE and TM fields into components along the orthogonal screen wires (Miller and Deadrick, 1973b). Note that this method fails for vertical antennas located at the center of a radial screen.

An alternative possibility is offered by the work of Astrakhan (1968) who derived reflection coefficients for infinite-plane wire grids. His results, given in terms of TE-TE, TM-TM, TE-TM, and TM-TE reflection coefficients can be modified to include the effect of the ground itself and used in the reflection-coefficient approximation. Of the above, only the radial-wire screen analysis has been implemented.

3.4.2 The Layered Ground

Reflection coefficients are of course available for a layered ground. For the special case of only two layers, and where the surface impedance approximation holds, the effective surface impedance is given by (Wait, 1962)

$$Z'_{surf} \approx Z_{surf} \frac{\sqrt{\varepsilon_1} + i\sqrt{\varepsilon_2} \tan kh\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2} + i\sqrt{\varepsilon_1} \tan kh\sqrt{\varepsilon_1}} ,$$

with ε_1 and ε_2 the relative permittivities of the two layers and h the thickness of layer 1.

3.4.3 Geometrical Theory of Diffraction

The geometrical theory of diffraction (GTD) does not have obvious application to antenna-ground problems. There are however, two areas where GTD may be beneficial: (1) ground-screen edge effects (diffraction) on input impedance and low-angle radiation; and (2) effects of large-scale terrain variations, e.g., diffraction at a cliff. Application of GTD to both areas has been studied by Thiele (1973). His approach was to combine GTD with the moment method to find the effect of the edge-diffracted field on the current distribution of a monopole antenna located on a wedge. This leads to an integral equation modified from that for free space by inclusion of the diffracted fields, given in terms of the antenna current, in the total tangential electric field on the antenna. Thus, no additional unknowns are involved. The far field is treated in a similar manner. Results obtained to date are encouraging, although use of the technique to analyze a real ground screen awaits derivation of diffraction coefficients for a perfectly conducting half-plane lying on a lossy interface.

3.4.4 Backscreen Evaluation

It is fairly common practice to employ backscreens to improve HF antenna performance. Backscreen are typically constructed of arrays of parallel, vertical wires whose spacing and diameter are selected to maximize the antenna's front-to-back ratio or some other measure of its performance. The backscreen parameters so chosen have necessarily been based on design criteria derived from experimental measurement and some simplified analysis for a limited number of cases (Moullin, 1949), and are thus unlikely to truly optimize the resulting antenna characteristics. Computer modeling offers some possibilities for improving this aspect of antenna design.

One approach that might be considered would be that of including the backscreen wires in the computer model. This could provide a very realistic representation of the overall antenna-backscreen system, but at a considerable increase in computer time, especially if extensive parametric studies were to be performed.

As an alternative, the parametric evaluation could be instead based on a two-dimensional integral equation, using infinite wires or strips for the backscreen and antenna members. The effects of spacing and size of the backscreen elements, backscreen width, antenna position, frequency, etc. could be much more efficiently studied, while many physical features important in the actual three-dimensional configuration could be retained. After identifying parameter values of greatest apparent interest, it might be then useful to perform limited calculations for the three-dimensional geometry to ensure the essential validity of the two-dimensional results. Another possible alternative which is worth mentioning is the application of image theory to the backscreen as well as to the interface. This is basically equivalent to a two-dimensional screen model, while it retains the three-dimensional aspects of the actual antenna.

3.4.5 Impedance Loading

In many cases of interest, the antenna may be connected to impedance loads of various kinds, or may even itself be lossy enough so that it cannot be accurately modeled as being perfectly conducting. These situations can be accommodated in the computer model by subtracting an appropriate voltage drop $Z_{ij}^{(L)} I_j$ from the source term E_i, where $Z_{ij}^{(L)}$ is the load impedance. When there are no mutual impedance effects, such as those due to transformer or transmission line interconnection, then $Z_{ij}^{(L)} = \delta_{ij} Z_{ij}^{(L)}$, i.e., the $\bar{\bar{Z}}$ matrix becomes diagonal. Lumped loads are simply specified in terms of their resistive and reactive components. Their treatment is similar to that accorded sources, since the two can be viewed as mathematically equivalent. Distributed loads which might be used to model wire losses can be derived from the wire properties (Cassidy and Fainberg, 1960).

3.4.6 Sheathed Wires

Another problem of relevance, especially for antennas located in lossy media such as ground or sea water, is that of a wire coated by a dielectric layer. It has been suggested, but not demonstrated, that the sheath could be modeled in the same way as a lossy wire, by a suitably derived impedance load (Miller et al., 1970). An alternative, more rigorous approach has been taken by Richmond (1973), who models the sheath with a radially directed polarization current, reasoning that the tangential field, being much smaller. is by comparison of negligible import. Since the radial sheath fields which determine this current are known in terms of the charge density on the wire, no additional unknowns are introduced. One simply obtains a modified integral equation which can be solved in the usual way.

3.4.7 Time Domain Analysis

Previous discussion has dealt exclusively with frequency-domain formulations. It is worthwhile to point out that these problems can also be attached from a time-dependent or time-domain viewpoint (Bennett and Weeks, 1968; Miller et al., 1973a; Poggio et al., 1973). As one outcome of such an effort, there can be derived time-dependent integral equations which correspond closely to their frequency-domain counterparts. The solution procedure, while also developed from the moment method, is significantly different in that a solution is obtained as an initial-value problem via time stepping. This leads to results which are valid for only a single incident field or source configuration but over a band of frequencies, in contrast with the more familiar frequency-domain approach of which the converse is true. Solutions may consequently be obtained more efficiently in the time domain than in the frequency domain for certain types of problem, especially for wire structures analyzed as antennas.

4. NUMERICAL RESULTS

In the context of practical applications, judgment on the relative merits of a particular computer model must ultimately depend upon the comparison of calculated results with independent data, preferably experimental, although other theoretically derived results may suffice. Unfortunately, reliable experimental data is not always available so that in many instances we may have to resort to various "computer experiments" and physical intuition when attempting to validate a numerical procedure. In the discussion which follows, we present numerical results for a variety of problems, accompanied where possible by measured data, to demonstrate the general applicability of the computer modeling approach for wire antennas. The order of presentation will follow that of Section 3 above.

4.1 INFINITE, HOMOGENEOUS, ISOTROPIC MEDIA

Problems which involve isolated antennas in infinite, homogeneous media are not frequently encountered, since, more often than not, environmental influences due to ground planes or the installation are important. Nevertheless, this kind of problem does provide a good, controlled check on the accuracy of computer calculations. Examples of such results are shown in Figs. 2 through 5 (Gee, Miller, and Selden, 1970). The input impedance as a function of frequency near resonance of a capacitively loaded circular loop antenna is compared with measured data in Fig. 2. Results are shown in Fig. 3 for the input impedance resonance frequency of a zig-zag antenna as a function of the wire angle, also compared with experiment. A comparison of two computed near-field results for a circular loop antenna, one obtained from a moment-method solution of Eq. (1) and the second from an alternative analysis (Fante, Otazo, and Mayhan, 1969) are presented in Fig. 4. Finally, in Fig. 5 we show a comparison of computed and measured pattern results for a 19-element foreshortened log-periodic antenna (MBAssociates, 1970).

4.2 PERFECTLY CONDUCTING HALF-SPACE

Some results for antennas located near perfectly conducting boundary planes are shown in Figs. 6 and 7. The computed input impedance for the top-loaded monopole, a LORAN-C antenna, is compared with measured results in Fig. 6 (MBAssociates, 1970). Although the actual antenna is located over a finitely

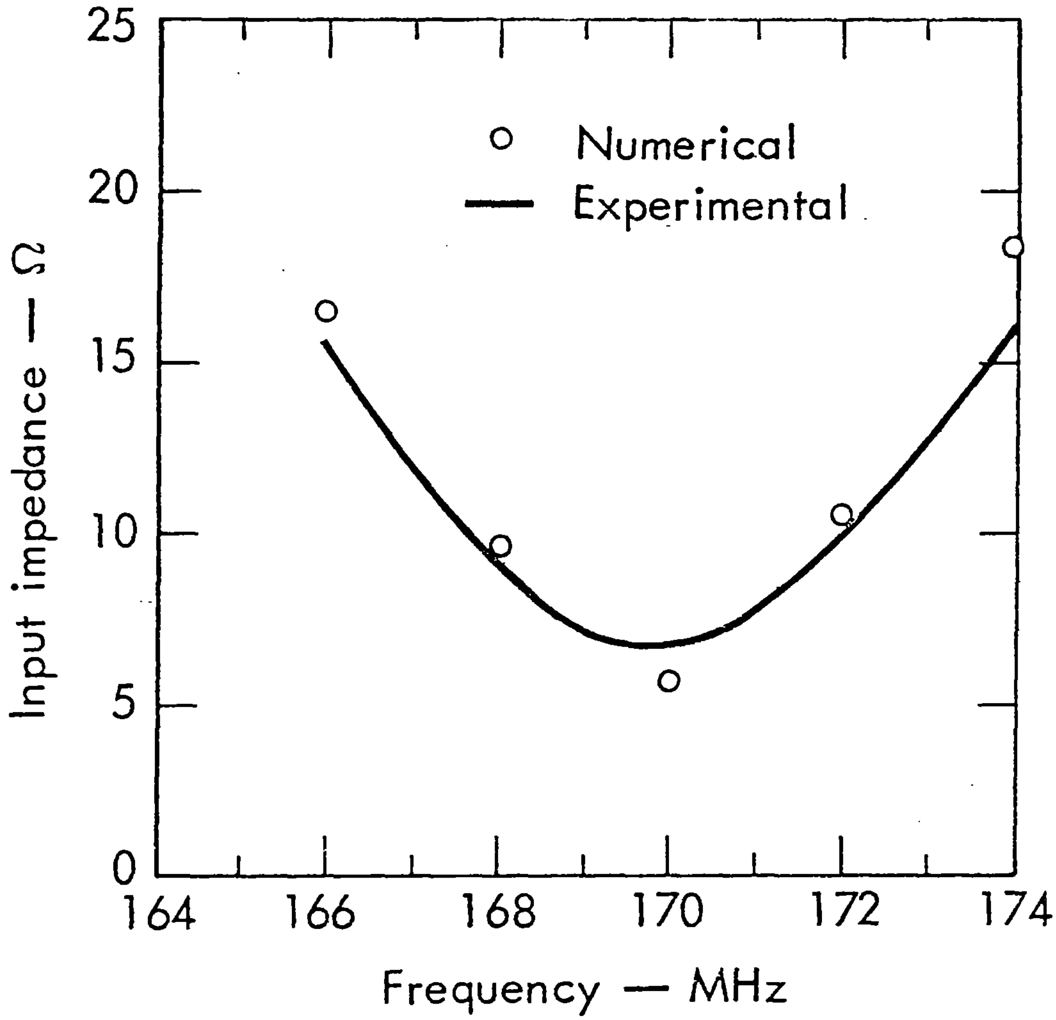

Figure 2. Input impedance of a capacitively loaded circular loop antenna (after Gee et al., 1970).

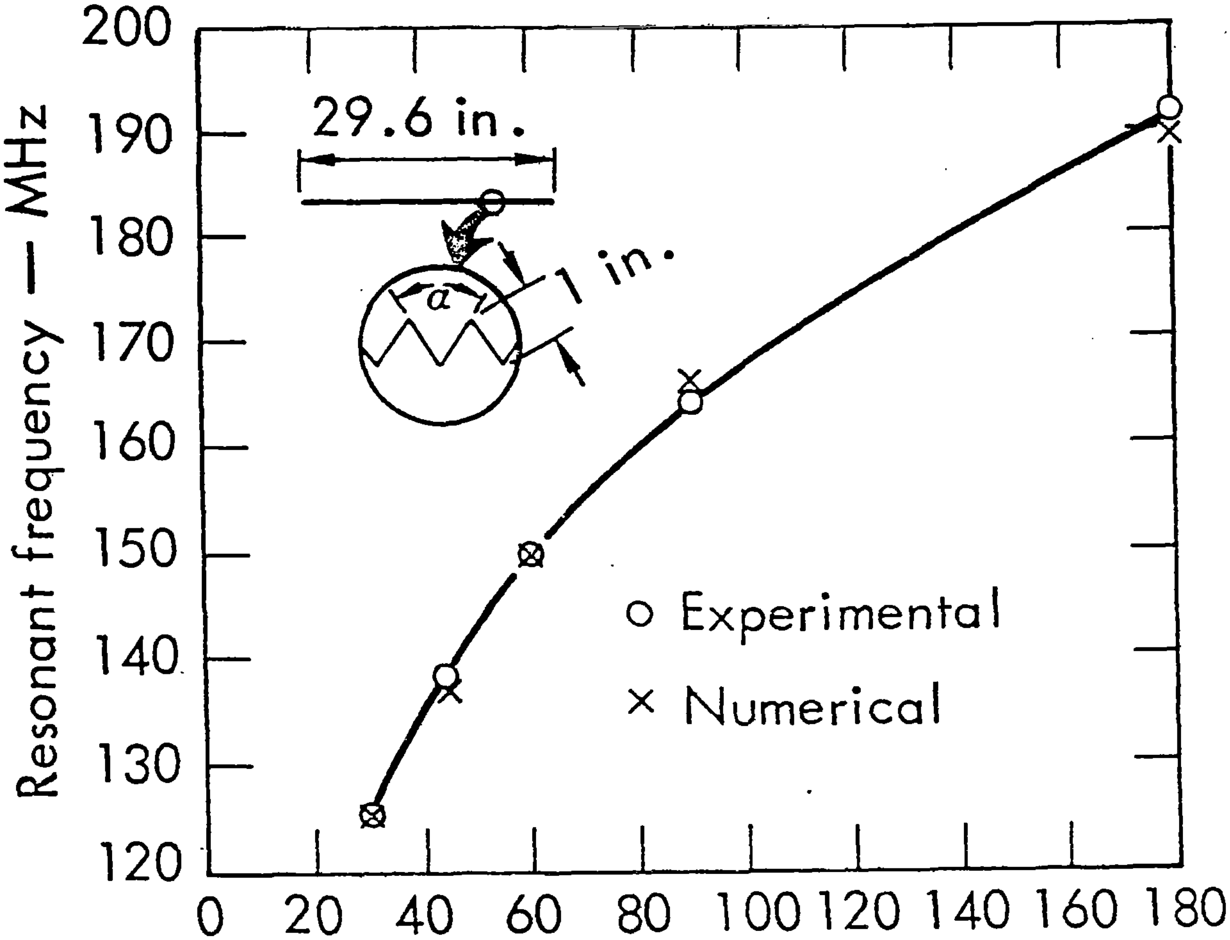

Figure 3. Resonance frequency of a zig-zag dipole (after Gee et al., 1970)

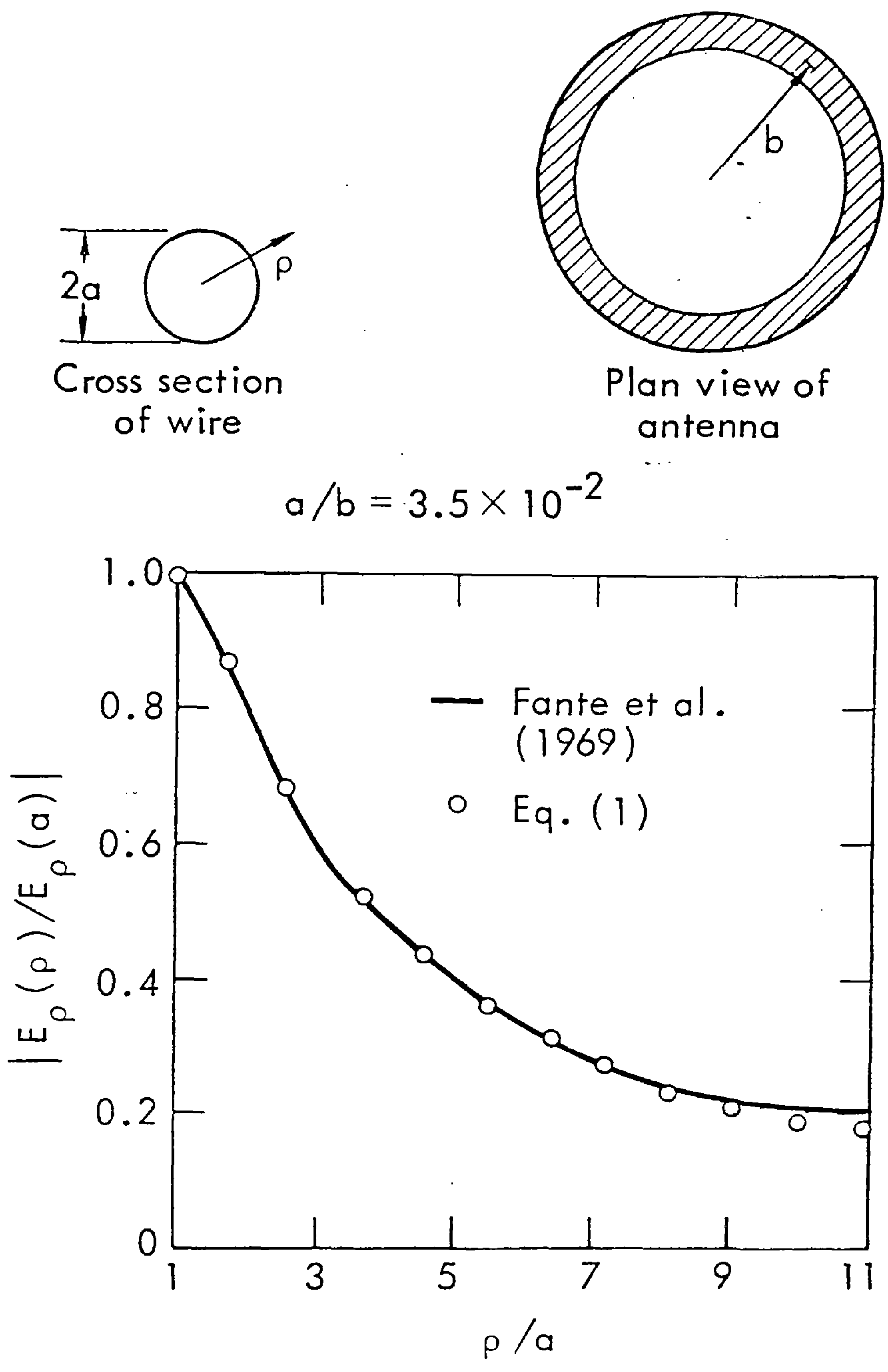

Figure 4. The near field of a circular loop antenna (after Gee et al., 1970).

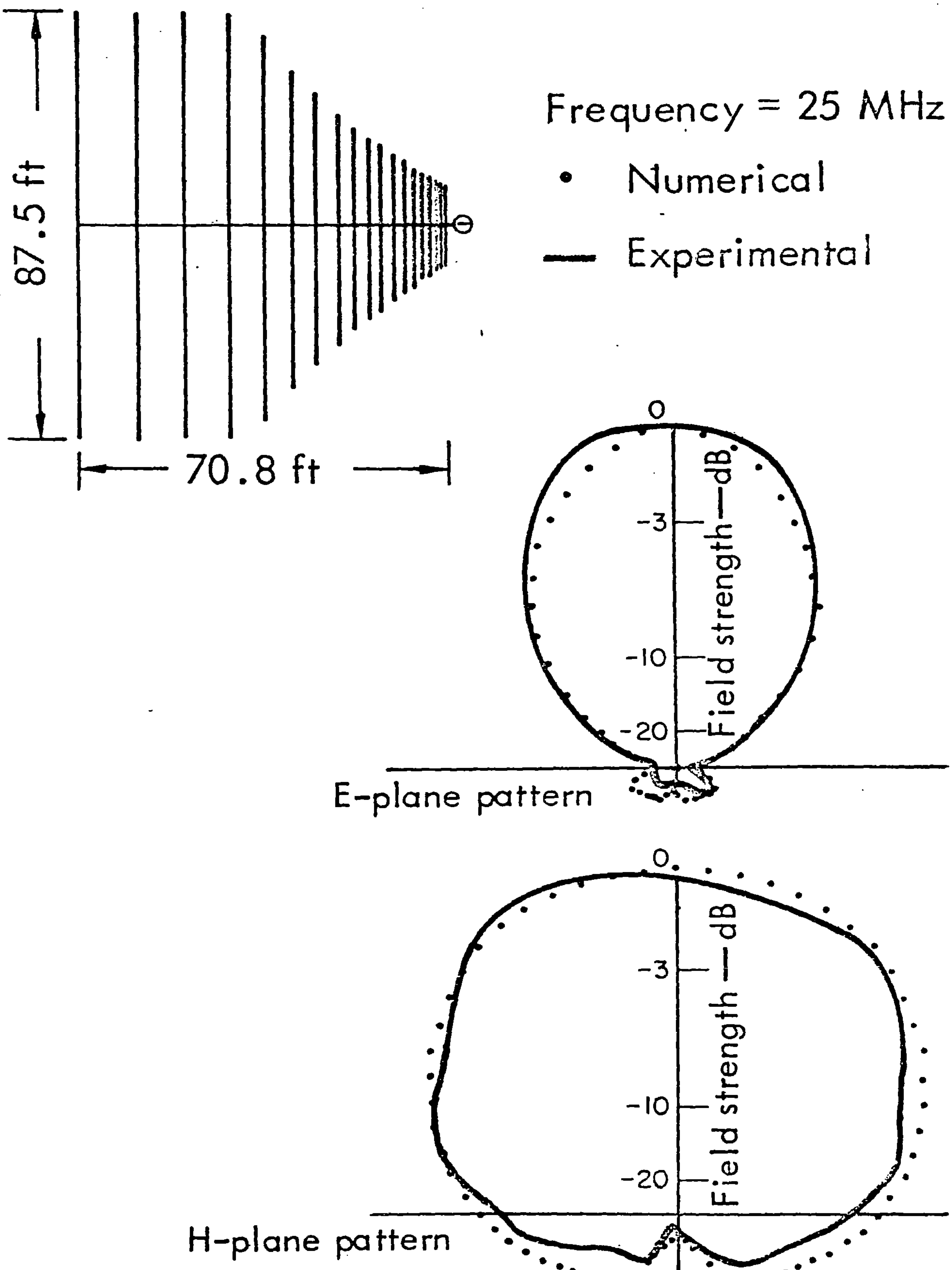

Figure 5. Radiation patterns of a foreshortened 19-element log-periodic antenna (after MB Associates, 1970).

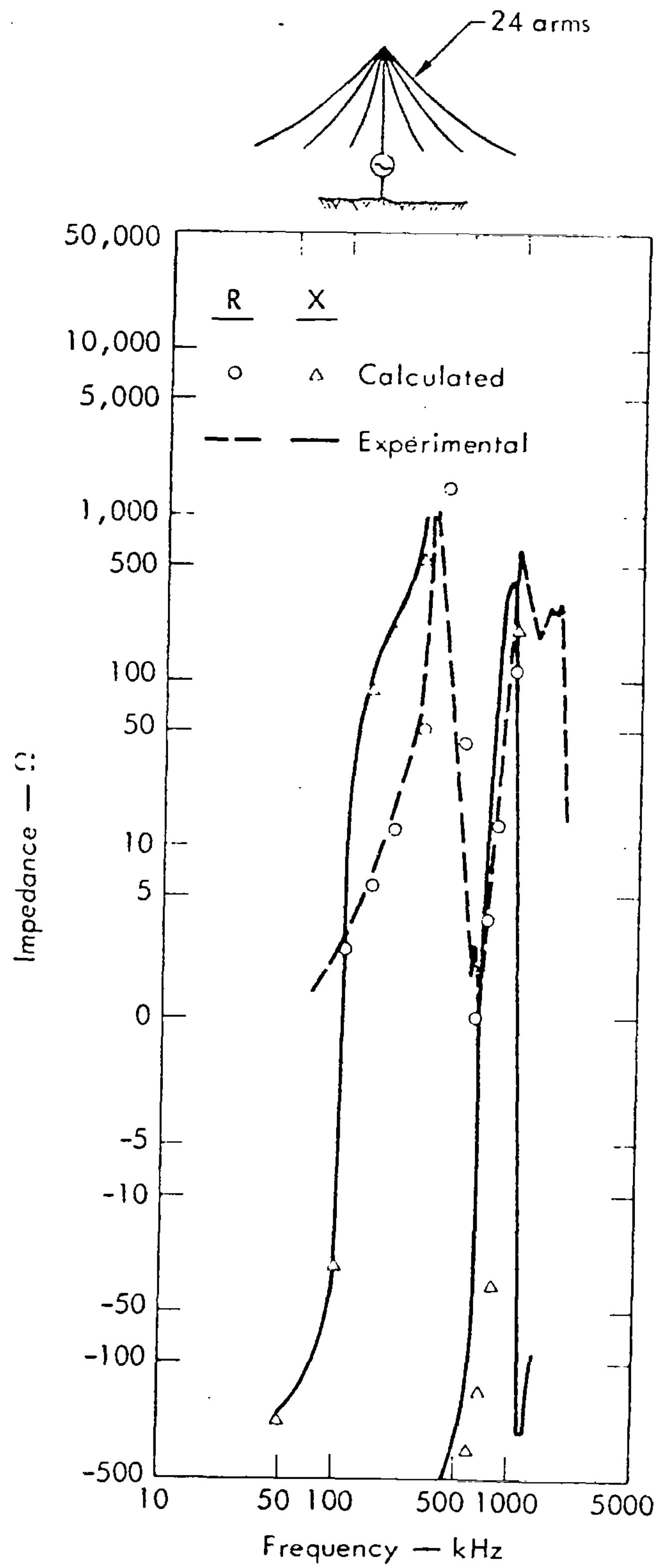

Figure 6. Input impedance of a top-loaded monopole antenna (after MB Associates, 1970).

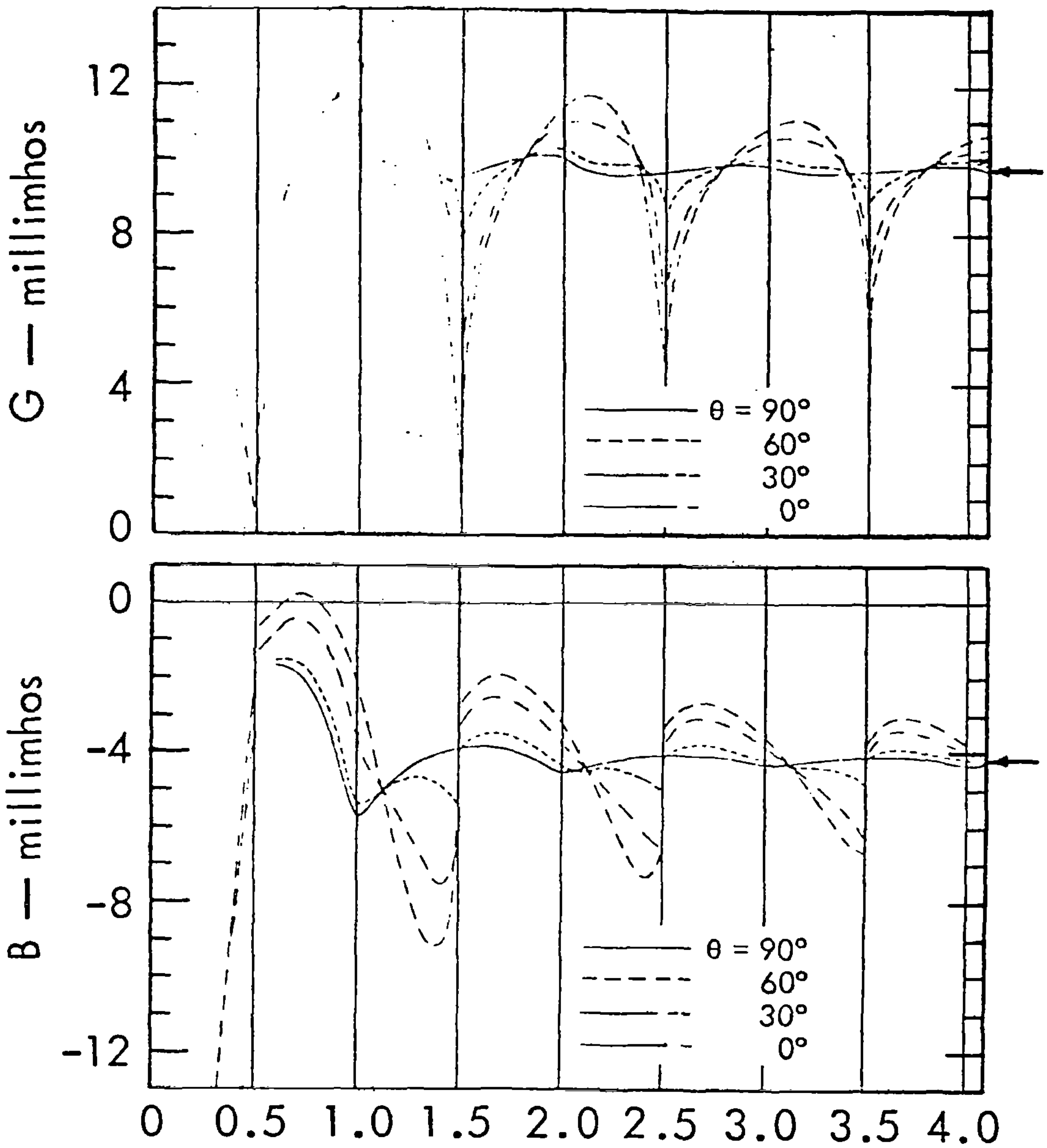

Figure 7. Input impedance of a half-wave dipole located midway between two infinite parallel plates (after Tesche, 1971). The arrow denote free-space values.

conducting ground, its ground screen (120 radial wires) simulates a perfect ground quite well, as demonstrated by the computed results, which are for a perfect ground. The input admittance for a half-wave antenna located midway between infinite, parallel, perfectly conducting plates is shown as a function of plate separation in Fig. 7, with the antenna angle relative to the plates a parameter (θ = 90° corresponds to perpendicular orientation) (Tesche, 1971).

4.3 IMPERFECTLY CONDUCTING HALF-SPACE

Representative results for antennas over lossy grounds are given in Figs. 8 through 13. Sommerfeld theory and reflection-coefficient approximation results for vertical and horizontal half-wave dipoles are compared in Figs. 8 and 9 (Miller et al., 1972a, 1972b). Results are included in Fig. 10 for the current distribution, impedance, and near-field variation of a Beverage antenna (Lytle et al., 1974). Comparison of current distributions obtained from image theory with that computed using the Sommerfeld approach reveals the potential limitations of the former for this case. The graph of Fig. 11 illustrates use of the approximate surface-wave mixed-path model, which is compared with the rigorous calculation due to Ott (1971). Figure 12 compares computed (using the reflection-coefficient approximation) and measured impedances of the sectionalized LORAN transmitting (SLT) antenna, including the effect of a radial-wire ground screen in the computations (Miller, Deadrick, and Henry, 1973b). The computed (reflection-coefficient approximation) and measured simulated EMP response of the fan-doublet antenna are shown in Fig. 13 (Landt et al., 1973). A compensation-theorem result for a vertical half-wave dipole is included in Fig. 8.

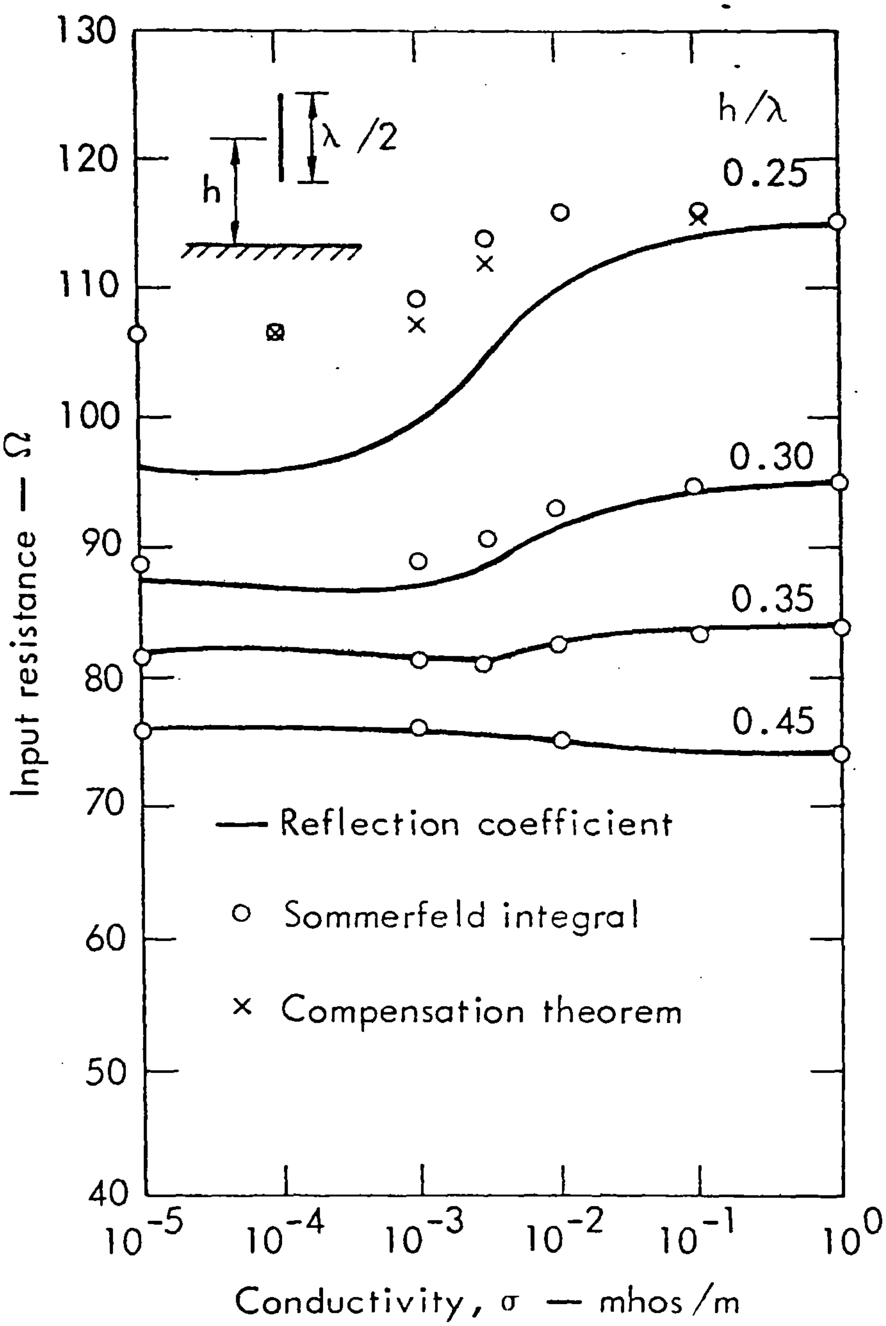

Figure 8. Results for a vertical half-wave dipole over a ground plane (after Miller et al., 1972a). f=3.0 MHz; ϵ_r = 10; a = 5 x $10^{-4}\lambda$.

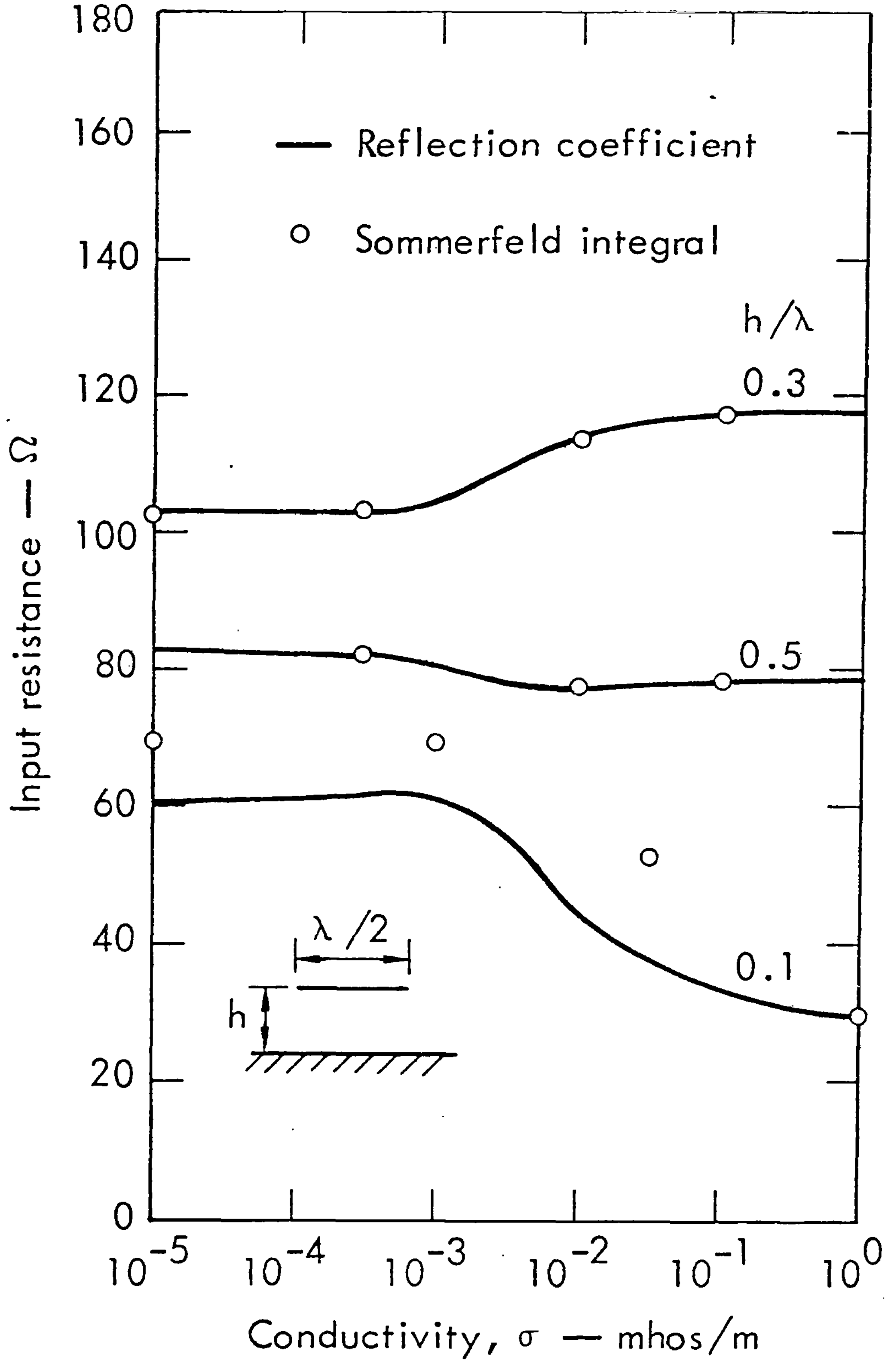

Figure 9. Results for a horizontal half-wave dipole over a ground plane (after Miller et al., 1972b). f = 3.0 MHz; ε_r = 10; wire radius/L = 0.01.

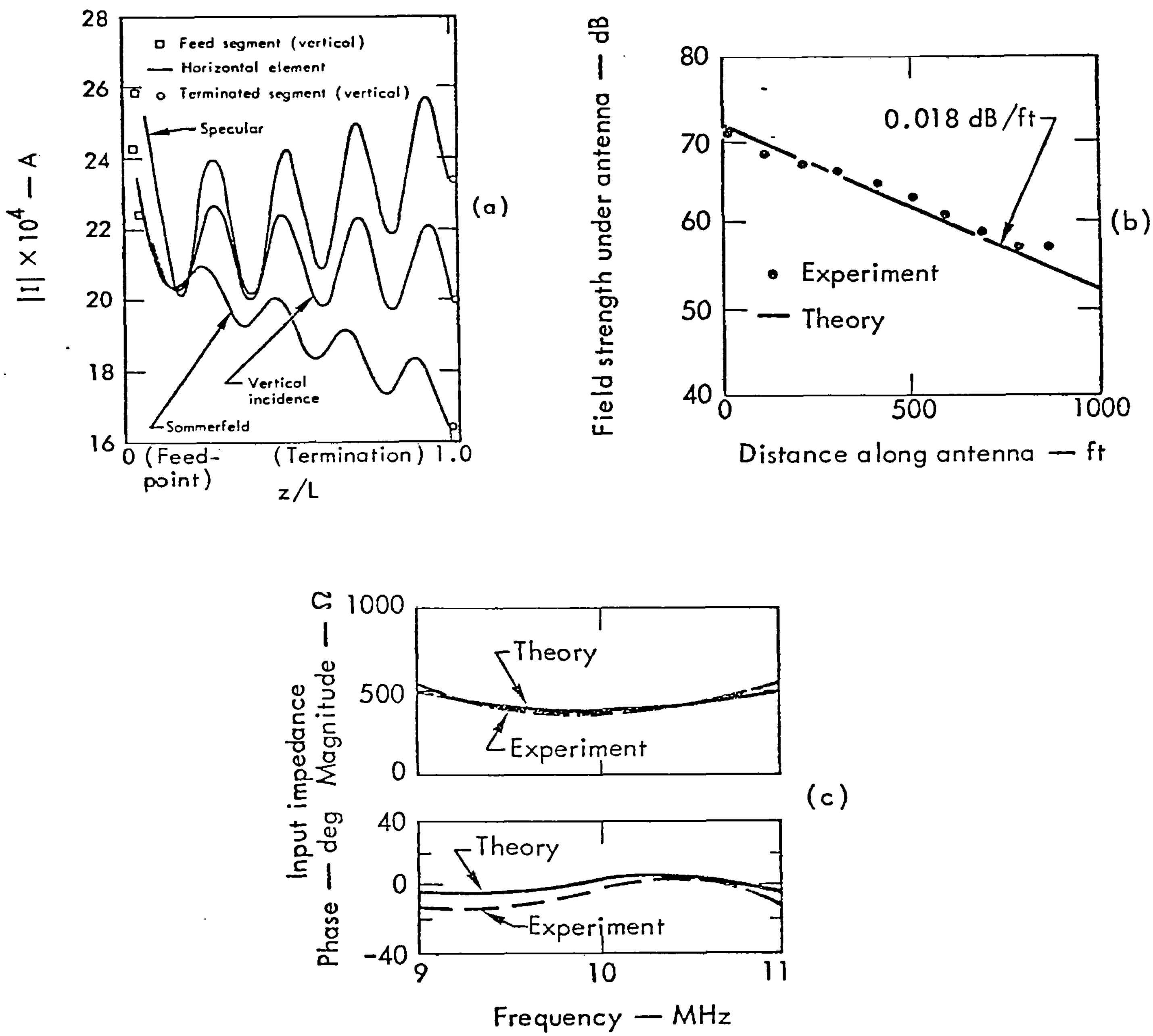

Figure 10. Results for a Beverage antenna (after Lytle et al., 1974). (a) Computed current distributions. f = 10 MHz; L = 215 ft; h = 2 m; Z_T = 452Ω; ϵ_r = 25; σ = 0.03. (b) Field strength. f = 10 MHz; L = 1000 ft; h = 2 m; Z_T = Ω. (c) Impedance.

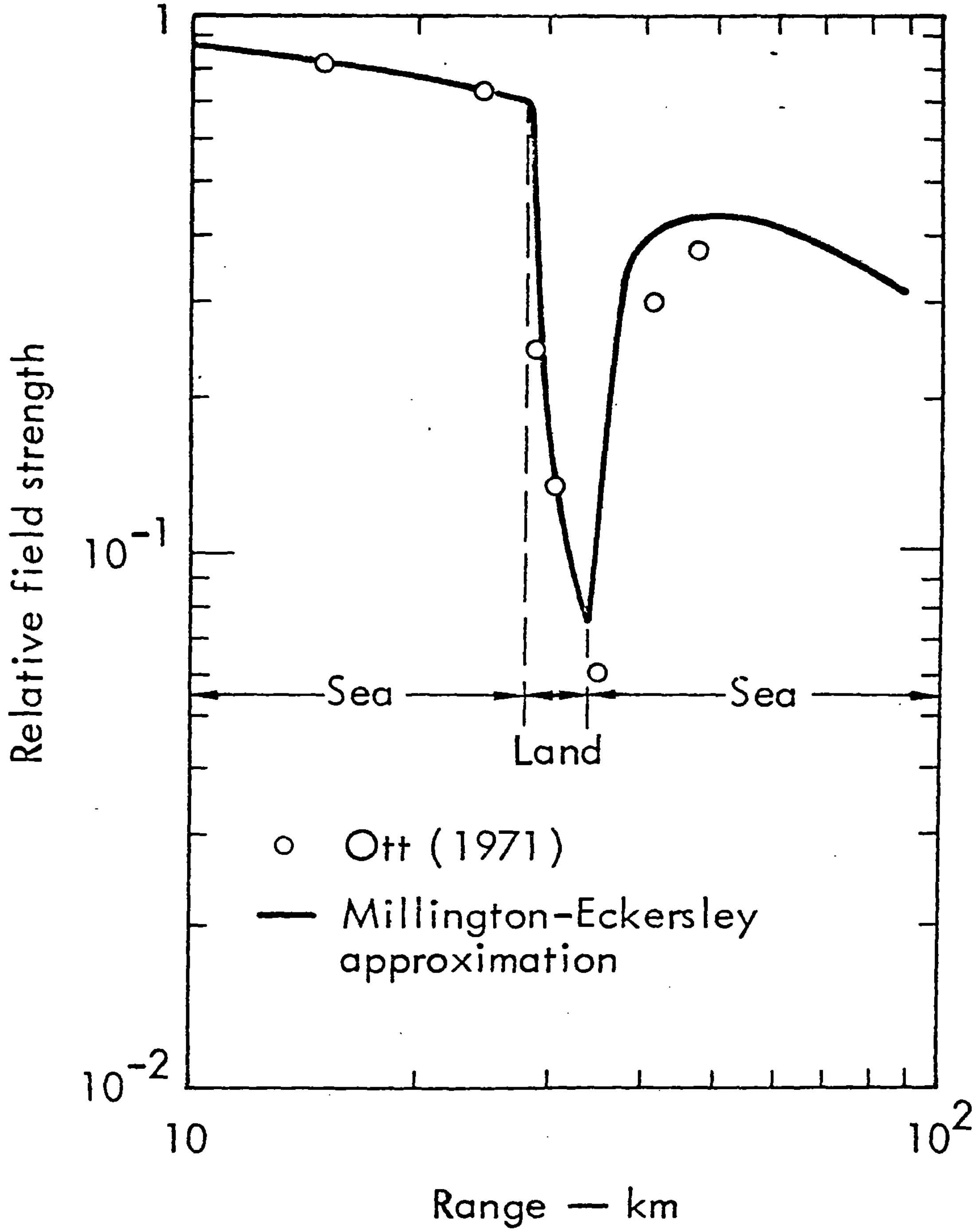

Figure 11. Surface-wave field strength over a sea-land-sea mixed path. f = 10 MHz. For land: σ = 0.002; ϵ_r = 15. For water: σ = 2; ϵ_r = 81.

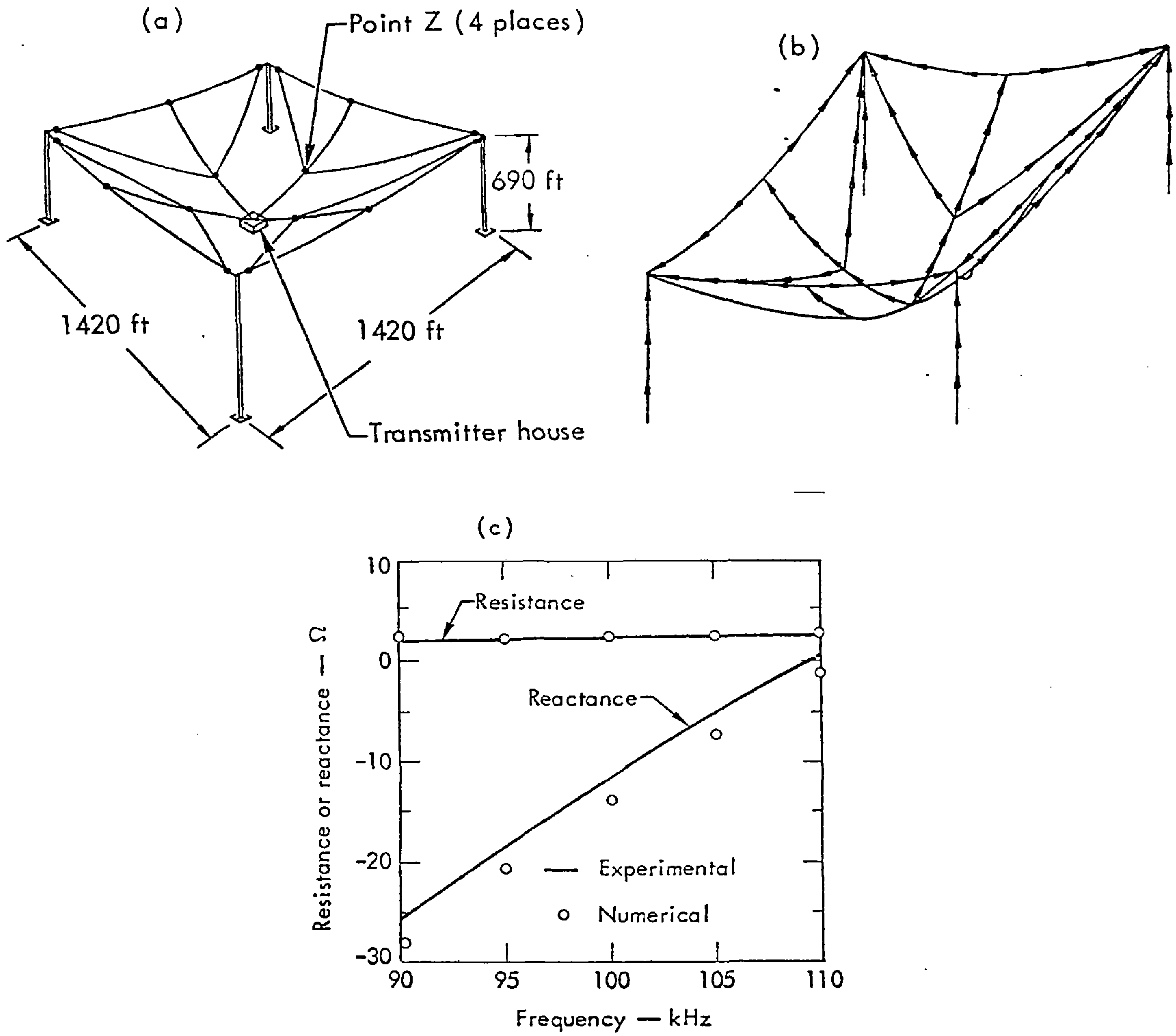

Figure 12. (a) SLT antenna geometry.
(b) Computer model.
(c) Experimental and numerical comparison of SLT antenna impedance. (After Miller et al., 1973b).

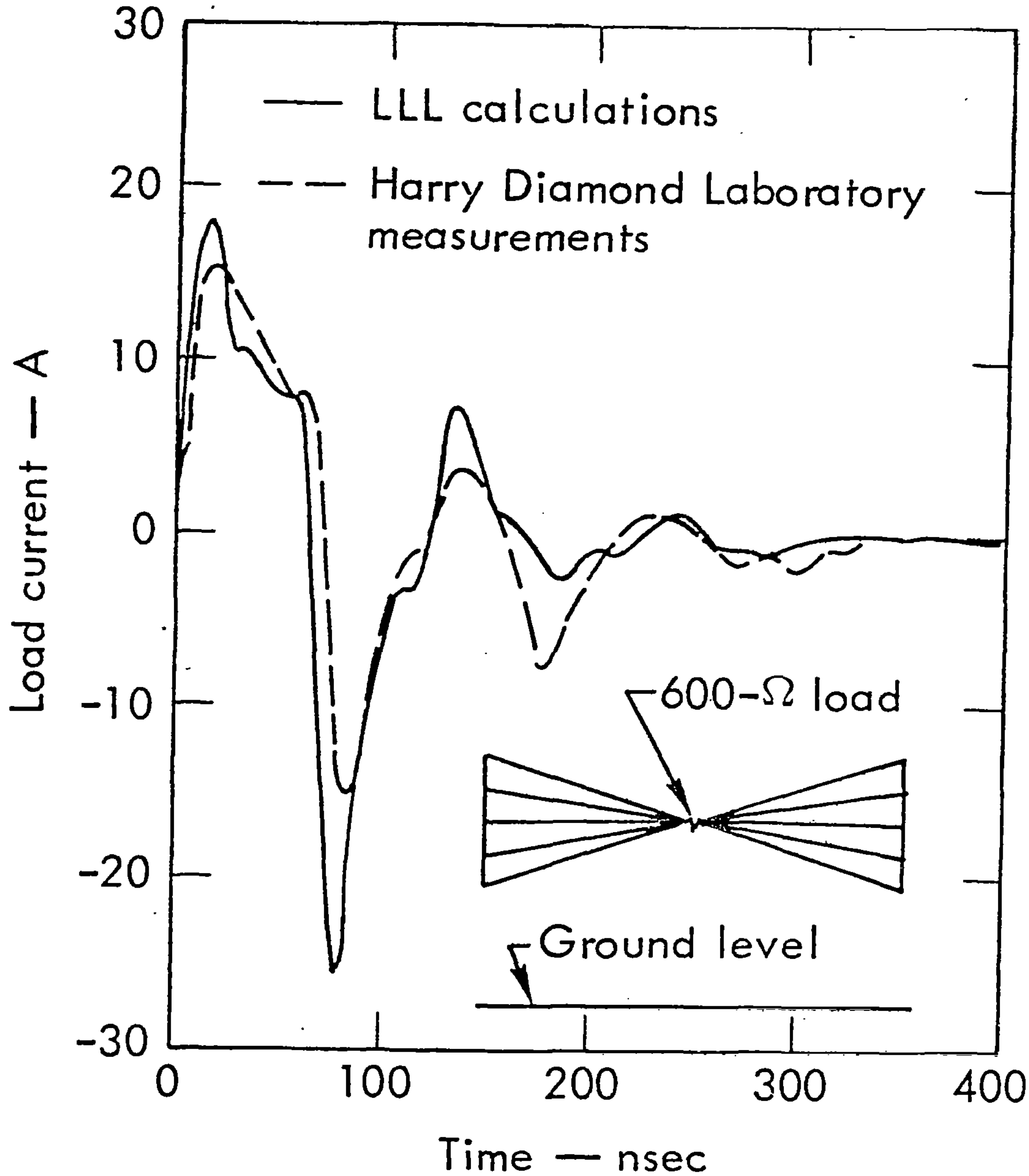

Figure 13. EMP simulated response of the fan doublet antenna (after Landt et al., 1973).

4.4 SPECIAL TOPICS

The effects of various wire ground screens and the influence of a sub-surface sea-water layer on the impedance of the SLT antenna are shown in Figs. 14 and 15 (Miller and Deadrick, 1973b). The effect of an octagonal ground plane on the input impedance of a monopole antenna computed using a combined moment-method and GTD approach due to Thiele (1973) is illustrated in Fig. 16.

The dependence of the front-to-back ratio of wire backscreens (two-dimensional model) with various numbers of wires upon wire spacing and size is demonstrated in Figs. 17 and 18 (Bevensee, 1974). Figure 19, which depicts the current distribution on a two-wire transmission line terminated in a matched load, is included to show an impedance load calculation (Miller and Deadrick, 1973b). The graph in Fig. 20 depicts results for a coated wire dipole due to Richmond (1973).

The concluding results of Fig. 21 demonstrate the transient feedpoint current response of a conical spiral antenna when it is excited by a Gaussian pulse and its input admittance derived from a Fourier transform of the current (Landt and Miller, 1974).

5. ACKNOWLEDGMENTS

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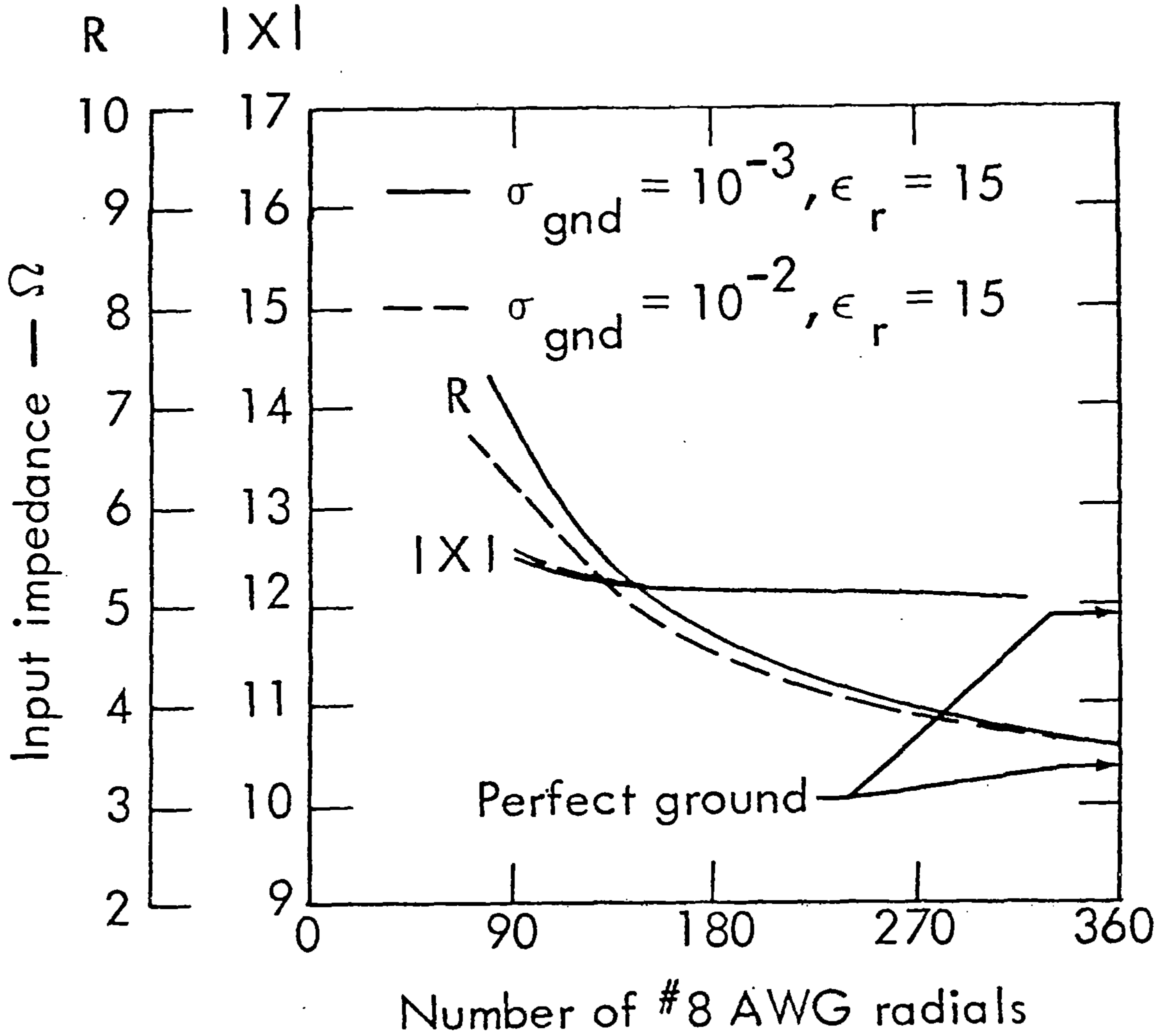

Figure 14. Influence of the ground screen on the sectionalized LORAN transmitting antenna (after Miller and Deadrick, 1973b).

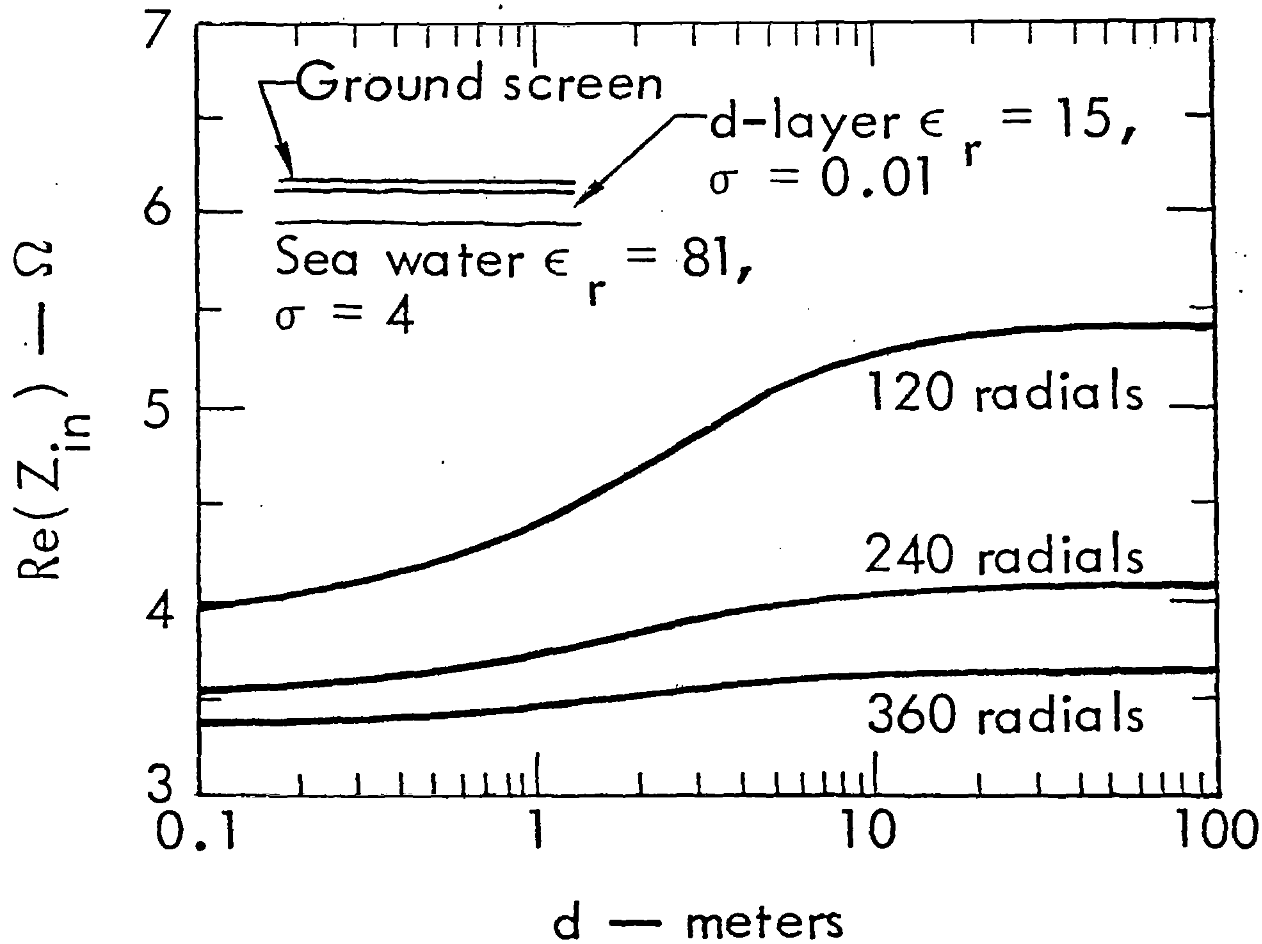

Figure 15. Effect of subsurface sea water on the sectionalized LORAN transmitting antenna (after Miller and Deadrick, 1973b).

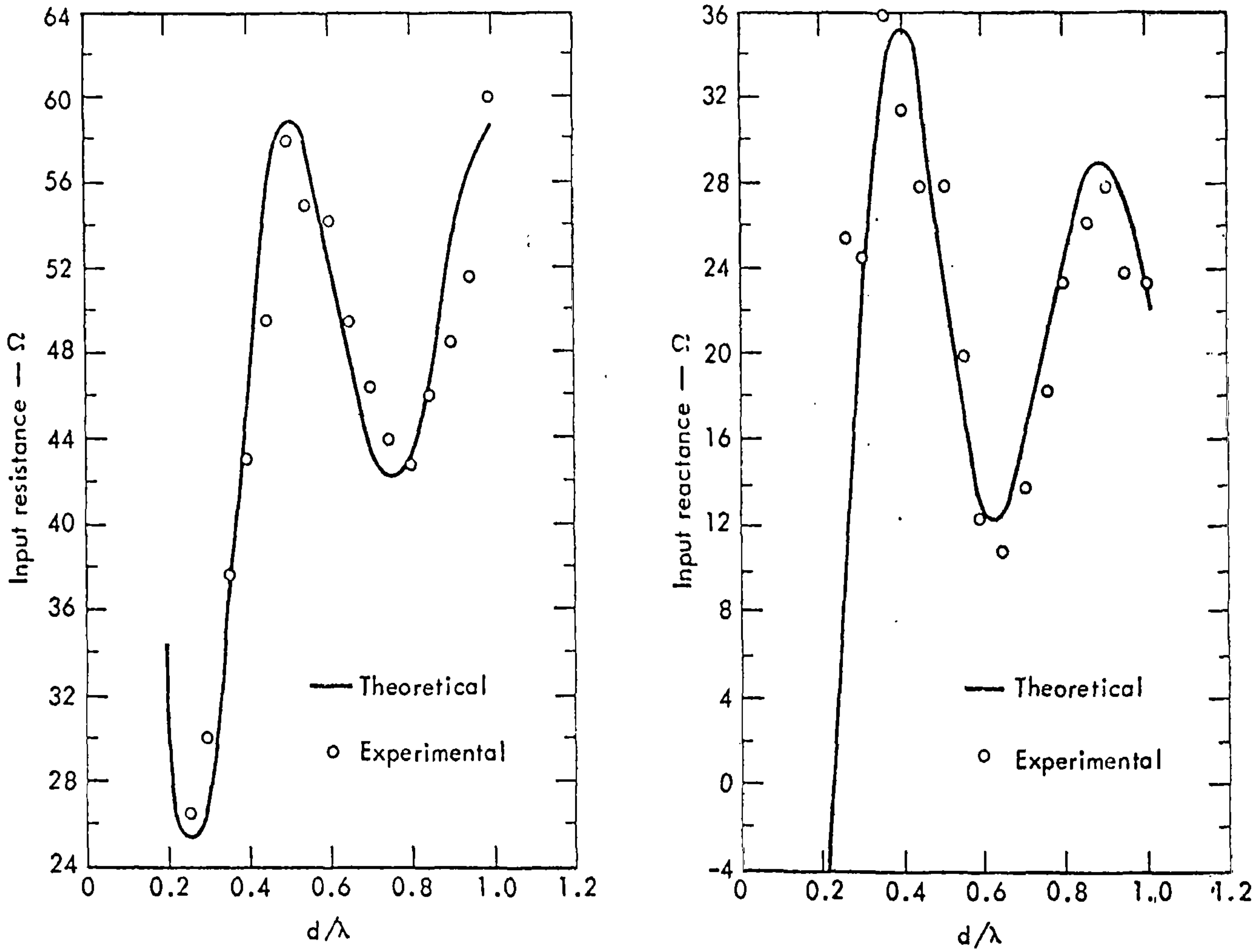

Figure 16. Application of the combined moment-method and GTD technique (after Thiele, 1973). These results are for an octagonal plate with a monopole at the center (fixed physical radius). h = 0.25λ; r = 0.1524 cm; WA = 0°.

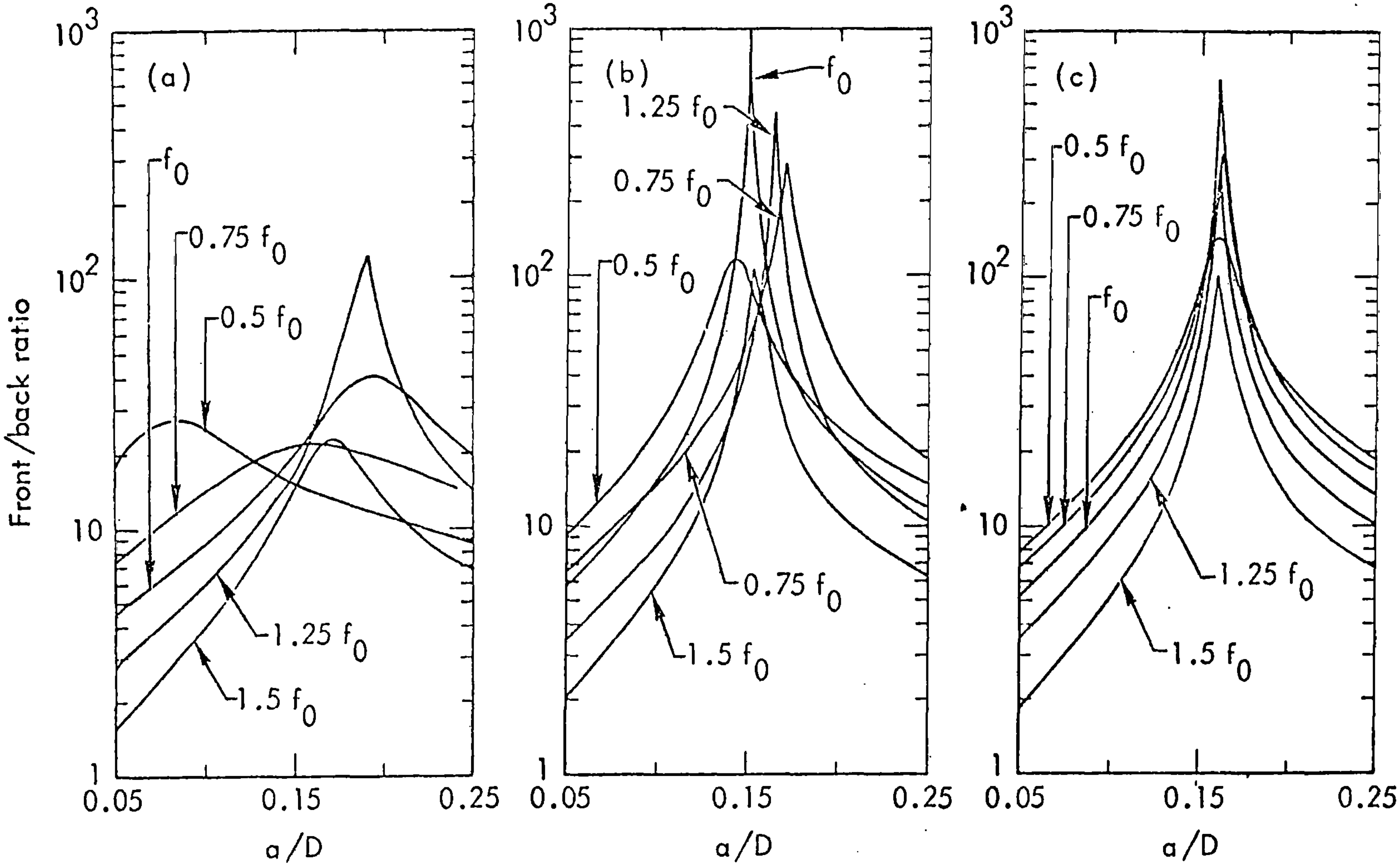

Figure 17. Some results from a backscreen optimization study (after Bevensee, 1974): Front/back ratio of screen antenna vs a/D, at various frequencies about f_0. The parameters are defined in Fig. 18. (a) 10 wires in screen. (b) 30 wires in screen. (c) 50 wires in screen.

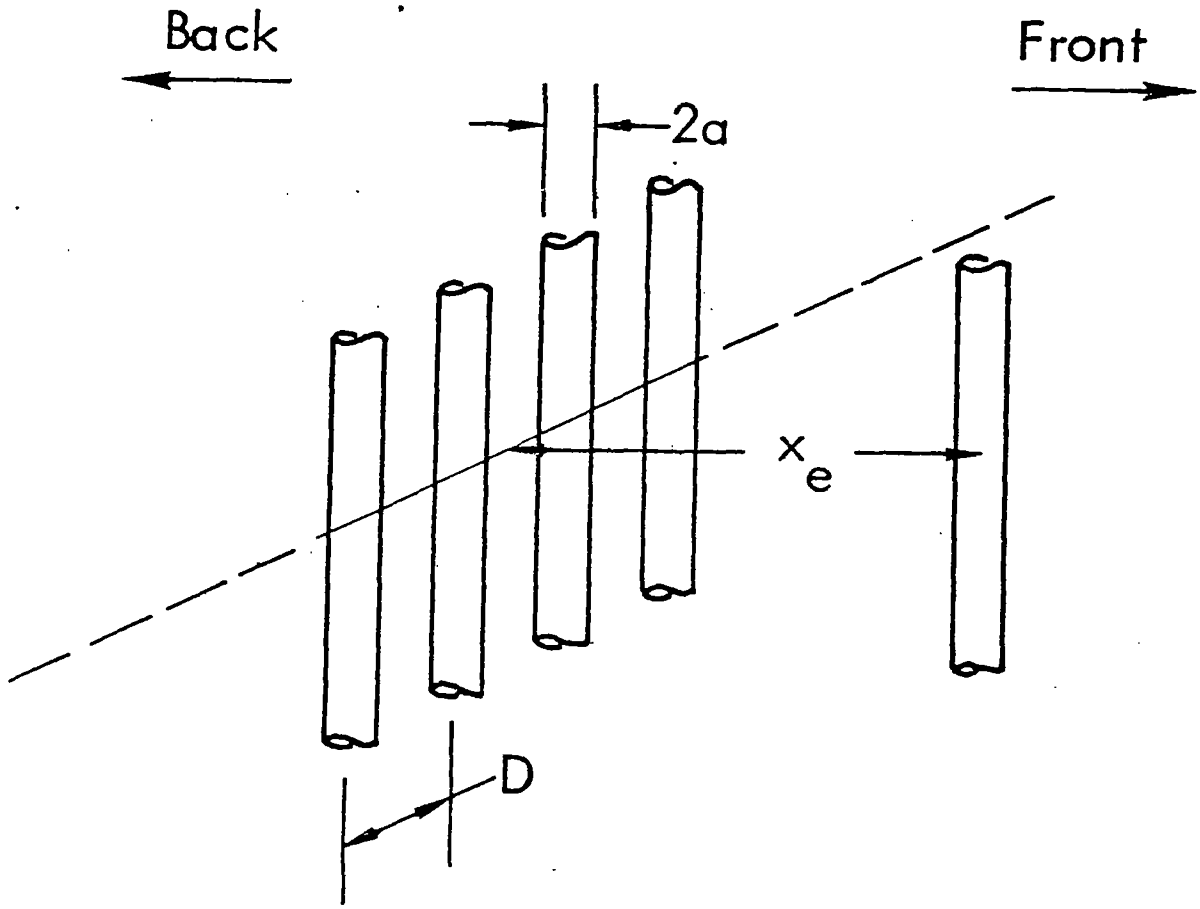

Figure 18. Definition of screen-antenna parameters plotted in Fig. 17. At f_0, $D/\lambda_0 = 0.16$, $x_e/\lambda_0 = 0.25$.

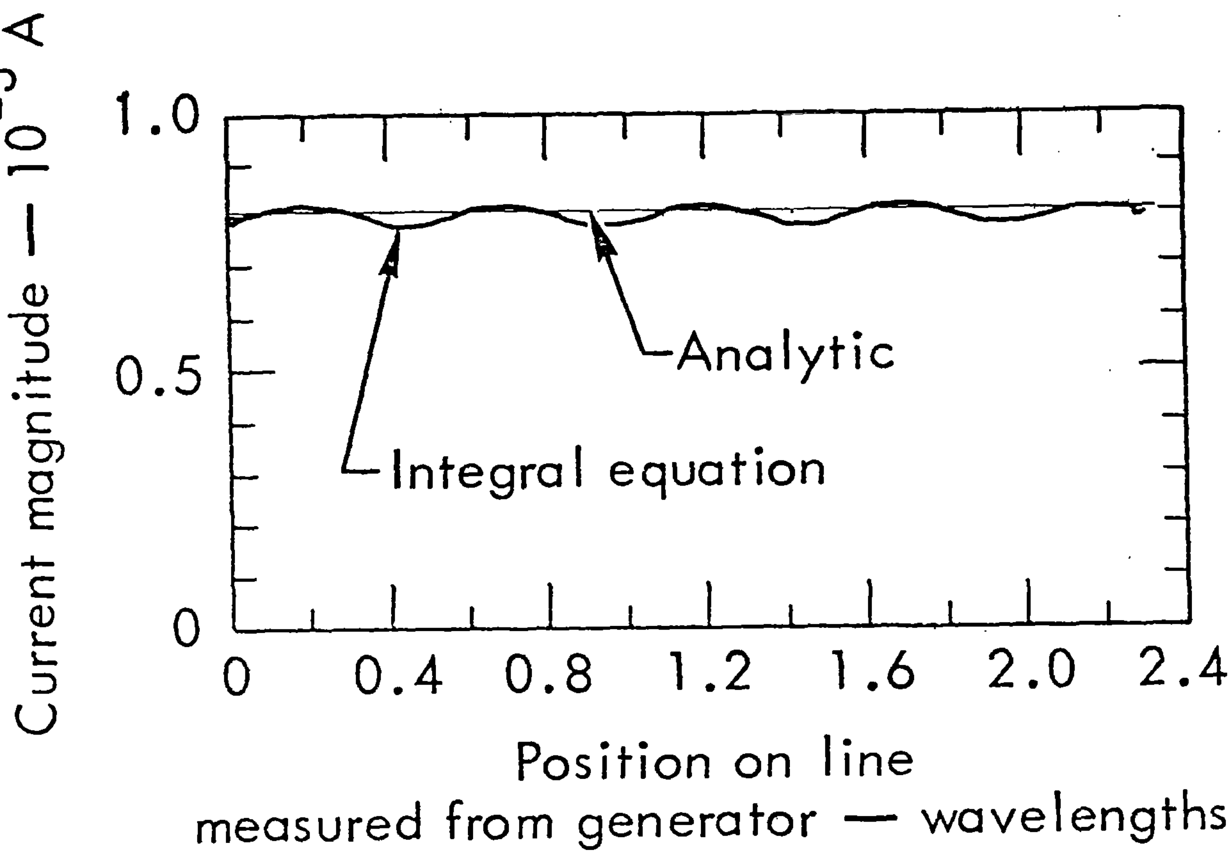

Figure 19. Integral-equation results for a matched load on a two-wire transmission line (after Deadrick and Miller, 1973b).

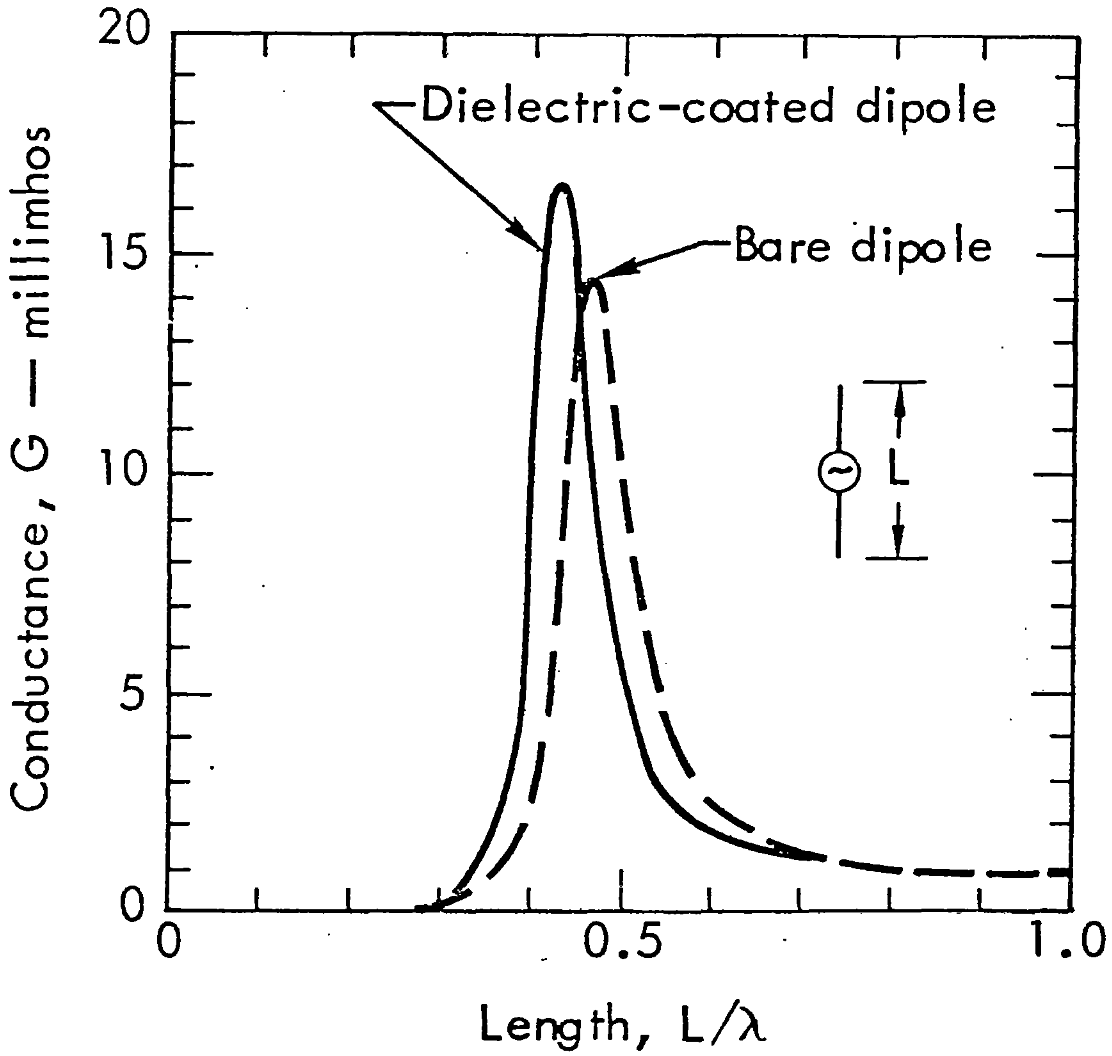

Figure 20. Results for a sheathed dipole (after Richmond, 1973). Wire diameter d = L/100; outer diameter of dielectric shell D = 2d; permittivity of shell $\varepsilon = 4\varepsilon_0$.

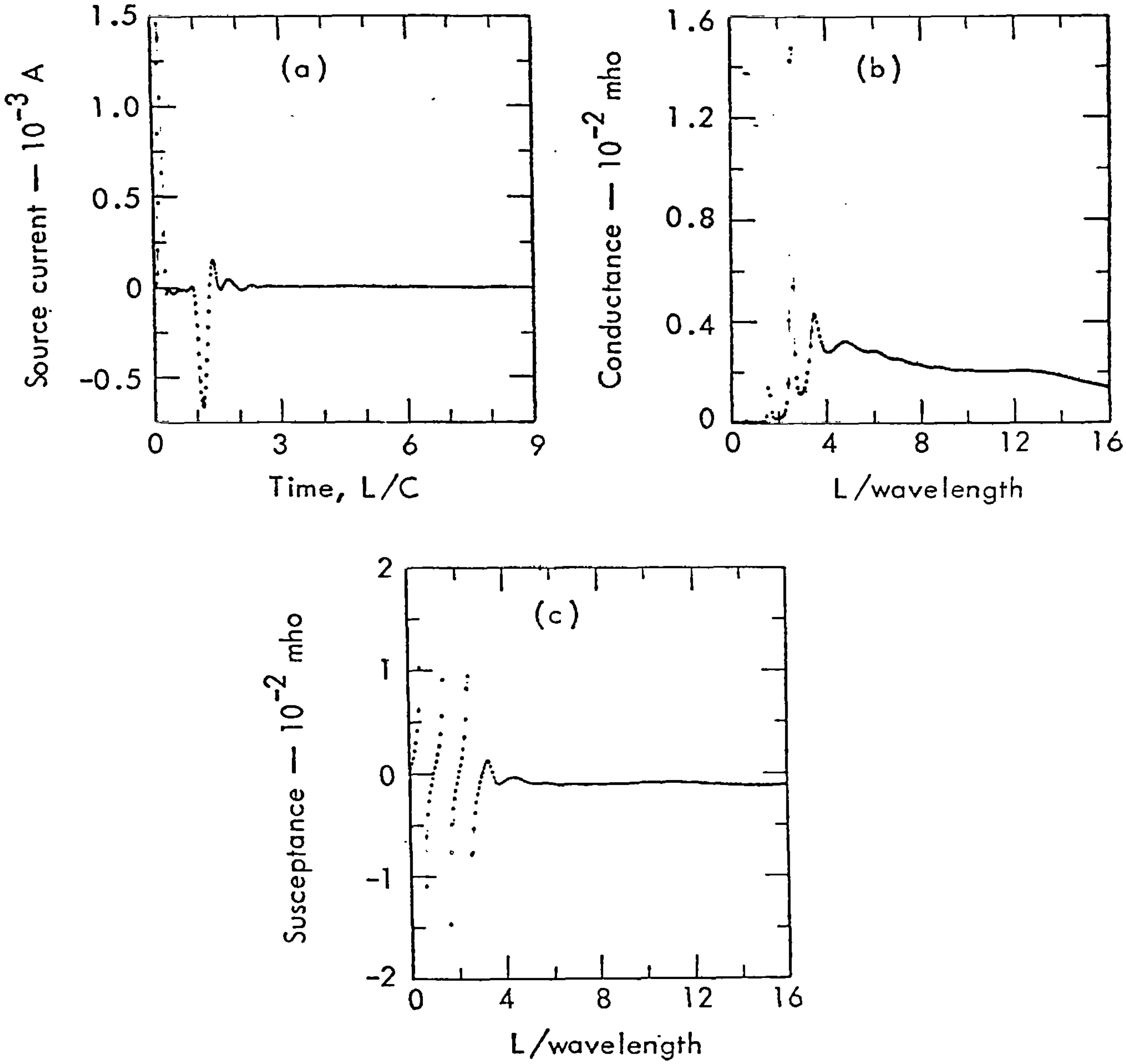

Figure 21. Time-domain computer results for a conical spiral: (a) Transient feedpoint current excited by a Gaussian pulse from a 350-Ω generator. (b) Conductance. (c) Susceptance. (b) and (c) are obtained from a Fourier transform of (a) with the generator impedance removed (after Landt and Miller, 1974).

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