INTERACTION NOTES
NOTE 249

STATISTICAL ANALYSIS OF CRITICAL LOAD EXCITATIONS INDUCED ON A RANDOM CABLE SYSTEM BY AN INCIDENT DRIVING FIELD: BASIC CONCEPTS AND METHODOLOGY

by

M.A. Morgan
F.M. Tesche

Science Applications, Inc.
Berkeley, California

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ABSTRACT

EMP internal interaction analysis is complicated by the presence of many seemingly random parameters which preclude the strictly deterministic solution for induced excitations at particular load points. The objective of this note is to briefly discuss a possible technique for the statistical analysis of load excitations on an unshielded N-wire random cable illuminated by an incident time-harmonic field using the reciprocity theorem in conjunction with subset representation of a statistical ensemble.

I. Introduction

The analysis of EMP internal coupling to critical electronic components and subsystems is complicated by the presence of many seemingly random parameters, such as the relative positions of bunched cables near POEs and the random positions of conductors in N-wire lines. These random parameters preclude the strictly deterministic solution for EMP induced excitations at particular load points. One can, of course, choose to analyze a single deterministic "average model" of the system in the hope that the excitations obtained will indicate expected excitations on any of several randomly different actual systems. If the random parameters strongly affect the coupling to certain critical system points the actual excitations may differ vastly from the deterministic predictions.

For the case of strong random effects in the system a statistical analysis should be performed in order to obtain a valid range of expected excitations.

The purpose of this paper is to give a brief discussion concerning a possible method for the statistical analysis of the load excitations on an unshielded N-wire random cable illuminated by an incident monochromatic field. The technique utilizes the concepts of time-harmonic EM field reciprocity and statistical representation of an ensemble by a subset. Although the discussion here is restricted to a limited class of structures (e.g. unshielded, unbranched N-wire cables), the method should be extendable to shielded and branched cables as well. In addition, it may be possible to conduct the analysis directly in the time-domain using Prof. Welch's time-domain reciprocity theorem, [1].

II. Probabilistic System Description

Consider an ensemble of possible N-wire cable configurations, where one such typical structure is shown in Fig. 1, for the case of N=3. The wires are assumed to run in the general direction of the z-axis. The random relative position of the K-th wire in a constant-z cross section is given by the vector $\overline{R}_k(z)$, as shown in Fig. 1.

The probabilistic system description can be formally represented as a stochastic random process where the random vector $\overline{R}_k$ is actually a function of two parameters, s and z, where s represents points in an abstract sample space, S, having a 1-1 relationship to the ensemble of possible cable configurations. To each subset of sample points (e.g. each subset of possible cable configurations), there is assigned a positive probability via the mapping into the real line segment (0,1] as illustrated in Fig. 2.

The joint probability distribution function of the positions of the wires is, in general, quite complicated.

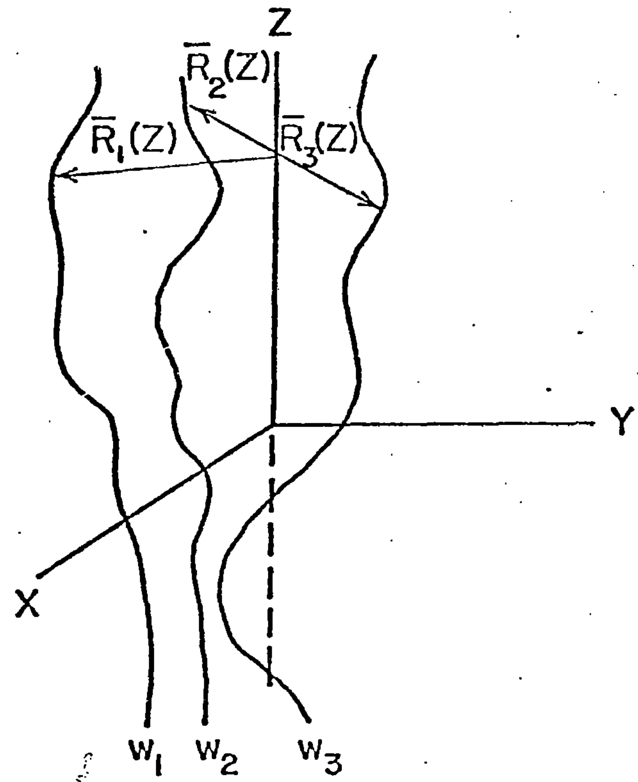

Figure 1. Random Wire Locations

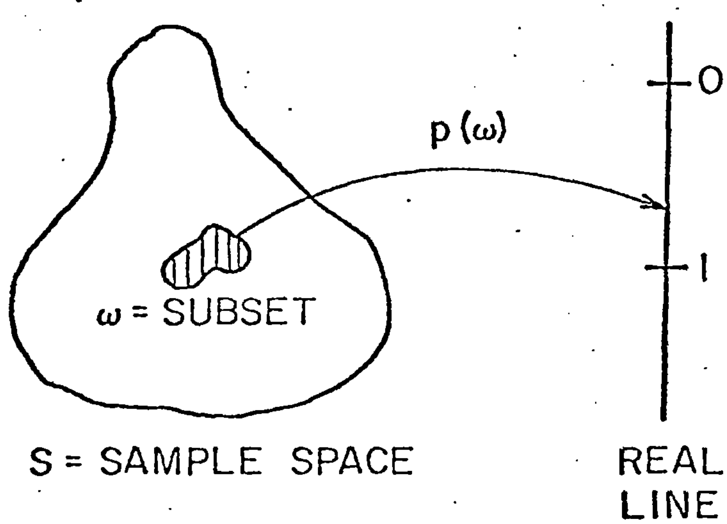

Figure 2. Probability Mapping

It depends upon all values of z since the position of each wire is affected by the positions of all wires everywhere on the line. The usual approximation that is made to simplify the statistical description of such a system is to assume that the random vectors (in this case the wire positions) are independently distributed. This allows one to define the joint probability distribution function (p.d.f) as the product of the p.d.f.s. of each wire. The assumption of independence may be quite erroneous for the case of a tightly bunched cable system and will not be employed in the statistical models to be presented.

A common method for describing an ensemble of parameter dependent random vectors (e.g. a stochastic process) is through a random coefficient basis function expansion, [2],

$$\bar{R}_k(z|s) = \sum_{j=1}^{\infty} \phi_j(z)\left[a_{jk}(s)\hat{x} + b_{jk}(s)\hat{y}\right] \qquad (1)$$

where $\left\{\phi_j(z)\right\}_{j=1}^{\infty}$ is a complete basis function set, $\hat{x}$ and $\hat{y}$ are the usual unit vectors, and $\left\{a_{jk}(s), b_{jk}(s)\right\}$ are coefficient functions of the sample point, s, in the parameter space related to the ensemble of possible cables. The probability distribution functions of the random coefficients will depend upon the basis set used and the type of cable configuration, such as straight wire parallel cable, helical twisted wire cable, or twisted pair cable, etc. Given the type of cable, it should be possible to deduce the form of the p.d.f. for the coefficients.

A second method for describing the random variation of the cable configuration is via a random walk process, [4]. In this type of description we approximate the continuous possible wire locations by a discretized model as shown in Figs. 3 and 4 having Q possible cable positions for the Q cables to occupy on a mutually exclusive basis.

At each periodic cross section, where jumps are allowed there are Q! possible cable arrangements. At the Z_2 cross section the probability of each cable arrangement is given in terms of the Q!×Q! transition matrix, $\underset{\sim}{P}(Z)$, where $P_{ij} = \text{Prob}\left\{\text{the ith configuration being realized at } Z \mid \text{given the jth configuration at } Z-\Delta Z\right\}$.

III. Use of the Reciprocity Theorem

To obtain the induced currents in the loads of a transmission line that is illuminated by some specified spatially distributed and oriented driving field one can solve either the original boundary value problem or the reciprocal problem, [3].

As an example of the use of the reciprocity theorem, consider an arbitrary N-wire line shown in Fig. 5 being illuminated by the discrete electric dipole source, $\overline{J}_1 = \overline{J}_S\, \delta(\overline{r}-\overline{r}_x)$, where $|\overline{J}_S| = 1$.

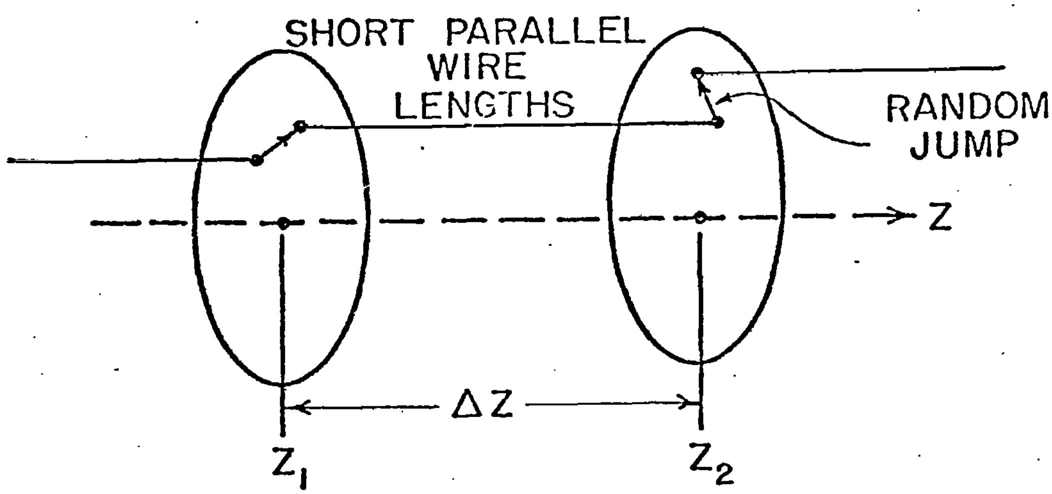

Figure 3. Stepwise Changes in Cable Model

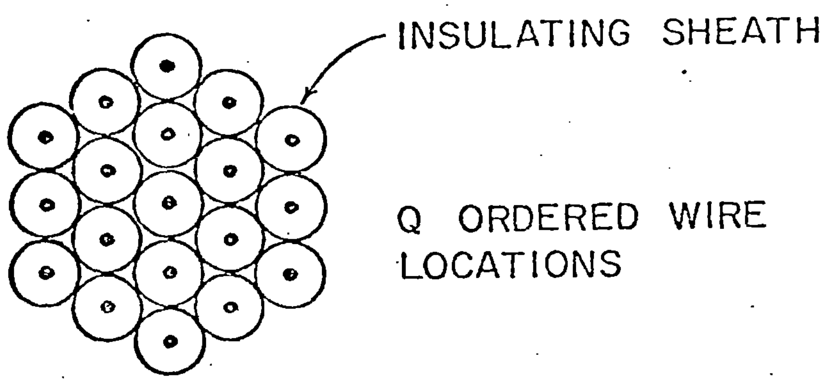

Figure 4. Bunched Cable Configuration in a Cross Section

To obtain the induced voltage across a load impedance, say for example Z_L, we place a reciprocal unit current source, J_2, across Z_L and compute the reciprocal source generated electric field at the original source point, $\overline{r_s}$. Then, using the Lorentz reciprocity theorem for a region containing all source distributions,

$$\int_{V^\infty} \overline{E}_1 \cdot \overline{J}_2 + \overline{H}_2 \cdot \overline{M}_1 \, dv = \int_{V^\infty} \overline{E}_2 \cdot \overline{J}_1 + \overline{H}_1 \cdot \overline{M}_2 \, dv \tag{2}$$

where $\overline{M}_1 = \overline{M}_2 = \overline{0}$, we obtain the induced load voltage via

$$V_L = \overline{E}_2 \cdot \overline{J}_s \tag{3}$$

For the case of a unit magnitude discrete magnetic dipole illuminating source, $\overline{M_1}$, we can determine the induced current in Z_L by placing a reciprocal discrete voltage source, as shown in Fig. 6, in series with Z_L and determining the $\overline{H_2}$ generated at $\overline{r_s}$. The load current induced in Z_L by $\overline{M_1}$ is given by

$$I_L = \overline{H_2} \cdot \overline{M_s} \tag{4}$$

The use of the reciprocity theorem to obtain induced load impedance currents and voltages requires the ability to compute the fields radiated by the arbitrary N-wire line when the line is excited at the load position by a discrete

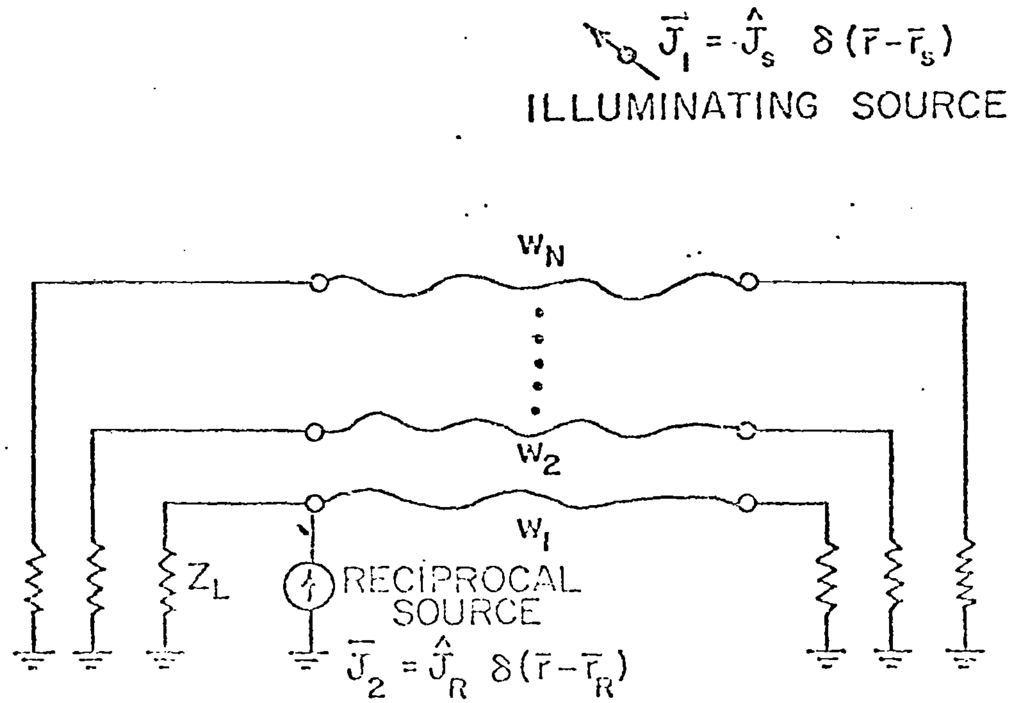

Figure 5. Arbitrary N-Wire Line with Reciprocal Excitations

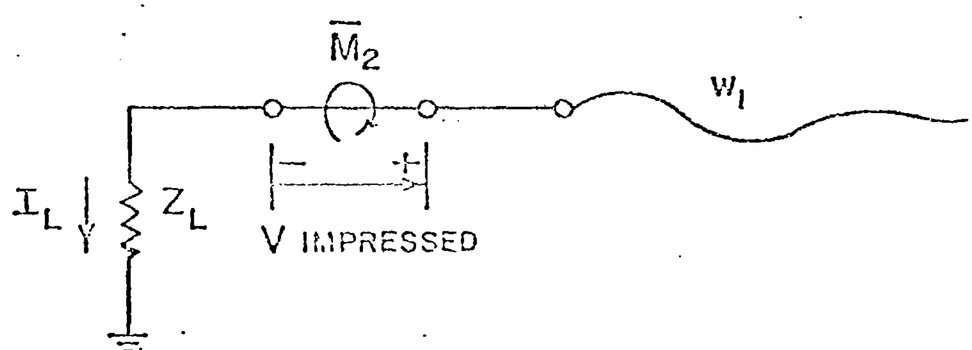

Figure 6. Reciprocal Voltage Source

current or voltage source. This may be quite a formidable task, expecially if the N-wire is located close to other obstacles such as arbitrarily oriented cables, bulkheads, etc., whose scattering properties must be considered.

IV. Minimum Subset Representation

To obtain useful knowledge about the expected induced load currents and their variances we must consider in detail the induced load currents in a finite subset of the continuous parameter ensemble sample space $\left\{s \varepsilon S\right\}$. That is, using the reciprocity theorem, we must compute the radiated fields generated by localized sources at the load positions. This can usually be done in two steps by assuming weak coupling to nearby scatterers (except massive ground plane structures). For the weak coupling case we can first compute the currents induced in the N-wire line by the localized sources, neglecting the effects of the scatterers. We then consider the field source to be the N-wire structure radiating in the presence of the scatterers and hopefully, using approximations, we can obtain at least an upper bound on the radiated field at the illuminating source point. We will only consider here the first step in this procedure.

In addition to considering the structures as being random, the incident field may also have to be treated as random. Some reasonable bounds on the maximum expected EMP induced excitations are the desired quantities of any internal interaction analysis. If the maximum drive incident

EMP field, (i.e. spatial distribution and field orientation), is not known a priori for a particular load point under consideration then the statistical expectations and variances of load responses will have to be averaged over all possible incident fields as well as possible structure variations.

We must compute the induced currents on a representative set of the ensemble of N-wire lines (possibly over a ground plane) generated by the localized reciprocal sources. A major question of great importance is: "what constitutes a representative subset of the given ensemble of N-wire lines being considered?" The detailed answer to that question will have to be found at a later date but will, of course, depend upon the type of cable configuration. At this time we will only consider some simplifications incurred by assuming certain subsets to be valid representatives of the total ensemble for statistical averaging purposes.

The simplest possible subset of random N-wire cables, as shown in Fig. 1, is the set of uniform straight parallel wires with random distances between wires. The transmission line currents on the cable can easily be computed in terms of the random vector distances between cable wires. These random variable currents will then set up random variable fields which can be obtained from a straightforward computation, [5]. Using multiple transformations of the known probability densities of the wire locations a density on the radiated fields can be obtained and the average and standard deviations of the fields can be obtained, at least in principle. The density of the physical locations of the wires in this simple model will usually be uniform for all positions within a given cylindrical boundary. The advantage of using this subset of random N-wire cables is that in most cases simple uniform transmission line theory can be used to obtain the reciprocal source induced line currents. However, the expectation and variance of the load excitations obtained using this simple model may not correctly represent the corresponding statistics for the entire random cable ensemble.

Another possible ensemble subset is the random coefficient expansion given in (1). The series will have to be trucated and the random coefficient density functions will need to be deduced. If the maximum longitudinal oscillation of the random line is slow enough the non-uniform transmission line theory can be applied to obtain the line currents in terms of the random coefficients. A direct solution for line currents can also be obtained using a coupled integral equation approach where the random wire positions appear in the kernels.

The random walk description of the cable wire positions can also be used to generate another ensemble subset.

In this case a series of piecewise uniform cables can be generated using a step by step numerical Monte Carlo random walk procedure. Each of the realized cables is then analyzed using multiple section transmission line theory.

In using the reciprocity theorem the random wire positions on the cable will not only play a part in determining the reciprocal source generated line currents but will also appear as independent parameters in the solution for the random radiated field.

V. Summary

A brief and somewhat informal discussion has been presented concerning the statistical analysis of load impedance excitations induced on a random N-wire cable by an incident time-harmonic field. The N-wire unshielded and unbranched random cable is modeled by a stochastic random process which can be approximately described using a variety of common techniques. The use of the reciprocity theorem is described for incident fields generated by discrete electric or magnetic dipole sources and its applicability is directly extendable to distributed sources. A qualitative discussion is given with regards to the subset representation of a statistical ensemble sample space. The motivation for this concerns the possible reduction in statistical modeling complexity by considering only certain geometrically simple subsets of a complex random system.

The statistical modeling approach applied to large and complex random cable structures is an extremely important topic with respect to EMP internal interaction analysis. Considerable work needs to be done to develop practically applicable methods for use on real cable systems. The emphasis here has been on a nonrigorous presentation of ideas which it is hoped will serve as a precursor for a more detailed study to be initiated in the near future.

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