C 2

INTERACTION NOTES

Note 266

# Electromagnetic Theory of the Loosely Braided Coaxial Cable, Part I

23 February 1976

James R. Wait*
C.I.R.E.S., University of Colorado/NOAA
Boulder, Colorado

ABSTRACT

A solution to Maxwell's equations subject to boundary conditions on counter-wound helical wires is achieved. The helices are contained in a cylindrical surface that is concentric to a perfectly conducting center conductor of circular cross section. The permittivity of the annular region may be different from that of the external region. The excitation is taken to by symmetrical about the cable which leads to a considerable simplification of the formulation. The key step is to recognize that the assumed form of the current on the thin helical wires is a spatial harmonic expansion that leads to a doubly infinite expansion, in such harmonics, for the resultant fields. The inherent complication of the problem results from the intercoupling betwen the spatial harmonics of the helix currents. Various generalizations of the theory are also indicated.



* Consultant to the Institute of Telecommunication Sciences, Office of Telecommunications, Boulder, CO

## INTRODUCTION

A braided coaxial cable can be envisaged as a composite counter-wound helical structure with a concentric center conductor. While the actual geometry varies greatly from one cable to another, the basic concept is that each helix carries a current that interacts with neighboring helices and with the center conductor and the insulating dielectrics. Much progress has been made in understanding the operation of braided coaxial cables by postulating equivalent circuit or transmission line parameters that characterize, in some sense, the mean electrical properties [1,2]. An example of this approach is to represent the braided wire sheath by a thin uniform cylindrical shell with a specified transfer impedance that relates the discontinuity of the tangential magnetic field [3,4]. Obviously, such a parameter has great utility when the performance of the cable in a complicated environment is to be determined. While the surface transfer impedance of the sheath and related parameters can be measured, it seems that a basic electromagnetic analysis of some idealized cases is badly needed. It is really surprising that such a general analysis has not been attempted before now although some related theoretical work in connection with travelling wave tubes has been performed [5]. Also, we should call attention to some important studies by Latham [6] and also by Lee and Baum [7] who put the transmission line theory on a firmer basis.

Our immediate purpose, then, is to formulate the problem of a cylindrical structure that consists basically of a dielectric coated conductor that is sheathed by a finite number of counter wound helices. Our first task will be to obtain the fields of a single helix that carries a filamental current that can be represented by a spatial harmonic expansion. We then add the fields of the counter-wound helix and the prescribed incident field.

An impedance boundary condition at the surface of the helical wires is then applied. The resulting infinite set of equations can be solved, in principle, for the amplitudes of the individual spatial harmonics of the filamental currents. In concept, this aspect of the problem is the same as used for determining the currents induced on a rectangular wire mesh by an incident plane wave [8,9]. Also, it should be mentioned, that Casey [10] has solved a similar problem as posed here but he assumed initially that the filamental currents were uniform. The validity of this assumption could be questioned in the general case of counter-wound helices.

## BASIC FORMULATION OF PROBLEM

With respect to a right-handed cylindrical coordinate system $(\rho,\phi,z)$, we can define a single thin-wire helix by the equation $\phi = (z/\rho_o)\tan\psi$. Here $\rho_o$ is the radius of the cylindrical surface that is common to the helix and $\psi$ is the pitch angle as illustrated in Fig. 1. The center conductor of radius a is assumed to be perfectly conducting. As indicated below, the helix wires may be imperfectly conducting and characterized by an appropriate impedance parameter that relates the filamental current to the tangential electric field. The region external to the helix (i.e. $\rho > \rho_o$) is taken to be free space with permittivity $\varepsilon_o$. An insulating dielectric of permittivity $\varepsilon$ is assumed to occupy the concentric region $\rho_o > \rho > a$. Thus we neglect any external dielectric jacket and possibly lossy external coatings although they would not introduce any new basic difficulties (just more complexity). The whole region external to the center conductor and the sheath wires is taken to have the same magnetic permeability $\mu$. In what follows, all field quantities will be taken to vary with time according to $\exp(i\omega t)$.

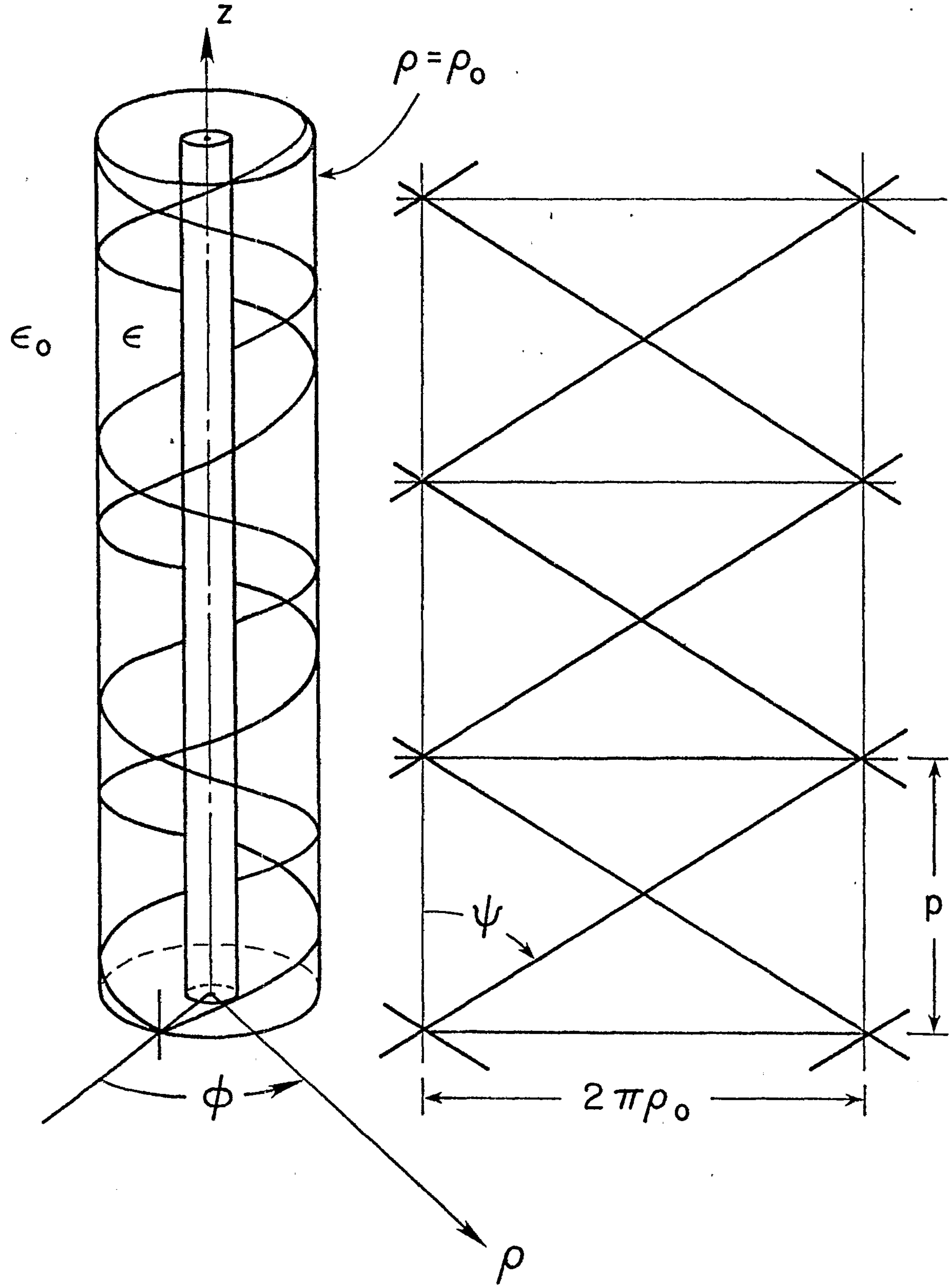


Fig. 1 Perspective view of counter-wound helices and planar development of the cylindrical surface.

In accordance with the previous discussion and for reasons that will become evident below, we adopt the following representation for the current $I(z)$ in the helix at the axial coordinate $z$:

$$I(z) = \sum_{m=-\infty}^{+\infty} I_m \exp(-i\beta_o z)\exp(-i \frac{2\pi m}{p} z) \tag{1}$$

where the summation over $m$ extends over all integers, including zero, from $-\infty$ to $+\infty$. We note here that $\beta_o$ is the mean propagation constant in the z direction for the current while $p$ is the axial period or pitch of the helix. The coefficients $I_m$ are to be determined later but for the time being we will consider the fields that result from this helical current.

To facilitate the analysis, we now observe that the components of the surface current density in the cylindrical surface at $\rho = \rho_o$ are

$$j_z(\phi,z) = I(z)\cos\psi(1/\rho_o)\delta\left(\phi - (2\pi/p)z\right) \tag{2}$$

and

$$j_\phi(\phi,z) = I(z)\sin\psi(1/\rho_o)\delta\left(\phi - (2\pi/p)z\right) \tag{3}$$

where the Dirac or impulse function can be written in its spectral form

$$\delta\left(\phi - (2\pi/p)z\right) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \exp in\left(\phi - (2\pi/p)z\right) \tag{4}$$

where the summation over $n$ extends over all integers including zero. Thus, on combining (1) to (4), we obtain

$$j_z(\phi,z) = \frac{\cos\psi}{2\pi\rho_o} \sum \sum I_m \exp(-i\beta_{m,n} z)\exp(in\phi) \tag{5}$$

and

$$j_\phi(\phi,z) = \frac{\sin\psi}{2\pi\rho_o} \sum\sum I_m \exp(-i\beta_{m,n}z)\exp(in\phi) \quad (6)$$

where $\beta_{m,n} = \beta_o + (2\pi/p)(m + n)$. The double infinite summations over $m$ and $n$, in (5) and (6), and in the subsequent equations are understood.

## FIELD REPRESENTATIONS

In general, for a homogeneous region, we can express the vector fields $\vec{E}$ and $\vec{H}$ in terms of Hertz vectors of the electric type $\vec{\Pi}$ and of the magnetic type $\vec{\Pi}^*$. Thus

$$\vec{E} = -i\mu\omega \text{ curl } \vec{\Pi}^* + (k^2 + \text{grad div})\vec{\Pi} \quad (7)$$

and

$$\vec{H} = i\varepsilon\omega \text{ curl } \vec{\Pi} + (k^2 + \text{grad div})\vec{\Pi}^* \quad (8)$$

where $k = (\varepsilon\mu)^{\frac{1}{2}}\omega$ is the wave number for the homogeneous region under consideration. For cylindrical structures, it usually seems most convenient to choose these so that $z$ components, denoted by $\Pi$ and $\Pi^*$ respectively, are non vanishing [11]. Then the field components can be obtained from

$$E_\rho = \frac{-i\mu\omega}{\rho} \frac{\partial\Pi^*}{\partial\phi} + \frac{\partial^2}{\partial\rho\partial z}\Pi \quad (9a) \qquad H_\rho = \frac{i\varepsilon\omega}{\rho}\frac{\partial\Pi}{\partial\phi} + \frac{\partial^2}{\partial\rho\partial z}\Pi^* \quad (9b)$$

$$E_\phi = i\mu\omega \frac{\partial\Pi^*}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2}{\partial\phi\partial z}\Pi \quad (10a) \qquad H_\phi = -i\varepsilon\omega \frac{\partial\Pi}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2}{\partial\phi\partial z}\Pi^* \quad (10b)$$

$$E_z = \left(k^2 + \frac{\partial^2}{\partial z^2}\right)\Pi \quad (11a) \qquad H_z = \left(k^2 + \frac{\partial^2}{\partial z^2}\right)\Pi^* \quad (11b)$$

These will be the appropriate forms to employ for the homogeneous region $a < \rho < \rho_o$. In the external region $\rho > \rho_o$, we replace $\varepsilon$ by $\varepsilon_o$ and $k$ by $k_o$ where $k_o = (\varepsilon_o\mu)^{\frac{1}{2}}\omega$.

Taking a hint from the forms adopted in (5) and (6), we choose

$$\Pi = \sum \sum \Pi_{m,n} \exp(-i\beta_{m,n} z)\exp(in\phi) \tag{12}$$

and

$$\Pi^* = \sum \sum \Pi^*_{m,n} \exp(-i\beta_{m,n} z)\exp(in\phi) \tag{13}$$

where $\Pi_{m,n}$ and $\Pi^*_{m,n}$ are functions of $\rho$ only. Since $(\nabla^2 + k_o^2)\Pi = 0$ in the region $\rho > \rho_o$, it is evident that an appropriate solution is

$$\Pi_{m,n} = A_{m,n} K_n (v_{m,n}\rho) \tag{14}$$

where $K_n$ is the modified Bessel function of the second type of order $n$ and

$$v_{m,n} = \left(\beta^2_{m,n} - k_o^2\right)^{\frac{1}{2}} = i\left(k_o^2 - \beta^2_{m,n}\right)^{\frac{1}{2}}$$

The coefficient $A_{m,n}$ is yet to be determined. In a similar fashion, for $\rho > \rho_o$, we can also write

$$\Pi^*_{m,n} = A^*_{m,n} K_n (v_{m,n}\rho) \tag{15}$$

In seeking the appropriate form of the solutions for the region $a < \rho < \rho_o$, we require that both $E_\phi$ and $E_z$ should vanish at $\rho = a$. This leads to the adoption of the following forms for this region:

$$\Pi_{m,n} = B_{m,n} Z_n (u_{m,n}\rho) \tag{16}$$

where

$$Z_n(u\rho) = I_n(u\rho) - [I_n(ua)/K_n(ua)]K_n(u\rho)$$

and

$$\Pi^*_{m,n} = B^*_{m,n} Z^*_n (u_{m,n}\rho) \tag{17}$$

where

$$Z^*_n(u\rho) = I_n(u\rho) - [I'_n(ua)/K'_n(ua)]K_n(u\rho).$$

Here $u_{m,n} = (\beta^2_{m,n} - k^2)^{\frac{1}{2}} = i(k^2 - \beta^2_{m,n})^{\frac{1}{2}}$ and we have also introduced the modified Bessel function of the first kind $I_n$. The prime over the Bessel functions indicates differeniation with respect to the indicated argument or more precisely $I'_n(ua) = dI_n(x)/dx$ evaluated at $x = ua$.

## APPLICATION OF SHEATH BOUNDARY CONDITIONS

Now the conditions at the sheath are that the tangential electric fields are continuous and that the tangential magnetic fields are discontinuous by the amount of surface current. An explicit statement is

$$E_z(\rho_o^-) = E_z(\rho_o^+) \tag{18a}$$

$$H_z(\rho_o^-) = H_z(\rho_o^+) + j_\phi(\phi,z) \tag{18b}$$

$$E_\phi(\rho_o^-) = E_\phi(\rho_o^+) \tag{18c}$$

$$H_\phi(\rho_o^-) = H_\phi(\rho_o^+) - j_z(\phi,z) \tag{18d}$$

these lead easily to the following set of equations

$$u^2ZB = v^2KA, \tag{19a}$$

$$-u^2Z^*B^* + v^2KA^* = J\sin\psi, \tag{19b}$$

$$i\mu\omega Z^{*\prime}B^* + (n\beta/\rho_o)ZB = i\mu\omega vK'A^* + (n\beta/\rho_o)KA, \tag{19c}$$

$$-i\varepsilon\omega uZ'B + (n\beta/\rho_o)Z^*B^*$$

$$+i\varepsilon_o\omega vK'A - (n\beta/\rho_o)KA^* = -J\cos\psi \tag{19d}$$

where $A = A_{m,n}$, $B = B_{m,n}$, $A^* = A^*_{m,n}$, $B^* = B_{m,n}$, $u = u_{m,n}$, $v = v_{m,n}$, $Z = Z_n(u_{m,n}\rho_o)$, $Z' = Z'_n(u_{m,n}\rho_o)$, $Z^* = Z^*_n(u_{m,n}\rho_o)$, $Z^{*\prime}(u_{m,n}\rho_o)$, $K = K_n(v_{m,n}\rho_o)$, $K'_n(v_{m,n}\rho_o)$, $\beta = \beta_{m,n}$ and $J = I_m/(2\pi\rho_o)$. The four linear equations (19a) to (19d) may be solved explicitly for the coefficients A, B, $A^*$ and $B^*$ in terms of J. Thus, for example,

$$A = \left[ i\mu\omega v \left( \frac{v}{u} \frac{Z^{*\prime}}{Z^*} K - K' \right) \left( \frac{n\beta}{u^2\rho_o} J \sin\psi - J \cos\psi \right) - \frac{n\beta}{\rho_o} K \left( \frac{v^2}{u^2} - 1 \right) \frac{i\mu\omega}{u} \frac{Z^{*\prime}}{Z^*} J \sin\psi \right] D^{-1} \tag{20}$$

and

$$A^* = \left[ \frac{k_o^2 v}{u} \frac{Z^{*\prime}}{Z^*} \left( \frac{\varepsilon}{\varepsilon_o} \frac{v}{u} \frac{Z'}{Z} K - K' \right) J \sin\psi - \left( \frac{n\beta}{u^2\rho_o} J \sin\psi - J \cos\psi \right) \frac{n\beta}{\rho_o} K \left( \frac{v^2}{u^2} - 1 \right) \right] D^{-1} \tag{21}$$

where

$$D = k_o^2 v^2 \left[ \frac{v}{u} \frac{Z^{*\prime}}{Z^*} K - K' \right] \left[ \frac{\varepsilon}{\varepsilon_o} \frac{v}{u} \frac{Z'}{Z} K - K' \right] - \left( \frac{n\beta}{\rho_o} K \right)^2 \left( \frac{v^2}{u^2} - 1 \right)^2 \tag{22}$$

The tangential electric fields in the region external to the sheath (i.e. $\rho > \rho_o$) are given by

$$E_\phi = \sum \sum \left[ i\mu\omega v_{m,n} A^*_{m,n} K'_n(v_{m,n}\rho) + \frac{1}{\rho} n\beta_{m,n} A_{m,n} K_n(v_{m,n}\rho) \right] \times \exp(in\phi) \exp(-i\beta_{m,n} z) \tag{23}$$

and

$$E_z = - \sum \sum v^2_{m,n} A_{m,n} K_n(v_{m,n}\rho)\exp(in\phi)\exp(-i\beta_{m,n}z) \tag{24}$$

where the coefficients $A_{m,n}$ and $A^*_{m,n}$ are given in terms of the current on the right-handed helix via (21) and (22). Also, we should remember that $v_{m,n} = (\beta_{m,n} - k_o^2)^{\frac{1}{2}}$ and $\beta_{m,n} = \beta_o + (2\pi/p)(m + n)$.

To obtain the fields of the current on the corresponding left-handed helix we can proceed precisely in the same fashion. This helix is defined by $\phi = -(2\pi/p)z$ at $\rho = \rho_o$. Also, for the case of usual concern, the current $I(z)$ on this helix will be the same as for the right-handed helix given by (1). The exception discussed later is when the excitation is not locally uniform about the cable. Thus, for this symmetrical situation, the sheath current densities, corresponding to (5) and (6), are

$$\hat{j}_z(\phi,z) = \frac{\cos\psi}{2\pi\rho_o} \sum \sum I_m \exp(-i\hat{\beta}_{m,n}z)\exp(in\phi) \tag{25}$$

and

$$\hat{j}_\phi(\phi,z) = - \frac{\sin\psi}{2\pi\rho_o} \sum \sum I_m \exp(-i\hat{\beta}_{m,n}z)\exp(in\phi) \tag{26}$$

where

$$\hat{\beta}_{m,n} = \beta_{m,-n} = \beta_o + (2\pi/p)(m - n)$$

As indicated, we place a circumflex over the quantity when it refers to the changed form needed for the left-handed helix. The tangential electric fields in the region external to the sheath, that are analogous to (23) and (24), are

$$\hat{E}_\phi = \sum \sum [i\mu\omega v_{m,-n}\hat{A}^*_{m,n}K'_n(v_{m,-n}\rho) + \frac{1}{\rho} n\beta_{m,-n}\hat{A}_{m,n}K_n(v_{m,-n}\rho)] \times \exp(in\phi)\exp(-i\beta_{m,-n}z) \tag{27}$$

and

$$\hat{E}_z = - \sum \sum v^2_{m,-n} \hat{A}_{m,n} K_n (v_{m,-n}\rho) \exp(in\phi) \exp(-i\beta_{m,-n} z) \tag{28}$$

The coefficients $\hat{A}$ and $\hat{A}^*$ are given by (20) and (21) with the reversed sign for $\psi$ (i.e. replace $\sin\psi$ by $-\sin\psi$). We also should note that $K$ and $Z$ are replaced by $\hat{K}$ and $\hat{Z}$ defined by $\hat{K} = K_n(v_{m,-n}\rho_o)$ and $\hat{Z} = Z_n(u_{m,-n}\rho_o)$.

It is useful now to note, according to (20) and (21), that

$$A_{m,n} = P_{m,n} I_m \; , \quad A^*_{m,n} = P^*_{m,n} I_m$$

and (29)

$$\hat{A}_{m,n} = \hat{P}_{m,n} I_m \; , \quad \hat{A}^*_{m,n} = \hat{P}^*_{m,n} I_m$$

where the P's are explicitly known in terms of the counter-wound helix geometry and the specified value of the axial wave number $\beta_o$.

## APPLICATION OF WIRE BOUNDARY CONDITION

We are now in the position to apply the impedance condition at the helix wires. Since the wires themselves have already been assumed to be very thin, the longitudinal electric field at the surface of the wires is sensibly uniform around the wire circumference. Thus, for convenience, we choose to apply the impedance condition at the top of the wires which by definition is the spiral $z = (p/2\pi)\phi + c/\sin\psi$ where c is the wire radius. This is indicated in the sketch in Fig. 2. Also, because of the assumed rotational symmetry we need only apply the condition on one helix. The corresponding condition on the other helix will be automatically satisfied. Thus, we need to apply

$$\left[\left(E_z + \hat{E}_z\right)\cos\psi + \left(E_\phi + \hat{E}_\phi\right)\sin\psi + E^p_z \cos\psi\right] = I(z) Z_w \tag{30}$$

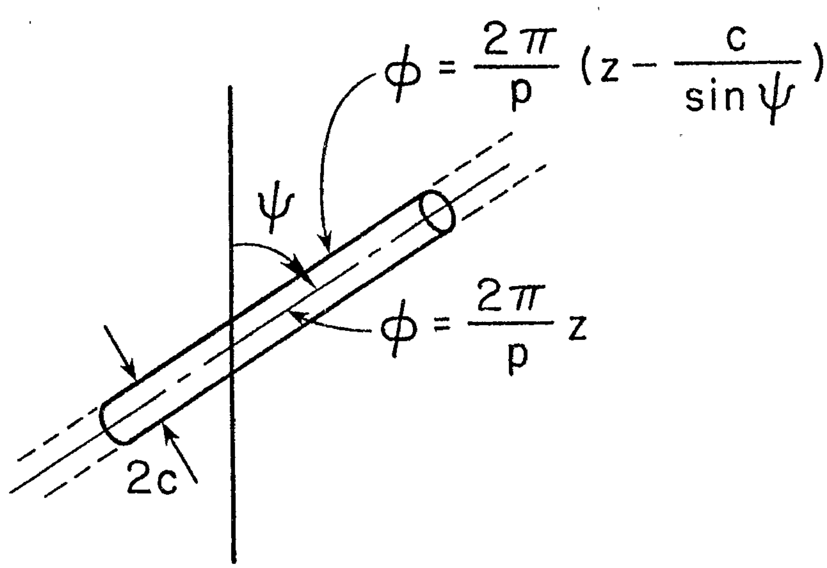


Fig. 2 Microscopic view of segment of helix wire.

at $\rho = \rho_o$ and $\phi = (2\pi/p)[z - c/(\sin\psi)]$. Here $E_z^p$ is the axial component of the "primary" field; it is the resultant field that would exist at the surface $\rho = \rho_o$ for the same cylindrical structure but in the absence of the helix wires. As mentioned above, $E_z^p$ can be regarded as invariant to $\phi$ when $\rho_o$ is much less than the free space wavelength (i.e. $k_o\rho_o << 1$). The series impedance per unit length $Z_w$ is determined by the local property of the wires and treated as if they were straight [11]. This appears to be justified always when $c << \rho_o$. Thus, if the electrical constants of the wires are $\sigma_w$, $\varepsilon_w$ and $\mu_w$ we would use the usual relation

$$Z_w = [\eta_w/(2\pi c)] I_o(\gamma_w c)/I_1(\gamma_w c) \tag{31}$$

where $\eta_w = [i\mu_w\omega/(\sigma_w + i\varepsilon_w\omega)]^{\frac{1}{2}}$ and $\gamma_w = [i\mu_w\omega(\sigma_w + i\varepsilon_w\omega]^{\frac{1}{2}}$. As $\omega \to 0$ this reduces to the expected DC form, namely $Z_w \to (\sigma_w\pi c^2)^{-1}$.

Using (23), (24), (27), (28), and (29), the impedance condition (30) now takes the form

$$\sum_n \sum_m I_m R_{m,n} \exp\left(-in \frac{2\pi c}{p \sin\psi}\right) \exp\left(-i \frac{2\pi}{p} mz\right)$$

$$+ \sum_n \sum_{m'} I_{m'} \hat{R}_{m',n} \exp\left(-in \frac{2\pi c}{p \sin\psi}\right) \exp\left(in \frac{4\pi}{p} z\right) \exp\left(-i \frac{2\pi}{p} m'z\right) \tag{32}$$

$$+ \sum_m E_z^p \delta_{m,o} \exp\left(-i \frac{2\pi m}{p} z\right) \cos\psi = Z_w \sum_m I_m \exp\left(-i \frac{2\pi m}{p} z\right)$$

where we have used $m'$ in place of $m$ in the second term for convenience in the subsequent manipulation. The coefficients $R$ and $\hat{R}$ are defined by

$$R_{m,n} = -v_{m,n}^2 \cos\psi\, P_{m,n} K_n(v_{m,n}\rho_o)$$

$$+ i\mu\omega v_{m,n} \sin\psi\, P_{m,n}^* K_n'(v_{m,n}\rho_o) + \frac{\sin\psi}{\rho_o} n\beta_{m,n} P_{m,n} K_n(v_{m,n}\rho_o) \tag{33}$$

and

$$\hat{R}_{m,n} = -v^2_{m,-n} \cos\psi \, \hat{P}_{m,n} K_n(v_{m,-n}\rho_o)$$

$$-i\mu\omega v_{m,-n} \sin\psi \, \hat{P}^*_{m,n} K'_n(v_{m,-n}\rho_o) - \frac{\sin\psi}{\rho_o} n\beta_{m,-n} \hat{P}_{m,n} K_m(v_{m,-n}\rho_o) \tag{34}$$

Now in (32) we replace $m'$ by $2n + m$ which means that factor $\exp\left(-i(2\pi/p)mz\right)$ is now common to all terms. Then, since (32) is to hold for all z we obtain

$$\left[\sum_{n=-\infty}^{+\infty} R_{m,n} \exp\left(-in \frac{2\pi c}{p\sin\psi}\right)\right] I_m + \left[\sum_{n=-\infty}^{+\infty} \hat{R}_{2n+m,n} \exp\left(-in \frac{2\pi c}{p\sin\psi}\right)\right] I_{2n+m}$$

$$+ E^p_z \, \delta_{mo} \cos\psi = Z_w I_m \tag{35}$$

which is to hold for all integer values of m from $-\infty$ to $+\infty$. This, of course, is then an infinite set of equations that must be solved, after appropriate truncation, for the current coefficients $I_m$.

## THE SPECIFICATION OF THE PRIMARY FIELD

At this point we should obtain the appropriate expression for the primary field $E^p_z$ that is mentioned above. The incident plane wave is defined by

$$\vec{E} = \vec{E}_o \exp[i\alpha_o \rho\cos\phi]\exp(-i\beta_o z) \tag{36}$$

where $\alpha_o = (k^2_o - \beta^2_o)^{\frac{1}{2}}$. Here we can identify $\beta_o$ with $k_o \cos\theta$ where $\theta$ is the angle subtended by the wave normal and the negative z axis. Now, in the vicinity of the cable, we can assume for purposes of simplicity that $\alpha_o\rho$ or $k_o\rho\sin\theta$ is much less than one. Also, under the same condition, we only need to be concerned with the axial component of the incident field since the transverse components have a negligible interaction. This leads us to use a quasi-static analysis [4] in order to determine the primary field $E_z^p$. Thus, the required field forms for the coaxial structure, in the absence of the helix wires, are

$$\left.\begin{aligned} E_z &= \alpha^2[P + \frac{2}{\pi} Q \ln 0.89\ \alpha\rho] \quad &(37)\\ H_\phi &= -(2/\pi)i\varepsilon\omega Q/\rho \quad &(38)\end{aligned}\right\} \text{for } a < \rho < \rho_o$$

and

$$\left.\begin{aligned} E_z &= E_{oz}[1 + R(1 - i(2/\pi)\ln 0.89\ \alpha_o\rho] \quad &(39)\\ H_\phi &= -E_{oz}(2/\pi)\varepsilon_o\omega R/(\alpha_o^2\rho) \quad &(40)\end{aligned}\right\} \text{for } \rho > \rho_o$$

where $\alpha = (k^2 - \beta_o^2)^{\frac{1}{2}}$. We now apply the boundary conditions that $E_z$ is zero at $\rho = a$ and that both $E_z$ and $H_\phi$ are continuous at $\rho = \rho_o$. Thus we readily deduce that

$$P = -(2/\pi)Q \ln 0.89\ \alpha a \tag{41}$$

$$Q = -i(\varepsilon_o/\varepsilon)(R/\alpha_o^2)E_{oz} \tag{42}$$

and

$$R = -\left[i\frac{2}{\pi}\frac{\varepsilon_o}{\varepsilon}\frac{\alpha^2}{\alpha_o^2}\ln\frac{\rho_o}{a} + 1 - i\frac{2}{\pi}\ln 0.89\ \alpha_o\rho_o\right]^{-1} \tag{43}$$

Then, in fact

$$E_z^p = E_z\Big|_{\rho=\rho_o} = \alpha^2\frac{2}{\pi}Q\ln\frac{\rho_o}{a} \tag{44}$$

or, more explicitly

$$\frac{E_z^p}{E_{oz}} = \frac{i\frac{2}{\pi}\frac{\varepsilon_o}{\varepsilon}\frac{\alpha^2}{\alpha_o^2}\ln\frac{\rho_o}{a}}{1 + i\frac{2}{\pi}\left(\frac{\varepsilon_o}{\varepsilon}\frac{\alpha^2}{\alpha_o^2}\ln\frac{\rho_o}{a} - \ln 0.89\ \alpha_o\rho_o\right)} \tag{45}$$

## GENERALIZATIONS AND CONCLUDING REMARKS

There is a generalization as illustrated in Fig. 3 that we can mention briefly. If we have Q right-handed and Q left-handed spirals, that are equi-spaced, the formulation is only slightly more involved. For example, the equation for the right-handed helices is

$$\phi = (2\pi/p)(z - (q/Q)) \quad \text{at} \quad \rho = \rho_o \tag{46}$$

where $q = 0,1,2,3,\ldots.\ Q - 1$. If the helices are all made of identical wires, then the current on each can still be given by (1) in view of the assumed azimuthal uniformity of the excitation. But now, for example, the z component of the surface current in the sheath for the right-handed helices has the form

$$j_z(\phi,z) = I(z)\,\frac{\cos\psi}{\rho_o}\sum_{q=0}^{Q-1}\delta\left(\phi - \frac{2\pi}{p}z - \frac{2\pi}{Q}q\right) \tag{47}$$

in place of (2). Then, using the spectral representation for the impulse functions, we find that

$$j_z(\phi,z) = \frac{\cos\psi}{2\pi\rho_o}\sum_{q=0}^{Q-1}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} I_m \exp(-i\beta_{m,n}z)\exp(-i2\pi nq/Q)\exp(in\phi) \tag{48}$$

with a similar form for $j_\phi(\phi,z)$. Thus, on comparing with (5) and (6), it is evident that the essential modification of the formulation is to introduce the factor $\exp(-i2\pi nq/Q)$ with a summation over the number of separate helices. In dealing with the left-handed helices, we introduce a corresponding factor $\exp(+i2\pi nq/Q)$.

The boundary condition indicated by (30) still may be applied at the one helix only. The resulting coupled equation to determine the coefficients $I_m$ is again given by the following modification of (35):

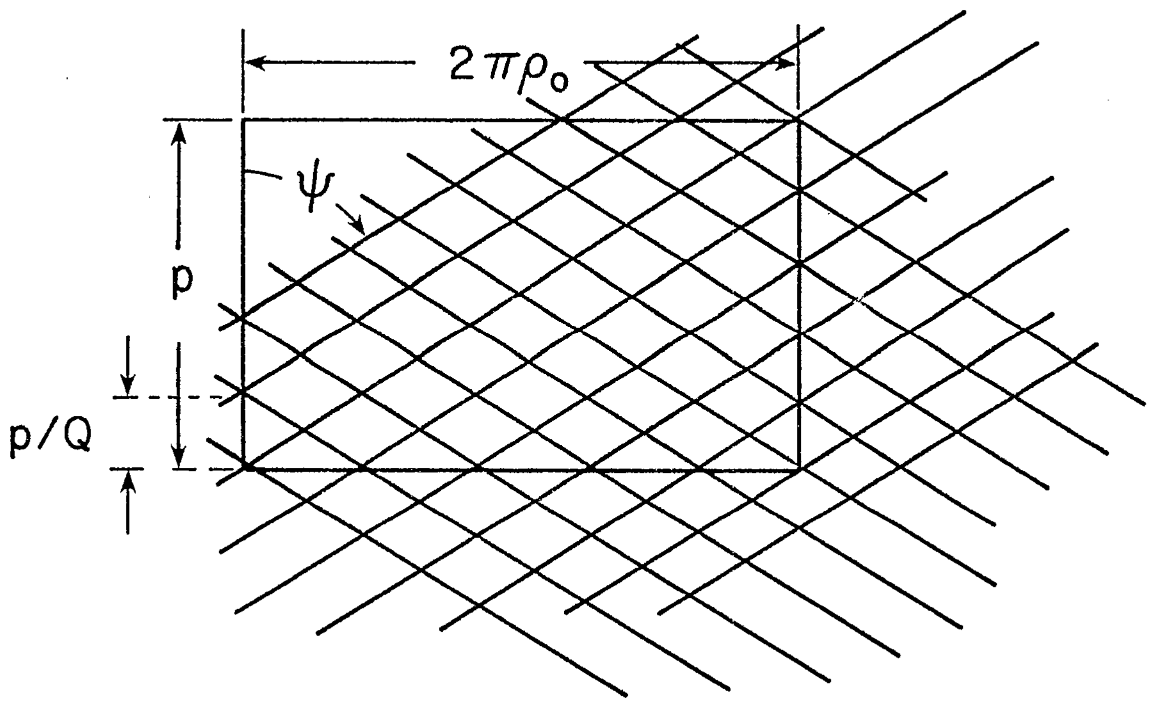


Fig. 3 Planar development of multi counter-wound helices (drawn for $Q = 5$).

$$\sum_{q=0}^{Q-1}\left\{\sum_{n=-\infty}^{+\infty} R_{m,n} \exp\left(-in \frac{2\pi c}{psinc}\right) \exp\left(-i \frac{2\pi nq}{Q}\right) I_m \right.$$

$$\left. + \sum_{n=-\infty}^{+\infty} R_{2n+m,n} \exp\left(-in \frac{2\pi c}{psinc}\right) \exp\left(+i\frac{2\pi nq}{Q}\right) I_{2n+m} \right\} \tag{49}$$

$$+ E_z^p \delta_{m,o} \cos\psi = Z_w I_m$$

Actually there is some simplification to (48) and (49) by noting that

$$\sum_{q=0}^{Q-1} \exp(\mp i 2\pi nq/Q) = \begin{cases} Q \text{ if } n = \ell Q \\ 0 \text{ if } n \neq \ell Q \end{cases} \tag{50}$$

where $\ell = 0, \pm 1, \pm 2, \pm 3 \ldots$

Another generalization that does not lead to any basic difficulty, at least for symmetrical excitation, is to remove the assumption of local uniformity of the excitation field. This amounts to representing $E_z^p$ itself as a harmonic expansion in the azimuth direction about the cable axis. Such a modification is only necessary, however, when the cable cross-section becomes comparable with a wavelength. In that case, we would also need to account for the presence of the transverse components of the exciting fields in which case the filamental currents on the counter-wound helices are no longer the same. In this situation, we would also need to be concerned with whether the counter-wound helices were bonded at their intersections. In analog to the work on planar wire meshes (e.g. Hill and Wait, [9]), the difference between bonded and unbonded wire intersections could be significant for the non-symmetrical component of the excitation field. One method to analyze this situation is to allow the right-handed and the left-handed helices to have slightly different radii. For the symmetrical excitation, which in fact is a good approximation at low frequencies, the final results would not be very sensitive to the difference between the helix radii. Thus, we should not expect the bonding to have a major influence on the low frequency performance of the cable. Nevertheless, this is a subject that should be investigated in a quantitative sense.

The influence of a dielectric jacket and/or lossy external coating on the cable can be considered in a straight-forward manner. Basically, this amounts to introducing radial wave functions in the region external to the sheath that satisfy the appropriate boundary conditions at the one or more new cylindrical interfaces. Finally, we should mention that the corresponding natural modes of propagation on the composite structure are obtained by simply letting the incident field be zero and then setting the (infinite) determinant of the coefficients of $I_m$ in (35) or (49) to zero and solving for the propagation constant(s) $i\beta_o$. Such solutions would include the surface waves that have their energy confined to the region of the sheath.

In Part II, we consider the numerical aspects of this general problem and the results are applied to specific cable configurations.

## APPENDICES

### a) A NOTE ON CONVERGENCE

The series over $n$ have the following form

$$S = \sum_{n=-\infty}^{+\infty} A_n \exp\left[-in \frac{2\pi c}{p \sin\psi}\right] \tag{51}$$

The convergence may be very poor since $c$ is small. Thus, as in other similar problems, there is some merit in summing the higher order terms in closed form by making use of the fact that the coefficient $A_n$ admits to an asymptotic expansion of the type

$$\lim_{(|n|\to\infty)} A_n \sim \pm n \left(A^{(0)} + \frac{A^{(1)}}{n^2} + \ldots\right) \tag{52}$$

This suggests that we write

$$\begin{aligned} S = A_o &+ \sum_{n=1}^{\infty} \left(A_n - nA^{(0)} - \frac{A^{(1)}}{n}\right) \exp\left[-in \frac{2\pi c}{p \sin\psi}\right] \\ &+ \sum_{n=1}^{\infty} \left(A_{-n} - nA^{(0)} - \frac{A^{(1)}}{n}\right) \exp\left[+in \frac{2\pi c}{p \sin\psi}\right] + \Delta S \end{aligned} \tag{53}$$

where

$$\Delta S = 2A^{(1)} \sum_{1}^{\infty} \frac{1}{n} \cos n \frac{2\pi c}{p \sin\psi} = - 2A^{(1)} \ln \left(2 \sin \frac{2\pi c}{p \sin\psi}\right) \qquad (54)$$

Here we have utilized the fact that $\sum_{n=-\infty}^{+\infty} n \exp(\pm inx) = 0$ for all real $x > 0$.

## b) FIELD AVERAGING

In general, a field component $\psi$ has the following doubly-infinite series representation

$$\psi = \sum_{n=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \psi_{m,n} \exp\left(-i\beta_{m,n} z\right) \exp(in\phi) \qquad (55)$$

where $\psi_{m,n}$ is a coefficient that does not depend on the coordinates $\phi$ and $z$. Now when dealing with a coaxial cable with electrically small radius and for the case where the axial period of the braid is small, the far-field scattered from the cable only involves the term for $n = m = 0$. It also follows rather simply that the "average" field $\psi$ near or at the cable also can be described by this term. This follows from the fact that

$$\overline{\psi} = \frac{1}{p} \int_{0}^{p} \left[\frac{1}{2\pi} \int_{0}^{2\pi} \psi d\phi\right] e^{i\beta_o z} \, dz = \psi_{o,o} \qquad (56)$$

In view of the above reasoning, it follows that a suitable definition of the *effective* axial impedance $Z_e(i\beta_o)$ of the cable is

$$Z_e(i\beta_o) = \overline{E}_z/(2\pi\rho\overline{H}_\phi)\Big|_{\rho=\rho_o}$$

$$\simeq \frac{E_{oz} - v^2_{o,o} A_{o,o} K_o(v_{o,o}\rho_o)}{2\pi i\varepsilon_o \omega A_{o,o}} \quad \text{where} \quad v_{o,o} = i\alpha_o \qquad (57)$$

This quantity is a useful description of the cable when its behavior in a more complicated environment is to be considered. It is stressed that $Z_e(i\beta_o)$ is a function of axial wavenumber.

## ACKNOWLEDGEMENT

I am grateful to Professor K.F. Casey of Kansas State University for comments on this paper and a useful exchange of views on this general subject.

## REFERENCES


[1] R.M. Whitmer, "Cable shielding performance and CW response", *IEEE Trans. Electromagn. Compat.*, Vol. EMC-15, No. 4, pp 180-187, Nov. 1973.

[2] E.F. Vance, "Shielding effectiveness of braided wire shields", Interaction Notes, Note 172, April 1974.

[3] P. Delogne and M. Safak, "Electromagnetic theory of the leaky coaxial cable", *The Radio and Electronics Engineer (Jour. of the Inst. of Elect. and Rad. Eng.*, London), Vol. 45, No. 5, pp. 233-240, May 1975.

[4] J.R. Wait and D.A. Hill, "Propagation along a braided coaxial cable in a circular tunnel", *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-23, No. 5, pp. 401-405, May 1975.

[5] S. Sensiper, "Electromagnetic wave propagation on helical structures", *Proc. IRE (now IEEE)*, Vol. 43, No. 2, pp. 149-161, Feb. 1955.

[6] R.W. Latham, "An approach to certain cable shielding calculations", *Interaction Note 90 (AFWL)*, Kirtland Air Force Base, New Mexico, Jan. 1972.

[7] K.S.H. Lee and C.E. Baum, "Application of modal analysis to braided wire shields", Interaction Notes, Note 132, January 1973.

[8] D.A. Hill and J.R. Wait, "Electromagnetic scattering of an arbitrary plane wave by two nonintersecting perpendicular wire grids", *Can. J. Phys.*, Vol. 52, No. 3, pp. 227-237, 1974.

[9] D.A. Hill and J.R. Wait, "Electromagnetic scattering of an arbitrary plane wave by a wire mesh with bonded junctions", *Can. J. Phys.*, Vol. 54, 1976 (in press).

[10] K.F. Çasey, "Effects of braid resistance and weather-proofing jackets on coaxial cable shielding", Interaction Notes, Note 192, 31 August 1974.

[11] J.R. Wait, *Electromagnetic Radiation From Cylindrical Structures*, Pergamon Press, Oxford, 1959.