Interaction Notes

Note 308

15 August 1976

A Review of Electromagnetic Penetration through Apertures in Conducting Surfaces for EMP Applications

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ABSTRACT

In designing hardened systems, one must be able to characterize as well as quantitatively determine the penetration of EMP signals through apertures of general shapes in structures of varying configurations. In this paper a tutorial review of a number of methods for analyzing such aperture problems is presented with an emphasis on techniques. The discussion presented herein is reasonably self-contained and is supplemented by references to classical as well as current approaches to the aperture problem. An extensive set of representative numerical results is included in the paper for completeness.

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I. INTRODUCTION

For EMP studies, it is desirable to characterize and quantitify electromagnetic penetration through apertures in conducting surfaces so that deleterious effects on electronic systems within aircraft, missiles, and communications centers, among other units, can be assessed. The various apertures of interest may be either electromagnetically small or large over the spectrum of the EMP. Furthermore, their existence may be intentional, e.g., windows, open access holes, and bombay doors, or they may be inadvertent as in the case of cracks around doors and plates covering access ports or poor electrical seams in outer skins. Interest in electromagnetic penetration through apertures of importance in EMP analyses ultimately centers upon the avoidance of destructive or other unwanted effects caused by currents in internal components and circuitry induced there by the EMP which enters the system in question through apertures such as those mentioned above. Obviously, harmful internal currents can be determined and avoided only if EMP penetration through apertures in outer skins is understood and can be computed.

Even though the classic problem of penetration of time-harmonic electromagnetic fields through an aperture in a planar conducting screen of infinite extent has been the subject of intensive research [1, 2] for many years, still the body of theory pertaining to this simplest of aperture problems remains a rather complicated subject, and only in the case of scalar diffraction—not electromagnetic diffraction—by a circular aperture are analytical results available [3] in the three-dimensional problem. Greater progress has been achieved for small* apertures in planar screens, where, in this context, small means that the maximum dimension across the aperture is small relative to the wavelength of the time-harmonic electromagnetic field, as well as in the two-dimensional problem of diffraction by an infinite slot of uniform width. For apertures in non-planar surfaces, far less progress has been made*. Fortunately, for many problems of practical interest, the introduction of an infinite, planar screen in place of a finite one, or one with gradual curvature relative to aperture size and, in some cases, wavelength, does not seriously degrade accuracy of the solution of a problem. Of course, practical problems pertinent to EMP studies involve not only apertures in curved surfaces but also the determination of currents induced on objects in partially closed regions behind such perforated surfaces.

The purpose of the present paper is to provide a tutorial review of aperture theory in its present state of maturity with emphasis upon those facets of the theory which lead to a better understanding of EMP penetration. A brief discussion

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*See references under appropriate heading in classified bibliography.

is given of the boundary value problem involving an aperture-perforated screen separating two homogeneous half spaces having the same electric properties, and integro-differential equations for this problem are formulated. Due to the importance of this fundamental but complex problem, these introductory developments are presented at an elementary level. Also, the properties of the fields in the two half spaces and the effect of the presence of the aperture are outlined. Next, these preliminary concepts are generalized and equations are derived for the problem of diffraction by a closed conducting surface in which an aperture has been cut. This general formulation is specialized to the case of an aperture in a front plate of a parallel-plate waveguide. Returning to the single-screen case, the authors discuss the modifications in the theory necessitated by the introduction of a material on one side of the screen which is electromagnetically different from that on the other side.

Because infinite slots of uniform width approximate reasonably well certain practical situations, this two-dimensional problem is treated briefly. Equations are formulated for TE and TM (to the slot axis) illumination, and representative data illustrative of slot diffraction properties are presented. The important, finite-length, narrow slot is mentioned briefly but no data are given for this problem due to the abundance of available information pertaining to a thin-wire, the dual of the narrow-slot.

In view of its importance in EMP analyses, appropriate attention is given in the paper to the concepts of electrically small aperture theory and to the determination of penetration through such small holes. Bethe's equivalent dipole moment representation of an aperture is reviewed, and integral equations of a form highly amenable to numerical solution procedures are formulated. These new equations are based on the so-called Rayleigh series.

In hopes of improving the reader's understanding of aperture theory and properties, the authors provide data for numerous example problems of a fundamental nature. Also, where available, sample data are given for problems of interest in EMP studies.

Due to space limitations, no attempt is made to include an extensive literature search in this paper. The interested reader is referred to the classic review paper of Bouwkamp [1] and to the reference list in a more recent paper by Egginmann [2]. As an alternative to an exhaustive review of the aperture literature, the authors provide a classified bibliography in which are listed papers that are not mentioned in the above two works.

II. GENERAL APERTURE/SCREEN EQUATIONS

The fundamental problem to be considered here is that of the electromagnetic interaction of the field due to impressed (specified) sources and a planar conducting screen having a hole (aperture) cut in it. As shown in Fig. 1, the screen is in a homogeneous medium of infinite extent that is electromagnetically characterized by $(\mu, \varepsilon)^*$ and is located, for convenience, in the xy plane of a Cartesian coordinate system. To facilitate the analysis which follows, the planar screen is assumed to be perfectly conducting, vanishingly thin, and of infinite extent. As usual, the sources $(\bar{J}^{i-}, \bar{M}^{i-})$ and $(\bar{J}^{i+}, \bar{M}^{i+})$, located in the left and right half spaces, respectively, as shown in Fig. 1, vary harmonically in time according to $e^{j\omega t}$, which factor is suppressed in subsequent equations. Equations are formulated for an aperture of general shape in a screen, and a brief discussion of the properties of aperture fields is provided.

The electromagnetic field on both sides of the screen must, of course, satisfy Maxwell's equations and the radiation condition. Furthermore, the tangential component of the total electric field must be zero on the screen, and both the electric and magnetic fields must be continuous along any path passing through the aperture. With the tangential component of the electric field zero on the screen and with the radiation condition satisfied, we know from the well-known uniqueness theorem that, if the transverse electric field (transverse to the z axis or to the normal to the screen) were known in the aperture, we could immediately calculate the total electromagnetic field everywhere on both sides of the screen. However, the

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*If desired, the medium may be lossy which condition may be accounted for by replacing ε by $\varepsilon - j\frac{\sigma}{\omega}$.

aperture electric field is not known until the problem has been solved; it is, in fact, the quantity which we ultimately treat as the unknown in the integro-differential equations formulated below.

The general procedure which leads to the desired aperture/screen integro-differential equations is briefly outlined below. First, we identify the transverse aperture electric field $\bar{E}_t^a$, the component of electric field in the aperture parallel to the screen, as the unknown to be determined. Next, we derive individual expressions for the magnetic field on both sides of the screen in terms of $\bar{E}_t^a$ (or, in terms of an equivalent magnetic current). The expressions which we derive for the magnetic field are formulated in terms of the electric vector potential, which ensures us that Maxwell's equations and the radiation condition are satisfied in both half spaces, and they are based upon image theory which ensures us that the boundary conditions on the screen itself are satisfied. Moreover, the magnetic field in each half space is written as a function of $\bar{E}_t^a$, which is common to the two half-space problems; thus the continuity of electric field through A is automatically ensured. Finally, the last remaining condition which must be met is that the magnetic field must be continuous through the aperture. Equating, in the aperture, the transverse component of magnetic fields, calculated from considerations of each half space individually, satisfies this last condition and leads to the desired equations.

Fig. 2 depicts a sequential procedure which one may employ to develop the expression for $\bar{H}^-$, the total magnetic field in the left half space. The original problem is seen in Fig. 2(a), while in 2(b) the aperture is short circuited, i.e., the conducting screen is made continuous, and the electric field is restored to its original value $\bar{E}_t^a$ at $z = 0^-$ by the equivalent surface magnetic current $\bar{M}_s$ $(= \hat{z} \times \bar{E}_t^a)$ placed over the region A on the short circuited screen. From Fig. 2(b) one obtains 2(c) directly by use of image theory.* Now, since all currents in Fig. 2(c) reside in a homogeneous space of infinite extent, one can write $\bar{H}^-$ in terms of the particular integral solutions of the wave equation for the electric vector potential:

$$\bar{H}^-(\bar{r}) = \bar{H}^{sc-}(\bar{r}) - j\frac{\omega}{k^2}\left[k^2\bar{F}(\bar{r}) + \nabla(\nabla\cdot\bar{F}(\bar{r}))\right] , \quad z<0 \qquad (1)$$

where $\bar{r}$ is the point of observation and where $k = 2\pi/\lambda$. $\bar{H}^{sc-}$ is the so-called short-circuit magnetic field [4] and is that field due to the sources $(\bar{J}^{i-}, \bar{M}^{i-})$ which would exist in the left half space with the aperture shorted. The remaining terms are the contributions from the equivalent magnetic current plus its image, and they account for the presence of the hole in the screen. The vector potential $\bar{F}$ is given by

$$\bar{F}(\bar{r}) = \frac{\varepsilon}{2\pi}\iint_A \bar{M}_s(\bar{r}')\,\frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}\,dS' . \qquad (2)$$

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*Fig. 2 is only schematic and so the vector directions shown should not be interpreted as actual directions of quantities.

In an analogous manner, one can write an expression for the right half-space magnetic field $\bar{H}^+$ as

$$\bar{H}^+(\bar{r}) = \bar{H}^{sc+}(\bar{r}) + j\frac{\omega}{k^2}\left[k^2\bar{F}(\bar{r}) + \nabla(\nabla\cdot\bar{F}(\bar{r}))\right] \quad , \; z>0 \qquad (3)$$

where, of course, $\bar{H}^{sc+}$ is the right half-space short-circuit magnetic field due to $(\bar{J}^{i+}, \bar{M}^{i+})$. The equivalent magnetic current for the right half-space problem is $-\bar{M}_s$, which accounts for the positive sign of the contribution from vector potential terms in (3).

$\bar{H}^+$ and $\bar{H}^-$ are calculated from the electric vector potential, so they satisfy Maxwell's equations and the radiation condition, and the use of image theory renders the tangential electric field on the screen zero. Since the electric field is automatically continuous along any path through the aperture, the final electromagnetic property is achieved by enforcing continuity of magnetic field:

$$\lim_{z\uparrow 0} (\bar{H}^-(\bar{r}) \times \hat{z}) = \lim_{z\downarrow 0} (\bar{H}^+(\bar{r}) \times \hat{z}) \quad , \; \bar{r}\varepsilon A \quad . \qquad (4)$$

In view of (1) and (3), (4) can be written

$$j\frac{\omega}{k^2}\left[k^2\bar{F} + \nabla_t\nabla_t\cdot\bar{F}\right] \times \hat{z} = \begin{cases} \frac{1}{2}(\bar{H}^{sc-} - \bar{H}^{sc+}) \times \hat{z} \quad , \\ \qquad \text{or} \\ (\bar{H}^{i-} - \bar{H}^{i+}) \times \hat{z} \end{cases} \quad \text{in } A \qquad (5)$$

where ∇_t is the transverse (to z) gradient operator and where one interprets (5) in the limiting sense of (4). $\bar{H}^{i-}$

and $\bar{H}^{i+}$ in (5) are the incident magnetic fields in the left and right half space, respectively, due to the respective specified sources radiating in the absence of the screen. Implicit in the right-hand-side terms of (5) is the fact that the short-circuit magnetic field on a planar screen is twice the tangential component of the incident magnetic field there, i.e., $\bar{H}^{sc} \times \hat{z} = 2\bar{H}^{i} \times \hat{z}$. Since $\bar{M}_s$ is in the xy plane, it has no z component, from which it follows that $\bar{F}$ has no z component. Thus, (5) embodies two scalar, coupled integro-differential equations with the two transverse (to z) components of $\bar{M}_s$, or, equivalently, of $\bar{E}^a_t$, as the unknown quantities.

In view of the well-known behavior of electric fields near edges [5], together with the relationship $\bar{M}_s = \hat{z} \times \bar{E}^a_t$, the component of $\bar{M}_s$ normal to the aperture/screen edge must approach zero at a point in A as the square root of the distance from this point to the screen, and the tangential component of $\bar{M}_s$ must be singular as the reciprocal of this square root of distance.

When $\bar{M}_s$ is available from the solution of (5) for a specified aperture problem, the magnetic fields on the two sides of the screen can be determined from (1) and (3) and the electric fields can be calculated from

$$\bar{E}^{\pm}(\bar{r}) = \bar{E}^{sc\pm}(\bar{r}) \pm \frac{1}{\varepsilon} \nabla \times \bar{F}(\bar{r}) \qquad (6)$$

where $\bar{E}^{sc\pm}$ represents the short-circuit electric fields on the two sides of the screen.

Properties of the Fields

From (3) and (5), one can show that the total transverse magnetic field in the aperture is equal to the transverse part of the incident field there:

$$\bar{H}^{\pm} \times \hat{z} = \frac{1}{2}(\bar{H}^{sc+} + \bar{H}^{sc-}) \times \hat{z} = (\bar{H}^{i+} + \bar{H}^{i-}) \times \hat{z} \quad \text{in A} . \qquad (7)$$

Also, the normal component of the total electric field in A is simply the normal part of the incident field in A:

$$\bar{E}^{\pm} \cdot \hat{z} = \frac{1}{2}(\bar{E}^{sc+} + \bar{E}^{sc-}) \cdot \hat{z} = (\bar{E}^{i+} + \bar{E}^{i-}) \cdot \hat{z} \text{ in A} . \qquad (8)$$

An interpretation of (7) and (8) is that the above components of fields are unchanged by the presence of the aperture-perforated screen from what they would be in the homogeneous space with no screen. In addition, the transverse component of the electric field and the normal component of the magnetic field, both due to the presence of the hole in the screen, are symmetric (even functions of z) with respect to the location of the screen ($z = 0$), while the normal component of the electric field and the transverse component of the magnetic field are antisymmetric with respect to the screen (odd functions of z). These symmetries can be expressed as

$$\hat{z} \times [\bar{E}^{-}(x,y,z) - \bar{E}^{sc-}(x,y,z)] = \hat{z} \times [\bar{E}^{+}(x,y,-z) - \bar{E}^{sc+}(x,y,-z)] \qquad (9a)$$

and

$$\hat{z} \cdot [\bar{E}^{-}(x,y,z) - \bar{E}^{sc-}(x,y,z)] = -\hat{z} \cdot [\bar{E}^{+}(x,y,-z) - \bar{E}^{sc+}(x,y,-z)] \qquad (9b)$$

plus

$$\hat{z} \times [\bar{H}^{-}(x,y,z) - \bar{H}^{sc-}(x,y,z)] = -\hat{z} \times [\bar{H}^{+}(x,y,-z) - \bar{H}^{sc+}(x,y,-z)] \tag{10a}$$

and

$$\hat{z} \cdot [\bar{H}^{-}(x,y,z) - \bar{H}^{sc-}(x,y,z)] = \hat{z} \cdot [\bar{H}^{+}(x,y,-z) - \bar{H}^{sc+}(x,y,-z)] \tag{10b}$$

where we emphasize that $(\bar{E}^{\pm} - \bar{E}^{sc\pm}, \bar{H}^{\pm} - \bar{H}^{sc\pm})$ is that part of the field which is due to the hole in the screen.

III. GENERAL APERTURE/CAVITY-WALL EQUATIONS

Because major achievements in aperture theory have been made for holes in infinite, planar screens and, also, because results for this model have proved useful as approximations to data needed in numerous applications, the previous section is devoted to a tutorial introduction to the theory of this important problem. However, all surfaces of interest in practice are not planar and the regions separated by such surfaces are not always empty half spaces. Therefore, at this point it is deemed appropriate to outline a theory which is generally applicable to a class of problems involving an aperture in a curved surface. Since a vector source radiates a vector field whose boundary conditions are vector in nature too, dyadic formalism is employed as a mathematical tool [6] for convenience. Below, equations are developed for the unknown electric field (transverse to the surface) in an aperture which exists in a general conducting shell. Also, as an example of this vector problem, the derived equations are specialized to the case of an aperture in an infinite, planar screen behind which is located another infinite screen parallel to the former.

General Formulation

The geometry of the structure under consideration in this section is a cavity with an aperture A in its shell S as shown in Fig. 3 where V_- and V_+ are used to indicate the exterior and interior regions of the cavity, respectively. It is assumed that V_- and V_+ are filled with homogeneous and isotropic materials (μ_-, ε_-) and (μ_+, ε_+), respectively, and that S is a perfectly conducting and vanishingly thin shell. The starting point is Maxwell's equations,

$$\begin{bmatrix} \nabla\times & j\omega\mu \\ -j\omega\varepsilon & \nabla\times \end{bmatrix} \begin{bmatrix} \bar{E} \\ \bar{H} \end{bmatrix} = \begin{bmatrix} -\bar{M}^i \\ \bar{J}^i \end{bmatrix} \tag{11}$$

which apply to V_- and V_+ individually. From (11) one arrives at the inhomogeneous vector wave equation for the electric field $\bar{E}$ in each region

$$(\nabla\times\nabla\times - k^2)\bar{E} = -\nabla\times\bar{M}^i - j\omega\mu\bar{J}^i \ . \tag{12}$$

Next we define the dyadic Green's function $\bar{\bar{G}}(\bar{r}|\bar{r}')$ in the usual way as the solution of

$$(\nabla\times\nabla\times - k^2)\bar{\bar{G}}(\bar{r}|\bar{r}') = \bar{\bar{I}}\delta(\bar{r}-\bar{r}') \tag{13}$$

subject to boundary conditions discussed subsequently, and where $\bar{\bar{I}}$ is the unit dyadic and δ is the Dirac delta distribution. It is noted from (13) that $\nabla\cdot\bar{\bar{G}}(\bar{r}|\bar{r}') \neq 0$. In order to establish a relationship between $\bar{E}$ and $\bar{\bar{G}}$, Green's Theorem in dyadic form [6] is used

$$\iiint\limits_{V}[\bar{E}\cdot\nabla\times\nabla\times\bar{\bar{G}} - \nabla\times\nabla\times\bar{E}\cdot\bar{\bar{G}}]dv = -\iint\limits_{\partial V}\hat{n}\cdot[\bar{E}\times\nabla\times\bar{\bar{G}} + (\nabla\times\bar{E})\times\bar{\bar{G}}]dS \tag{14}$$

where V denotes the domain of the volume integration, ∂V designates the closed surface surrounding the volume V and $\hat{n}$ is the outward unit normal to the boundary ∂V.

Since the shell S is assumed to be a perfectly conducting material, the electric field satisfies the following boundary condition

$$\hat{n}_{\mp}\times\bar{E}^{\mp} = \bar{0} \qquad \bar{r}\varepsilon S \tag{15}$$

where "-" and "+" are used to denote the quantities in V_- and V_+, respectively. The proper boundary condition to be imposed on the interior and exterior dyadic Green's functions is

$$\hat{n}_{\mp}\times\bar{\bar{G}}^{\mp}(\bar{r}|\bar{r}') = \bar{\bar{0}} \qquad \bar{r}\varepsilon SUA \quad . \tag{16}$$

Furthermore, for the exterior region, one requires the satisfaction of the radiation condition, viz.,

$$\lim_{r\to\infty} r\left(\nabla\times\begin{Bmatrix}\bar{E}^-\\ \bar{\bar{G}}^-\end{Bmatrix} + jk_-\hat{r}\times\begin{Bmatrix}\bar{E}^-\\ \bar{\bar{G}}^-\end{Bmatrix}\right) = \begin{Bmatrix}\bar{0}\\ \bar{\bar{0}}\end{Bmatrix} \qquad \bar{r}\varepsilon V_- \quad . \tag{17}$$

Employing (12) - (14) in regions V_- and V_+ and using (15) - (17), we obtain the following representation for the interior and exterior fields

$$\bar{E}^{\mp}(\bar{r}) = \iiint_{V_{\mp}} \left(-\nabla' \times \bar{M}^{i\mp} - j\omega\mu_{\mp} \bar{J}^{i\mp}\right) \cdot \bar{\bar{G}}^{\mp}(\bar{r}'|\bar{r})\, dv'$$

$$- \iint_{A} \hat{n}_{\mp} \times \bar{E}^{a} \cdot \nabla' \times \bar{\bar{G}}^{\mp}(\bar{r}'|\bar{r})\, dS' \qquad \bar{r} \varepsilon V_{\mp}, \tag{18}$$

where, clearly, $\hat{n}_+ = -\hat{n}_-$ and $\bar{E}^-(\bar{r}) = \bar{E}^+(\bar{r}) = \bar{E}^a$ for $\bar{r}\varepsilon A$. It should be mentioned that the exterior and interior Green's functions, respectively, $\bar{\bar{G}}^-$ and $\bar{\bar{G}}^+$, generally speaking are different in form. Equation (18) also reveals the fact that, from knowledge of the tangential electric field in the aperture and the Green's functions, one can construct the field everywhere else. Our goal is therefore to construct an integral equation for the unknown tangential electric field in the aperture, i.e., $\hat{n}\times\bar{E}^a$. This is done by first deriving the proper Green's functions for the geometry of interest and then enforcing the condition,

$$\hat{n} \times \bar{H}^- = \hat{n} \times \bar{H}^+ \quad \text{for } \bar{r}\varepsilon A \tag{19}$$

where $\bar{H}$ is determined by substituting (18) into (11). This procedure is demonstrated by an example in the following section.

Aperture in Front Screen of Two-Parallel Screens

In this section, we focus our attention on the problem of penetration into a parallel-plate region and we construct the appropriate integral equation for this structure. The geometry of the two-parallel-plate structure is shown in Fig. 4 where one sees two perfectly conducting, parallel plates separated by a distance w. A Cartesian coordinate system with its z axis normal to, and its xy plane parallel to, the plates is erected as shown in the figure. The plate at $z = 0$ is perforated by an arbitrarily shaped aperture and it is further assumed that a monochromatic wave $\bar{E}^{i-}$ and $\bar{H}^{i-}$ originating from a source situated in the half space V_- is incident on the structure.

The total electromagnetic field $(\bar{E}^{\mp}, \bar{H}^{\mp})$ at any point in either space is partitioned into an incident field $(\bar{E}^{i-}, \bar{H}^{i-})$, a reflected field $(\bar{E}^{r-}, \bar{H}^{r-})$ associated with the reflected wave which exists when the aperture is closed, and a diffracted field $(\bar{E}^{d\mp}, \bar{H}^{d\mp})$ due to the aperture. In V_-, the total electric field can be written

$$\bar{E}^{-} = \bar{E}^{i-} + \bar{E}^{r-} + \bar{E}^{d-} \tag{20a}$$

and in V_+

$$\bar{E}^{+} = \bar{E}^{d+} \tag{20b}$$

with similar expressions for the magnetic fields. The reflected field can, in general, be constructed from knowledge of the incident field and the reflecting surface. It should be recognized that $\bar{E}^{i-} + \bar{E}^{r-}$ of (20a) is the same as $\bar{E}^{sc-}$ defined above. For an incident plane wave

$$\bar{E}^{i-} = \left(\hat{x}\, E^{i}_{0x} + \hat{y}\, E^{i}_{0y} + \hat{z}\, E^{i}_{0z}\right) e^{-jk(\alpha x + \beta y + \gamma z)} \tag{21a}$$

and the field reflected from the plate at $z = 0$ is

$$\bar{E}^{r-} = \left(-\hat{x}\, E^{i}_{0x} - \hat{y}\, E^{i}_{0y} + \hat{z}\, E^{i}_{0z} \right) e^{-jk(\alpha x + \beta y - \gamma z)} \tag{21b}$$

where

$$\alpha = \sin\theta^{i} \cos\phi^{i} \quad , \quad \beta = \sin\theta^{i} \sin\phi^{i} \quad \text{and} \quad \gamma = \cos\theta^{i}$$

are the direction cosines of the incident wave vector, and θ^{i} and ϕ^{i} are the corresponding elevation and azimuthal angles. The diffracted fields $\bar{E}^{d-}$ and $\bar{E}^{d+}$ are obtained from (18):

$$\bar{E}^{d\mp} = \mp \iint_{A} \hat{z} \times \bar{E}^{a} \cdot \nabla' \times \bar{\bar{G}}^{\mp}(\bar{r}' | \bar{r})\, dS' \quad . \tag{22}$$

In (22), $\bar{\bar{G}}^{+}$ is the dyadic Green's function of the parallel-plate region and $\bar{\bar{G}}^{-}$ is the dyadic Green's function of the half space z<0. These dyadic Green's functions take the following form [7]

$$\bar{\bar{G}}^{\mp}(\bar{r}' | \bar{r}) = \left(\bar{\bar{I}} - \frac{1}{k^{2}} \nabla' \nabla \right) \begin{Bmatrix} g^{-} - g^{+} \\ G^{+} - G^{-} \end{Bmatrix} + 2\, \hat{z}\hat{z} \begin{Bmatrix} G^{+} \\ G^{-} \end{Bmatrix} \tag{23}$$

where

$$G^{\mp} = \sum_{n=-\infty}^{\infty} \frac{e^{-jk \sqrt{(x-x')^{2} + (y-y')^{2} + (z \pm z' + 2nw)^{2}}}}{4\pi \sqrt{(x-x')^{2} + (y-y')^{2} + (z \pm z' + 2nw)^{2}}} \tag{24a}$$

and

$$g^{\mp} = \frac{e^{-jk \sqrt{(x-x')^{2} + (y-y')^{2} + (z \pm z')^{2}}}}{4\pi \sqrt{(x-x')^{2} + (y-y')^{2} + (z \pm z')^{2}}} \quad . \tag{24b}$$

Substituting (23) into (22) and using the fact that

$$\nabla' \times \bar{\bar{G}}^{\mp}(\bar{r}'|\bar{r}) = \nabla' \left\{ \begin{matrix} G^{+} - G^{-} \\ g^{-} - g^{+} \end{matrix} \right\} \times \bar{\bar{I}} + 2\nabla' \left\{ \begin{matrix} g^{-} \\ G^{-} \end{matrix} \right\} \times \hat{z}\hat{z} \qquad (25)$$

and that

$$G^{-}\Big|_{z'=0} = G^{+}\Big|_{z'=0} \quad , \quad \frac{\partial}{\partial z'} G^{\mp}\Big|_{z'=0} = \mp \frac{\partial}{\partial z} G^{\mp}\Big|_{z'=0} \quad , \qquad (26)$$

one finally obtains the simplified form of (22), viz.,

$$\bar{E}^{d\mp} = \pm 2 \frac{\partial}{\partial z} \iint_A \left[E_x^a \hat{x} + E_y^a \hat{y}\right] g^{\infty}(\bar{r}'|\bar{r})\, dS'$$

$$\mp 2\, \hat{z} \left(\frac{\partial}{\partial x} \iint_A E_x^a\, g^{\infty}(\bar{r}'|\bar{r})\, dS' + \frac{\partial}{\partial y} \iint_A E_y^a\, g^{\infty}(\bar{r}'|\bar{r})\, dS' \right) \qquad (27)$$

where

$$g^{\infty}(\bar{r}'|\bar{r}) = G^{-}\Big|_{z'=0} = G^{+}\Big|_{z'=0} \quad ; \quad g^{o}(\bar{r}'|\bar{r}) = g^{-}\Big|_{z'=0} = g^{+}\Big|_{z'=0} \qquad (28)$$

Defining the equivalent surface magnetic current in the aperture as

$$\bar{M}_s = \hat{z} \times \bar{E}^a \qquad (29)$$

and introducing the vector potential

$$\bar{F}^{\mp} = 2\varepsilon \iint_A \bar{M}_s(\bar{r}')\, g^{\infty}(\bar{r}'|\bar{r})\, dS' \quad , \qquad (30)$$

we can readily express the total field (20) in the following form with the help of (27)

$$\bar{E}^{\mp} = \bar{E}^{sc\mp} + \frac{1}{\varepsilon} \nabla\times\bar{F}^{\mp} \qquad (31)$$

where $\bar{E}^{sc\mp} = \bar{E}^{i\mp} + \bar{E}^{r\mp}$. From Maxwell's equation (1) and (31) the magnetic field is determined as

$$\bar{H}^{\mp} = \bar{H}^{sc\mp} \mp j\frac{\omega}{k^2}\left[k^2\bar{F}^{\mp} + \nabla(\nabla\cdot\bar{F}^{\mp})\right] . \qquad (32)$$

To establish an integral equation for the unknown aperture field, or equivalently for $\bar{M}_s$, we enforce the continuity of the tangential magnetic field in A (4). It is noticed that the continuity of the tangential electric field is guaranteed by (27). Substituting (32) into (4) and simplifying the result, one finally arrives at the following conventional integro-differential equation for $\bar{M}_s$,

$$j\frac{\omega}{k^2}\left[k^2\bar{F} + \nabla_t\nabla_t\cdot\bar{F}\right]\times\hat{z} = (\bar{H}^{sc-} - \bar{H}^{sc+})\times\hat{z} \qquad (33)$$

where $\bar{F}$, which is entirely transverse, is defined as

$$\bar{F} = (\bar{F}^{-} + \bar{F}^{+})\Big|_{z=0} . \qquad (34)$$

<u>An Alternate Integral Equation</u>

For the case of plane wave excitation described in (21a), one can construct an alternate integral equation which has many desirable features when numerical techniques are considered. To this end, the continuity condition (4) is enforced in an indirect fashion. We employ the fact that the continuity of the normal component of the electric field together with the continuity of the normal derivative of the tangential electric field in the aperture do ensure the continuity of the tangential magnetic field in A. In other words we require that the following conditions be satisfied:

$$\lim_{z\uparrow 0} (\bar{E}^{-}\cdot\hat{z}) = \lim_{z\downarrow 0} (\bar{E}^{+}\cdot\hat{z}) \text{ and } \lim_{z\uparrow 0} \frac{\partial}{\partial z} (\bar{E}^{-}\times\hat{z}) = \lim_{z\downarrow 0} \frac{\partial}{\partial z} (\bar{E}^{+}\times\hat{z})$$

$$\bar{r}\varepsilon A \quad . \tag{35}$$

The continuity of the normal component of electric field can be expressed as

$$\left[\frac{\partial}{\partial x} F_x - \frac{\partial}{\partial y} F_y\right] = \varepsilon\hat{z}\cdot(\bar{E}^{sc-} - \bar{E}^{sc+}) \quad , \bar{r}\varepsilon A \tag{36}$$

where $\bar{F}$ is defined in (34). The continuity of the normal derivative leads to

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right]\bar{F} = \varepsilon\left[\hat{z}\times\frac{\partial}{\partial z}(\bar{E}^{sc-} - \bar{E}^{sc+})\right]_{z=0} \quad , \bar{r}\varepsilon A \tag{37}$$

where (31) and (35) are employed for the derivation of (37).

For the case of a plane wave incident from the sources in V_-, i.e., $\bar{E}^{sc-} = \bar{E}^{i-} + \bar{E}^{r-}$ defined in (21), and $\bar{E}^{sc+} = \bar{0}$, a solution of (37) subject to (36) may be written as follows [7]

$$\iint_A \bar{M}_s(\bar{r}')g(\bar{r}'|\bar{r})dS' = \frac{1}{jk\gamma} \hat{z}\times\bar{E}_0^i e^{-jk(\alpha x + \beta y)} + \bar{h} \quad , \rho\varepsilon A \tag{38}$$

where $\gamma \neq 0$, $\bar{h} = h_x\hat{x} + h_y\hat{y}$ is the homogeneous solution of the operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)$, and

$$g(\bar{r}'|\bar{r}) = \left[g^{o}(\bar{r}'|\bar{r}) + g^{\infty}(\bar{r}'|\bar{r})\right]_{z=0}$$

$$= \sum_{n=0}^{\infty} \frac{e^{-jk|\bar{r}-\bar{r}' + (2nw)\hat{z}|}}{2\pi|\bar{r}-\bar{r}' + (2nw)\hat{z}|} \quad . \tag{39}$$

The components of $\bar{h}$ can be expressed as

$$h_{\substack{x\\y}} = \frac{\pi}{k}\left\{\begin{matrix} j \\ -1 \end{matrix}\right\} \sum_{n=-\infty}^{\infty} C_n \left[j^{n+1} e^{j(n+1)\phi} J_{n+1}(k\rho) + \left\{\begin{matrix} -1 \\ 1 \end{matrix}\right\} j^{n-1} e^{j(n-1)\phi} J_{n-1}(k\rho)\right] , \tag{40}$$

where J_n is the Bessel function and C_n's are newly introduced unknown constants yet to be determined. Equation (38) is an integral equation for the unknown $\bar{M}_s(\bar{r}')$ or, equivalently, the tangential component of the electric field in A. This equation is solved in conjunction with the following condition for the determination of the C_n's,

$$\hat{c}\cdot\bar{M}_s(\bar{r}) = 0 \quad \text{or} \quad \hat{\tau}\cdot\bar{E}^a(\bar{r}) = 0 \qquad \bar{r}\varepsilon C \tag{41}$$

where C is the rim of the aperture, and $\hat{c}$ and $\hat{\tau}$ are the unit vectors normal and tangent, respectively, to the rim. Comparing (33) and (38) with equations appearing in linear antenna theory, one finds that (33) and (38) are the counterparts of Pocklington's and Hallen's equations, respectively. Although (33) and (38) apply to the problem represented by Fig. 4, they can be used for that represented by Fig. 1 simply by replacing g by g^o. Beginning with (36) and (37), Dunaway and Wilton [8] have developed the homogeneous solution $\bar{h}$ in another form also useful in obtaining numerical solutions for the aperture problem.

IV. SMALL APERTURES

In many applications, apertures of interest are electromagnetically small, a property which leads to very useful simplifications in computations. Diffraction by small circular and elliptic apertures has been investigated by numerous workers employing a wide variety of different approaches. Lord Rayleigh [9] proposed a solution in the form of a series in ascending powers of the wavenumber k ($= 2\pi/\lambda$) where λ is the wavelength. Bethe [10] obtained results for the leading terms in the Rayleigh series expansion by means of a scalar potential approach. Later, Bouwkamp [1,3] investigated the same problem using a more complete set of coupled integro-differential equations and pointed out some errors in Bethe's solution. A comprehensive review of articles pertaining to aperture diffraction in general is given in [3], and an extensive bibliography is accumulated in [2]. The low frequency scalar diffraction problem also has been analyzed by Van Bladel [11]. Recently, attempts have been made to use an integral equation approach for aperture diffraction problems with the goal of attacking non-separable geometries, e.g., rectangular apertures [12 - 15].

One can utilize (37) plus the auxiliary condition (35) and the boundary condition (41), or employ (38) subject to (36) and (41), to analyze the problem of electromagnetic diffraction by small apertures. Unlike some of the earlier work referred to above, the formulation discussed below is not restricted in its application to separable geometries only, and its use is particularly suitable when numerical techniques are considered. Here we mention two closely related procedures, both based upon Stevenson's method [16, 17] and both involving an expansion of the unknown magnetic current $\bar{M}_s$ in a so-called Rayleigh series

$$\bar{M}_s(\bar{r}) = \sum_{m=0}^{\infty} \bar{M}_s^{(m)}(\bar{r})k^m \quad . \tag{42}$$

One procedure, advanced by Rahmat-Samii and Mittra [18], is based upon an expansion of both sides of (38) in Rayleigh series and an incorporation of (36) through constraints on constants in the homogeneous solution $\bar{h}$ (in expanded form). An individual integral equation for each $\bar{M}_s^{(m)}$ is obtained by equating coefficients of like powers of k on the two sides of the expanded version of (38). Of course, (41) must be applied to each $\bar{M}_s^{(m)}$. The details of this procedure are given in [18].

In a related procedure [12,13], (42) is substituted into (36) and (37), which are expanded in Rayleigh series. Equating like powers of k leads directly to a set of integro-differential equations for the coefficients $\bar{M}_s^{(m)}$, and, again, (41) must be enforced for each m. By making use of the concepts of potential theory, one can obtain homogeneous and particular solutions to the differential operator equations and subsequently convert the integro-differential equations to integral equations in a manner reminiscent of the way one can convert Pocklington's thin-wire equation to Hallen's equation. A discussion of the latter procedure can be found in [12,13].

Both procedures [12,13,18] alluded to above lead to integral equations which are well-suited for numerical methods and from which, in principle, the coefficients $\bar{M}_s^{(m)}$ for all m can be determined for any aperture whose maximum dimension is less than $\lambda/2$. To solve the integral equation for $\bar{M}_s^{(m)}$, one must know $\bar{M}_s^{(m-1)}$; $\bar{M}_s^{(0)}$ can be determined directly. The integral equation for each $\bar{M}_s^{(m)}$ is somewhat simpler than (37) since the kernel of the former equation is of the electrostatic type and since no differential operators are involved. Another interesting feature of the equations of both procedures is that coupling between the two vector components of $\bar{M}_s^{(m)}$ is realized through relationships among constants in the homogeneous solutions of the differential operator equations and not through the operators themselves.

For a circular aperture of radius a, leading coefficients $\bar{M}_s^{(m)}$ can be determined exactly from solutions of the equations discussed above; these are found to be [18]

$$M_{s_\rho}^{(0)} = 0 \tag{43a}$$

$$M_{s_\phi}^{(0)} = \frac{2\rho}{\pi\left(a^2 - \rho^2\right)^{\frac{1}{2}}} E_{0z}^{i} \tag{43b}$$

where $\bar{M}_s^{(0)} = M_{s_\rho}^{(0)}\,\hat{\rho} + M_{s_\phi}^{(0)}\,\hat{\phi}$. Similarly, the next higher order terms are

$$M_{s_\rho}^{(1)} = -\frac{8j}{3\pi}\gamma\left(E_{0y}^{i}\cos\phi - E_{0x}^{i}\sin\phi\right)\left(a^2-\rho^2\right)^{\frac{1}{2}}$$

$$-\frac{4j}{3\pi}\left(\alpha\sin\phi - \beta\cos\phi\right)\left(a^2-\rho^2\right)^{\frac{1}{2}} E_{0z}^{i} \tag{44a}$$

$$M_{s_\phi}^{(1)} = \frac{2j}{3\pi}\gamma\left(E_{0x}^{i}\cos\phi + E_{0y}^{i}\sin\phi\right)\left[4\left(a^2-\rho^2\right)^{\frac{1}{2}} + \frac{2\rho^2}{\left(a^2-\rho^2\right)^{\frac{1}{2}}}\right]$$

$$-\frac{4j}{3\pi}\left(\alpha\cos\phi + \beta\sin\phi\right)\frac{\rho^2+a^2}{\left(a^2-\rho^2\right)^{\frac{1}{2}}} E_{0z}^{i} \quad . \tag{44b}$$

For normally incident illumination, one can show that $M_\phi^{(2)} = M_\rho^{(2)} = 0$; thus, for these cases, the low frequency expansion (42), correct up to the k^2 term, may be written as

$$M_{s_\rho} = -\frac{8j}{3\pi}\left(E_{0y}^{i}\cos\phi - E_{0x}^{i}\sin\phi\right)\left(a^2-\rho^2\right)^{\frac{1}{2}} k + 0(k^3) \tag{45a}$$

$$M_{s_\phi} = \frac{2j}{3\pi}\left(E_{0x}^{i}\cos\phi + E_{0y}^{i}\sin\phi\right)\left[4\left(a^2-\rho^2\right)^{\frac{1}{2}} + \frac{2\rho^2}{\left(a^2-\rho^2\right)^{\frac{1}{2}}}\right]k$$

$$+ 0(k^3) \quad . \tag{45b}$$

Equivalent Dipole Moments and Polarizabilities of Small Apertures

Even for the fundamental problem of an aperture in a single, planar screen, the aperture equations must be solved numerically, which, in general, is very demanding upon computer time and storage. Electrically small apertures are of practical importance and have been studied extensively, and one can obtain useful information from the literature about fields diffracted by such holes without having to solve the aperture equations.

Hence, in this subsection, we summarize the main features of the equivalent dipole representation of a small aperture, which embodies this information.

Preliminary to the introduction of an aperture-perforated screen, we consider a magnetic surface current density $\bar{M}_s$, in a small planar region R_s about the origin in the xy plane and residing in a homogeneous medium with properties (μ , ε). In R_s there is also a magnetic surface charge density m_s:

$$m_s = \frac{j}{\omega} \nabla_t \cdot \bar{M}_s \quad . \tag{46}$$

For R_s sufficiently small and at a point $\bar{r}$ sufficiently remote from this source region, the electromagnetic field due to this magnetic source can be approximated [1,19] by the radiation from an electric dipole of moment $\bar{p}_e$ and a magnetic dipole of moment $\bar{p}_m$, both located at (0 , 0 , 0), where

$$\bar{p}_e = - \frac{\varepsilon}{2} \iint_{R_s} \bar{r}' \times \bar{M}_s(\bar{r}')\, dS' \tag{47a}$$

and

$$\bar{p}_m = \frac{1}{\mu} \iint_{R_s} \bar{r}' m_s(\bar{r}')\, dS' = - \frac{j}{\omega\mu} \iint_{R_s} \bar{M}_s(\bar{r}')\, dS' \tag{47b}$$

with $\bar{r}'$ in R_s. If the dipole moments (47) are known for a magnetic source, one can readily compute the approximate field by means of simple formulas for dipole radiation. If the moments are known for the magnetic sources residing on a conducting surface, the presence of the surface must be accounted for in the calculation of the field from the equivalent dipoles.

Returning now to the aperture/screen problem, we note that the equivalent dipole moments of a given small aperture in a screen are related to the specified excitation by the so-called aperture polarizabilities [1,4,10,19]. Knowing the polarizabilities for an aperture and the illumination of the perforated screen, one can determine dipole moments and, subsequently, the diffraction caused by the presence of a small aperture in the screen. For a small aperture A in a screen, the electric polarizability α_e is defined

$$p_e^{\pm}\hat{z} = \pm\,\varepsilon\alpha_e\left(E_z^{sc-}(\bar{0}) - E_z^{sc+}(\bar{0})\right)\hat{z} \tag{48a}$$

and the magnetic polarizability $\bar{\bar{\alpha}}_m$, a dyadic,

$$\bar{p}_m^{\pm} = \mp\,\bar{\bar{\alpha}}_m\cdot\left(\bar{H}^{sc-}(\bar{0}) - \bar{H}^{sc+}(\bar{0})\right). \tag{48b}$$

The polarizabilities above are defined in such a way that $p_e^{+}\hat{z}$ and $\bar{p}_m^{+}$ are the moments for the equivalent dipoles <u>in the presence of the screen</u> for the right half space, i.e., the dipoles are located at $(0, 0, 0^{+})$ on the short circuited screen, while $p_e^{-}\hat{z}$ and $\bar{p}_m^{-}$ are the equivalent moments for the left half space. Polarizabilities are available in the literature for several small apertures and values are listed in Table I for three shapes which are of practical interest and for which values are expressible in closed form.

At points far from A, relative to the maximum dimension across the small aperture, the electric fields in the two half spaces are approximately

$$\bar{E}^{\pm} = \bar{E}^{sc\pm} + \bar{E}_e^{\pm} + \bar{E}_m^{\pm} \tag{49}$$

where $\bar{E}_e^{\pm}$ and $\bar{E}_m^{\pm}$ are due to the equivalent electric and magnetic dipoles, respectively, and are given

$$\bar{E}_e^{\pm}(\bar{r}) = \frac{p_e^{\pm}}{2\pi\varepsilon}\left(k^2 \hat{z} + \nabla \frac{\partial}{\partial z} \right) g(\bar{r}) \tag{50a}$$

and

$$\bar{E}_m^{\pm}(\bar{r}) = -j \frac{k\eta}{2\pi} \nabla g(\bar{r}) \times \bar{p}_m^{\pm} \tag{50b}$$

with

$$g(\bar{r}) = \frac{e^{-jk|\bar{r}|}}{|\bar{r}|} \quad . \tag{51}$$

Of course, corresponding expressions for the magnetic fields are available from the dipole moments too.

V. SLOTTED SCREEN

Very long slots of uniform width, subject to illumination whose electric field is transverse to the slot axis (TE), often can be approximated in typical practical applications by infinitely long slots. In some cases long slots subject to TE illumination can be approximated by infinitely long slots, but in usual cases such an approximation is poor and one must address the finite-length slot problem directly.

Infinite-Length Slot

If the aperture discussed in Section II is an infinite slot of uniform width w, one may readily obtain appropriate equations by specialization of Eq. (5). Let us consider the two above-mentioned cases of excitation: (i) transverse electric (TE) in which the incident electric field is entirely transverse to the slot axis (y axis) and in which the illumination of the slotted screen is independent of y (Fig. 6(a)) and (ii) transverse magnetic (TM) in which the incident magnetic field is entirely transverse to the slot axis with the illumination again independent of y (Fig. 6(b)).

<u>TE Case</u>. With TE excitation of the uniform-width slot (Fig. 6(a)), $\bar{E}_t^a$ has only an x component and depends only upon x. Hence, $\bar{E}_t^a = E_x^a(x)$ and so $\bar{M}_s = M_{s_y}(x)\,\hat{y}$ with $M_{s_y} = E_x^a$. Because $\bar{M}_s$ has only a y component and is independent of y, $F_x = 0$ and $\frac{\partial}{\partial y} F_y = 0$ so that (5) reduces to

$$j\omega F_y = \left(H_y^{i-} - H_y^{i+}\right) \quad \text{in slot} \tag{52a}$$

with

$$F_y(x) = \frac{\varepsilon}{2\pi} \int_{x'=-w/2}^{w/2} M_{s_y}(x') \int_{y'=-\infty}^{\infty} \frac{e^{-jk\left[(x-x')^2 + y'^2\right]^{\frac{1}{2}}}}{\left[(x-x')^2 + y'^2\right]^{\frac{1}{2}}}\, dy'dx' . \tag{52b}$$

In view of the integral representation of the zero-order Hankel function of the second kind $H_0^{(2)}$ [4], (52) becomes

$$\frac{k}{2\eta} \int_{x'=-w/2}^{w/2} M_{s_y}(x') H_0^{(2)}(k|x-x'|)\,dx' = \left(H_y^{i-}(x) - H_y^{i+}(x)\right) , \quad x\varepsilon(-w/2 , w/2) \tag{53}$$

which is a first-kind integral equation for the TE slot problem. Since E_x^a must obey the edge condition at the upper and lower edges of the screen, it follows that M_{s_y} must exhibit singularities of the form $[w/2 - x]^{-\frac{1}{2}}$ at the upper edge and $[w/2 + x]^{-\frac{1}{2}}$ at the lower edge [5].

TM Case. With the incident electric field parallel to the slot axis and having no y variation in the xy plane, $\bar{E}_t^a$ is independent of y and parallel to the slot axis. For the incident electric field in the negative y direction as shown in Fig. 6(b), $\bar{E}_t^a = -E_y^a(x)\,\hat{y}$ and $\bar{M}_s = M_{s_x}(x)\,\hat{x}$ with, of course, $M_{s_x} = E_y^a$. Due to the properties of the equivalent magnetic current, F_y and $\frac{\partial}{\partial y} F_x$ are zero and (5) simplifies to

$$j\frac{\omega}{k^2}\left(\frac{d^2}{dx^2} + k^2\right)F_x = \left(H_x^{i-} - H_x^{i+}\right) \quad \text{in slot}$$

from which follows the final integro-differential equation for the TM Case:

$$\frac{1}{2k\eta}\left(\frac{d^2}{dx^2} + k^2\right)\int_{x'=-w/2}^{w/2} M_{s_x}(x')H_0^{(2)}(k|x-x'|)\,dx' = \left(H_x^{i-}(x) - H_x^{i+}(x)\right),$$

$$x\varepsilon(-w/2\,,\,w/2) \qquad (54)$$

E_y^a in the slot is parallel to the perfectly conducting screen so it, hence M_{s_y}, must approach zero as $[w/2 - x]^{\frac{1}{2}}$ at the upper edge of the slot and as $[w/2 + x]^{\frac{1}{2}}$ at the lower edge.

The present slot analysis can be extended readily to include illumination which varies along the slot axis. Also, we observe that (53) is of the form of the equation for TM scattering from a flat strip (duality) while (54) is of the form of the equation for the TE strip problem.

Narrow Slot Approximation. The surface current density $\bar{J}_s$ induced on an infinite, perfectly conducting screen (no aperture) by a uniform incident plane wave is in the direction of the projection of $\bar{E}^i$ upon the plane. This induced current is altered far less by a narrow slot cut parallel (TM case) to $\bar{J}_s$ than by a slot cut perpendicular (TE case) to $\bar{J}_s$. Furthermore, the field reflected from a screen with a very narrow slot parallel to $\bar{J}_s$ is approximately equal to that reflected from the unslotted screen, whereas, from a screen with a slot perpendicular to $\bar{J}_s$, the reflected field is significantly different. Since the equivalent magnetic current introduced in the above formulations accounts for the difference in the total field and the short-circuit field in a half space, it is clear for narrow slots that M_{s_x} is very small in the case of TM illumination while M_{s_y} is indeed significant in the TE case. In summary, one can conclude that the problem of the TM-illuminated slotted screen can be approximated in some applications by a continuous screen if $kw \ll 1$, whereas in the TE case desired field quantities must be determined from solutions of (53).

In usual applications, a narrow slot subject to TM excitation can be ignored, whereas for one subject to TE illumination we must solve (53). For kw<<1, $H_0^{(2)}(k|x-x'|)$ can be replaced by its small argument approximation [21] and (53) reduces to

$$-j\frac{k}{\eta\pi}\int_{x'=-w/2}^{w/2} M_{s_y}(x')\,\ell n|x-x'|\,dx' = \left(H_y^{i-}(x) - H_y^{i+}(x)\right) + H^f \qquad (55a)$$

where $\gamma = 0.5772156649$ is Euler's constant and where H^f is the constant

$$H^f = j\frac{k}{\eta\pi}\left(\gamma + \ell n\frac{k}{2} + j\frac{\pi}{2}\right)\int_{x'=-w/2}^{w/2} M_{s_y}(x')\,dx' \quad . \qquad (55b)$$

Wilton and Govind [22] have carefully investigated numerical solution techniques applicable to (53) and they give the solution to the dual of (55) in closed form for a constant forcing function. With $(H_y^{i-} - H_y^{i+}) = H_0^i$, their result adapted to (16) yields

$$M_{s_y} = j\,H_0^i\,\frac{\frac{\eta}{k}}{\left[\gamma + \ell n\left(\frac{kw}{8}\right) + j\frac{\pi}{2}\right]}\cdot\frac{1}{\left[\left(\frac{w}{2}\right)^2 - x^2\right]^{\frac{1}{2}}} \qquad (56)$$

as the solution to the narrow slot problem with constant TE excitation.

Finite-Length, Narrow Slot

We next consider the aperture to be an electrically narrow, rectangular slot of width w and length L, which for convenience is centered about (0 , 0 , 0) with its longer axis oriented along the y axis. Narrow-slot assumptions, similar in principle to those invoked in thin-wire theory [23], can be employed here to simplify the analysis. When the slot is very narrow relative to the wavelength and long compared to its width, the electric field in the slot is principally transverse to the longer slot dimension and, away from the slot ends, takes on a variation of the form of (56) provided the slot excitation does not possess an appreciable component which is an odd function with respect to x. Ignoring the small axial (y component) electric field, or transverse equivalent magnetic current, in the slot, which, of course, implies $F_x = 0$, and evaluating quantities of interest along the slot axis (x = 0, z = 0), one reduces the surviving component of the electric vector potential F_y to

$$F_y(0,y,0) = \frac{\varepsilon}{2\pi} \int_{y'=-L/2}^{L/2} I_m(y') \int_{x'=-w/2}^{w/2} \xi(x') \frac{e^{-jk\left[(y-y')^2 + x'^2\right]^{\frac{1}{2}}}}{\left[(y-y')^2 + x'^2\right]^{\frac{1}{2}}} dx'dy' \tag{57}$$

where $I_m(y)$ is the unknown axial variation of the magnetic current and $\xi(x)$ is the known transverse variation (normalized)

$$\xi(x) = \frac{\frac{1}{\pi}}{\left[\left(\frac{w}{2}\right)^2 - x^2\right]^{\frac{1}{2}}} \tag{58a}$$

such that the approximate magnetic current is $\bar{M}_s = M_{s_y}(x,y)\,\hat{y}$ with

$$M_{s_y}(x,y) = \xi(x)\, I_m(y) \quad . \tag{58b}$$

Subject to (58) and the integration variable transformation,

$$y' = \frac{w}{2} \sin\frac{\alpha}{2} \quad ,$$

Eq. (57) becomes

$$F_y(0,y,0) = \frac{\varepsilon}{2\pi} \int_{y'=-L/2}^{L/2} I_m(y')\, K(y-y', w/4)\, dy' \qquad (59)$$

where

$$K(\zeta,a) = \frac{1}{2\pi} \int_{\alpha=-\pi}^{\pi} \frac{e^{-jk\left[\zeta^2 + 4a^2\sin^2\frac{\alpha}{2}\right]^{\frac{1}{2}}}}{\left[\zeta^2 + 4a^2\sin^2\frac{\alpha}{2}\right]^{\frac{1}{2}}}\, d\alpha \quad . \qquad (60)$$

With $F_x = 0$ in the narrow slot and with F_y given by (59), the general aperture/screen equation (5) reduces to

$$\frac{j}{2\pi k\eta}\left(\frac{d^2}{dy^2} + k^2\right) \int_{y'=-L/2}^{L/2} I_m(y')\, K(y-y', w/4)\, dy' = \left(H_y^{i-}(y) - H_y^{i+}(y)\right),$$
$$\text{on slot axis} \;, \qquad (61)$$

which, in view of (60), is seen to be of the form of the thin-wire equation for a wire of radius equal to w/4. Since (61) is essentially the same as the thin-wire equation, one can employ the many solution techniques available for the latter to solve (61) for I_m. If the incident field does not vary greatly over the narrower dimension of the slot, then the results based upon solutions of (61) are good approximations to quantities of interest associated with the finite-length slot problem. Of course, one can extend the present analysis and draw upon the extensive thin-wire literature to develop methods of handling problems involving configurations of slots, i.e., crossed slots and arrays of slots.

VI. EXCITATION OF AN OBJECT THROUGH AN APERTURE IN A SCREEN

In this section, attention is turned to the important problem of calculating the current induced on an object by the field which penetrates a screen through an aperture [24]. The structure and quantities of interest are illustrated in Fig. 7. The object or scatterer is perfectly conducting and of general shape and the only sources are in the lower half space, i.e., $(\bar{J}^{i+}, \bar{M}^{i+}) = (\bar{0}, \bar{0})$. Desired equations for this problem are achieved by modifying both (5) and the equation for the scatterer in the presence of a continuous—shorted aperture—screen. These modifications are outlined below [24].

We extend Eq. (5) to include the effect of the scatterer by treating the field scattered back to the aperture by the object as part of the forcing function of (5):

$$j\frac{\omega}{k^2}\left(k^2\bar{F} + \nabla_t\nabla_t\cdot\bar{F}\right)\times\hat{z} = \begin{cases} \frac{1}{2}[\bar{H}^{sc-} - \bar{h}^{sc}]\times\hat{z} \\ \quad\text{or} \\ [\bar{H}^{i-} - \bar{h}^{i}]\times\hat{z} \end{cases}, \text{ in A} \qquad (62)$$

where $(\bar{e}^{sc}, \bar{h}^{sc})$ and $(\bar{e}^{i}, \bar{h}^{i})$ are, respectively, the short-circuit and "incident" fields in the upper half space due to scattering by the object and where $\bar{F}$ of (62) is given in (2). Since the current $\bar{J}_s$ induced on the object is due to fields which penetrate the aperture and since $\bar{h}^{sc}$ and $\bar{h}^{i}$ are entirely due to $\bar{J}_s$, we view this portion of the forcing function of (62) as excitation from a dependent "generator":

$$\bar{h}^{i}(\bar{r}) = \frac{1}{4\pi} \nabla\times \iint_{S_B} \bar{J}_s(\bar{s}') \frac{e^{-jk|\bar{r}-\bar{s}'|}}{|\bar{r}-\bar{s}'|} dS' \qquad (63)$$

where S_B represents the surface of the object and $\bar{s}'$ locates a point on S_B.

In addition to (62), an equation must be available which characterizes the scatterer in the presence of the shorted screen subject to illumination from the aperture equivalent magnetic current $\bar{M}_s$. The electric field in the upper half space due to $\bar{M}_s$ is given by (6) (with $\bar{E}^{sc+} = \bar{0}$) so, for the perfectly conducting scatterer, one requires the tangential electric field on the surface S_B of the object to be zero:

$$\left(j \frac{\omega}{k^2} [k^2\bar{A} + \nabla\nabla\cdot\bar{A}] + \frac{1}{\varepsilon} \nabla\times\bar{F}\right)\times \hat{n} = \bar{0} \quad \text{on } S_B \qquad (64)$$

where $\hat{n}$ is the outward unit normal at a point $\bar{r}$ on S_B and where the magnetic vector potential due to the surface current $\bar{J}_s$ on the object in the presence of the screen is

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \iint_{S_B} \bar{J}_s(\bar{s}')\cdot\bar{\bar{g}}(\bar{r},\bar{s}')dS' \qquad (65a)$$

with

$$\bar{\bar{g}}(\bar{r},\bar{s}') = \hat{\hat{I}} \frac{e^{-jk|\bar{r}-\bar{s}'|}}{|\bar{r}-\bar{s}'|} + (2\hat{z}\hat{z} - \hat{\hat{I}}) \frac{e^{-jk|\bar{r}-\bar{s}' + 2(\bar{s}'\cdot\hat{z})\hat{z}|}}{|\bar{r}-\bar{s}' + 2(\bar{s}'\cdot\hat{z})\hat{z}|} \qquad (65b)$$

in which $\hat{\hat{I}}$ is the unit dyadic.

The above formulation can be specialized to the case in which the scatterer is a wire [25], and numerical data are available for the further specialization that the aperture be a narrow slot of finite length [25]. Also, for small apertures of shapes whose polarizabilities are known and with the scatterer not close to A relative to the maximum dimension across A, a simple formulation for an equation characterizing $\bar{J}_s$ has been devised [24]. The resulting equation is of the order of difficulty to solve as would be the equation for the same scatterer in the presence of the screen with no aperture.

VII. APERTURE-PERFORATED SCREEN SEPARATING DIFFERENT MEDIA

The formulation of the general equation for the problem of an aperture-perforated screen separating two half spaces whose electromagnetic properties are different parallels that outlined in Section II for the same-media case. With properties of the left and right half-space media characterized by (μ_-, ε_-) and (μ_+, ε_+), respectively, Butler and Umashankar [26] show that $\bar{M}_s$ satisfies

$$j\frac{\omega}{2}\left[\bar{F}^+ + \bar{F}^- + \nabla_t\nabla_t\cdot\left[\frac{\bar{F}^+}{k_+^2} + \frac{\bar{F}^-}{k_-^2}\right]\right]\times\hat{z} = \begin{cases} \frac{1}{2}\left[\bar{H}^{sc-} - \bar{H}^{sc+}\right]\times\hat{z} \\ \text{or} \\ \left[\bar{H}^{i-} - \bar{H}^{i+}\right]\times\hat{z} \end{cases}, \text{ in A} \tag{66a}$$

where $k_\pm^2 = \omega^2\mu_\pm\varepsilon_\pm$ and where

$$\bar{F}^\pm(\bar{r}) = \frac{\varepsilon_\pm}{2\pi}\iint_A \bar{M}_s(\bar{r}')\,\frac{e^{-jk_\pm|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}\,dS' \quad . \tag{66b}$$

For $(\mu_-, \varepsilon_-) = (\mu_+, \varepsilon_+)$, Eq. (66) is seen to be the same as (5). For TE and TM illumination of a uniform-width slot, equations corresponding to (53) and (54) can be obtained readily. Butler and Umashankar [26] point out that the equation for the TM-excited slot in a screen between different media can be solved numerically but that special care must be exercised, while the equation for the TE-excited slot can be solved routinely.

Narrow, TE-Excited Slot

By making use of the integral equation given in [26] for the TE-excited slot (Fig. 6) in a screen separating different media and the small argument approximation for $H_0^{(2)}(\xi)$, one can show that the equivalent magnetic current $\bar{M}_s = M_{s_y}(x)\,\hat{y}$ in the two-media problem satisfies

$$-\frac{j}{2\pi}\left(\frac{k_+}{\eta_+} + \frac{k_-}{\eta_-}\right) \int_{x'=-w/2}^{w/2} M_{s_y}(x')\,\ln|x-x'|\,dx' = H^{f2} + \left(H_y^{i-}(x) - H_y^{i+}(x)\right), \quad x\varepsilon(-w/2\,,\,w/2) \tag{67a}$$

whenever the slot is very narrow relative to the wavelength in both media, i.e., whenever $k_\pm w \ll 1$, where

$$H^{f2} = \frac{j}{2\pi}\left[\left(\frac{k_+}{\eta_+} + \frac{k_-}{\eta_-}\right)\left[\gamma + j\,\frac{\pi}{2}\right] + \frac{k_+}{\eta_+}\,\ln\frac{k_+}{2} + \frac{k_-}{\eta_-}\,\ln\frac{k_-}{2}\right] \int_{x'=-w/2}^{w/2} M_{s_y}(x')\,dx' \,. \tag{67b}$$

It is of interest to note that the logarithmic kernel of (67) implies that the singularity of M_{s_y} in the two-media case is the same as that of the single-medium case exhibited by Eq. (56);

Meixner has demonstrated by his classic procedure that the singularities in these two problems are the same [5]. Furthermore, one can show [22] that for $(H_y^{i-}(x) - H_y^{i+}(x)) = H_0^i$, a constant, the solution to (67) is

$$M_{s_y}(x) = H_0^i \frac{M_0}{\left[\left(\frac{w}{2}\right)^2 - x^2\right]^{\frac{1}{2}}}, \quad k_\pm w << 1 \quad , \tag{68a}$$

where

$$M_0 = \frac{2j}{\left(\frac{k_+}{\eta_+} + \frac{k_-}{\eta_-}\right)\left[\gamma + j\frac{\pi}{2} + \ell n \frac{w}{4}\right] + \frac{k_+}{\eta_+}\ell n \frac{k_+}{2} + \frac{k_-}{\eta_-}\ell n \frac{k_-}{2}} \quad . \tag{68b}$$

In a manner similar to that found in Section V for the single-medium case, one can obtain an approximate integro-differential equation for the long, narrow slot in a screen separating different media. The equation is more complex than is that for the single-medium problem but it can be solved numerically [27]. Finally, we point out that the concepts of Section VI can be generalized [24] to handle the two-media counterpart of the problem of excitation of an object through an aperture in a screen.

VIII. EXAMPLE FREQUENCY-DOMAIN DATA

Ultimate interest in EMP investigations lies, of course, in the time history of the electromagnetic field at critical points in a system under evaluation. Usually such a time history is computed (via Fourier inversion) from knowledge of the corresponding time-harmonic field over a frequency spectrum of practical limits. For this reason and, also, due to the present paucity of time-domain electromagnetic field data that are of utility in EMP studies, it is of value to become familiar with available information in the frequency domain. To this end, a summary of time-harmonic results is provided in this section.

The discussions below are based upon the fundamentals outlined in Sections II - VII and the data presented were calculated from numerical solutions of the equations developed there. Such numerical methods are important in present-day work but space limitations do not allow for coverage of this subject here. Original sources are cited and should be consulted by those interested in numerical techniques appropriate for a given type of problem.

Square and Circular Apertures in Planar Screens

In Figs. 8 and 9 are displayed the magnitudes of the components of the transverse electric field in a $1\lambda \times 1\lambda$ square aperture in an infinite conducting screen, excited by a normally incident plane wave in the left half space with $\bar{E}^{i} = E^{i}\hat{y}$. The singularity in the field at the aperture/screen edges is clearly exhibited, and one should note that the field components are different in both peak magnitude and distribution across the aperture. These data were computed by Rahmat-Samii and Mittra [7] from numerical solutions of (38) - (41). We wish to point out that Wilton and Glisson [28] have devised a clever scheme for solving (5) very efficiently from which these results are obtainable too. Others [29,30] have employed moderately reliable solution techniques which are founded upon an approximate wire-grid model of a conducting plate, the Babinet equivalent of an aperture in a planar screen.

In Fig. 10 is illustrated a comparison of a component of the electric field in square and circular apertures excited by a normally incident plane wave. The results for the square aperture were obtained from integral equation solutions while those for the circular hole were measured by Robinson [31]. The fields in the two apertures are seen to be quite comparable, even near the edges. The close agreement of the field distributions in circular and square apertures is obtained only when one compares the fields along the principal axes of the square aperture, which is not altogether unexpected. Fig. 11 shows penetrated fields obtained from integral equation solutions, from the Kirchhoff approximation, and from measurements made by Andrews [32] for a circular aperture. The three curves exhibit essentially the same behavior for $z/\lambda > 1.5\lambda$.

Small Apertures

Over the practical spectrum of the EMP, many apertures of interest are electromagnetically small while others fall into this category over a significant portion of the spectrum. Far more data are available for small apertures than for those whose maximum dimension is a sizable fraction of the wavelength.

In Fig. 12 are found plots of the dominant component of electric field in small square and circular apertures, due to a normally incident plane wave ($\bar{E}^{i} = E^{i}\,\hat{y}$). The square-aperture data were obtained from numerical solutions of (38) - (41) with $g = g^{o}$ [18], while those for the circular case were computed from (45) directly. We observe from this set of curves that the electric field along the principal axes of the small square aperture is quite close to that in the (inscribed) circular aperture.

In Fig. 13 is displayed E_y as a function of D/λ, at the center of the square aperture, together with the field obtained from (45) evaluated at the center of the inscribed circular aperture. Again the excitation is a normally incident plane wave in the left half space. We observe that for almost the entire range $0 \leq D/\lambda \leq .1$ the amplitude curves are linear and the phase is constant at 90^{o}. This is, of course, predictable analytically for the circular apertures from the formula (45), and it is interesting to note that the numerical results for the rectangular aperture behave similarly. One can take advantage of this fact and derive an empirical formula by solving the rectangular aperture problem numerically at a single frequency in the range where the linear relationship for the amplitude and constancy of the phase pattern are valid, thus saving a considerable amount of computer time which would otherwise be required to determine the center field over a range of frequencies.

For an electrically small aperture one may approximate the aperture-produced fields by making use of (50) and the dipole moments calculated for the given aperture. How good the dipole moment approximation to the actual fields is depends upon the electrical size of the aperture, the distance from the aperture to the point at which the field is evaluated, and the choice of the coordinate origin with respect to which the dipole moments are calculated. Therefore, since one wishes to take advantage of the simplicity afforded by use of dipole moments to characterize the electromagnetic behavior of an aperture, it is of interest to determine fields directly from the numerically calculated magnetic current $\bar{M}_s$ as well as from the moments and then to compare values so obtained. Such comparisons enable one to assess the accuracy of fields calculated from moments.

Fig. 14 shows the electric field which penetrates a small square aperture ($2a = 2b = 0.15\lambda$) subject to normally incident illumination with a 1 volt/meter electric field directed along the y axis. In this case the electric dipole moment is zero so the total field is approximated by that of a magnetic dipole. These approximate values of fields together with exact values determined from computed $\bar{M}_s$ are both displayed for comparison. One sees good agreement at a radial distance $r = 10a$ but sees significant differences at 3a and 2a. In Fig. 15 are displayed the fields which penetrate the same aperture subject to edge-on incident illumination with $E_z^{i-}(0) = 1$ volt/meter and with the direction of propagation along either the x axis or the y axis.

The primary reason for the departure of the two results is the approximate nature of the dipole moment calculation, which incorrectly predicts an infinite field for $z/\lambda \to 0$. Thus, one must exercise caution in using the dipole moment approach to compute the diffracted field close to an aperture.

Now we return to observations of similarities in small circular and square apertures. In particular we compare diffracted fields computed exactly for the square aperture from integral equation solutions and approximately for the circular hole from dipole moments. We note from Fig. 16, which displays the behavior of the E_y field as a function of z/λ, that the phases coincide and that the amplitude curves also agree with each other for $z/\lambda > .08$. However, for closer points, i.e., $0 < z/\lambda < .08$, the dipole moment calculations for the circular aperture deviate substantially from the numerically exact solution for the square aperture.

Slots

If an infinite slot is very narrow and the excitation is a normally incident plane wave TE to the slot axis, one can compute the slot electric field or equivalent magnetic current directly from (56) (or from (68) in the two-media case). For wider slots (53) can be solved numerically for the equivalent magnetic current, and for TM excitation (54) must be solved.

The computed magnetic current, due to normally incident illumination, in a one wavelength slot (relative to the wavelength λ_- of the left half space), is displayed in Fig. 17 for various values of contrast in the media on the two sides of the screen. Fig. 18 shows the far field pattern of the magnetic field which passes through the slot. The contrast in material on the two sides of the screen is seen to have a strong effect upon these field patterns.

If a second conducting screen is placed parallel to and behind the slotted screen as shown in Fig. 19, one finds [33] that, in a narrow slot subject to normally incident TE illumination, the field retains the essential distribution (like (17)) of that found in a slot in a single screen. However, as is evident from Fig. 20, the strength of the slot field differs from that of the isolated case and, as expected, depends upon the separation of the two conducting planes. For comparison, the values of $E_x(0)$ in the same slot cut in an isolated plane are indicated by the dashed lines in Fig. 20. For $h < \lambda/2$, only the TEM mode exists in the guide remote from the slot. The electric field of this propagating TEM mode, apart from the factor $e^{\pm jkx}$, is given as a function of h in Fig. 21.

Turning attention to slots subject to TM illumination, we first mention that, for a given slot, less energy reaches the shadow side due to this excitation than due to TE excitation. Fig. 22 illustrates the magnetic current in slots of width $\lambda/2$, λ, and 3λ subject to normally incident TM illumination; these data were obtained from numerical solutions of (54) with $H_x^{i+} = 0$. In the two-media TM case, the magnetic current in a slot can vary markedly in both distribution and strength with varying media contrast, whereas, in the TE case, the contrast primarily effects the strength [26]. In Fig. 23 is depicted the far field pattern of the shadow-side electric field due to normally incident TM illumination in the left half space with different contrasts.

No data are provided here for the finite-length, narrow slot due to its duality with the thin wire for which copious results can be found.

Wire Scatterer behind a Slotted Screen

As a special but practically important case of the type problem discussed in Section VI, we let the scatterer be a finite-length, thin wire parallel to the screen and the aperture be a narrow slot of finite length as depicted in Fig. 24. The wire center is designated (x_c , y_c , z_c), and the slot center is at $(0 , 0 , 0)$. And, in the following discussion of current induced on the wire, the angular rotation of the wire about its center is in a plane parallel to the screen and is measured by the angle β defined as the angular displacement from the y axis. Both thin wire and narrow slot simplifications are utilized and the resulting equations, found in [25], for the unknown magnetic current in the slot and unknown total axial current I on the wire can be solved by numerical methods. Current on the wire is given in Figs. 25 - 30 for several cases of interest which are described in Table II; in all cases, the excitation is a plane wave normally incident upon the screen, the wire radius a is 0.001λ, and the slot width w is 0.05λ.

Fig. 25 shows the current on a half-wavelength wire, as a function of position along the wire, induced by the field which penetrates a quarter-wavelength slot; the center of the wire is on the z axis, $\lambda/4$ behind the screen, and I is given for selected values of the angle β. When $\cos\beta = 1$, the wire and slot are perpendicular, and the coupling is seen to be maximum as expected, while there is no coupling when the wire and slot are parallel ($\cos\beta = 0$). For resonant length ($L = \lambda/2$), the wire current distribution is essentially a cosine function as one would expect. As seen in Fig. 26 the current is greater when the wire is brought closer ($z_c = \lambda/8$) to the screen. With the wire returned to the original position ($z_c = \lambda/4$) but with the slot length increased to $\lambda/2$, the wire current, as seen in Fig. 27, is far greater than that in Fig. 26 due to the fact that the penetration through a half-wavelength slot is much greater than that through a quarter-wavelength slot.

In Fig. 28 and 29 are given data for the situation of a half-wavelength slot and a one-wavelength wire. The current displayed in Fig. 28 is on a wire whose center is on the z axis ($0, 0, \lambda/8$) so, for any value of β, the slot radiation causes an even-function excitation of the wire which, in turn, produces even-function current. If the wire center is displaced from the z axis to the point ($\lambda/8, \lambda/4, \lambda/4$), the current is quite different (Fig. 29). With the wire center not above the slot axis, the wire excitation is never an even function and a strong antiresonant current is excited for all angles β. Even though the wire is much closer to the slot in the former than in the latter case, the peak current on the wire is larger in the latter due to the fact that the antiresonant current can be excited only in this case.

Distribution of wire current and slot magnetic current are very sensitive to the location of the wire center and to β

when the length of both the slot and the wire is one wavelength. If the wire center is on the z axis, the one-wavelength anti-resonant current is not excited for any value of cosβ and the wire current is again an even function with smaller magnitude than in cases for which the slot is half-wavelength (Fig. 30). The dominant illumination of the slot is the incident field with a smaller excitation caused by scattering from the wire. The normally incident illumination is an entirely even-function excitation of the slot and, thus, causes a slot magnetic current having only a forced response. In the absence of the strong, odd-function antiresonant component of magnetic current, the resulting wire excitation due to penetration through the slot is relatively weak as can be seen from a comparison of Figs. 28 and 30.

As pointed out by Butler and Umashankar [25], the energy scattered back into the aperture from the wire can be quite significant and can strongly influence the aperture fields. Calculation of aperture fields under the assumption that the wire is not present can lead to serious errors in certain cases.

IX. EXAMPLE TIME-DOMAIN DATA

As an example of the response of an aperture to an EMP, we compute the time history $\bar{e}^{+}(t,\bar{r})$ of the field which passes through a 115 cm × 1.3 cm rectangular aperture in a planar screen. The excitation of the aperture/screen is a normally incident, double-exponential, plane-wave EMP represented by

$$\bar{e}^{i}(t,\bar{r}) = \hat{y}\, E_0 \left\{ \left(e^{-\alpha(t - z/c)} - e^{-\beta(t - z/c)} \right) \theta(t - z/c) \right. \tag{69}$$
$$\left. - \left(e^{-\alpha(t - \tau - z/c)} - e^{-\beta(t - \tau - z/c)} \right) \theta(t - \tau - z/c) \right\}$$

where

$$c = 3.0 \times 10^{8} \text{ m/sec}$$

$$\alpha = 6.0 \times 10^{6} \text{ sec}^{-1}$$

$$\beta = 2.0 \times 10^{8} \text{ sec}^{-1}$$

$$\tau = 2.04189 \times 10^{-9} \text{ sec}$$

$$E_0 = 10^{3} \text{ v/m} \quad .$$

θ - unit step function

One computes the spectrum of (69) and, subject to this excitation, determines frequency-domain quantities for the aperture problem; then the time history of each quantity is available from Fourier inversion. The time domain response $\bar{e}^{+}(t,\bar{r})$ is computed from knowledge of $\bar{E}^{+}(f,\bar{r})$ via standard transform means and the integrals involved in this process may be handled by the FFT algorithm [34] which is known to be efficient for such purposes. Also, using the fact that $\bar{e}^{+}(t,\bar{r})$ is real, one can show [35] that either the real or the imaginary part of $\bar{E}^{+}(f,\bar{r})$ is sufficient to determine $\bar{e}^{+}(t,\bar{r})$, an observation which lessens computational labor. For an accurate evaluation of the intergrals, $\bar{E}^{+}(f,\bar{r})$ must be adequately sampled in the frequency range of interest so that the criterion suggested by sampling theorem [36] is not violated. One finds that, according to this criterion, it is necessary to evaluate $\bar{E}^{+}(f,\bar{r})$ at no fewer than 1024 points in the interval $0 - 10^{9}$ Hz beyond which $|\bar{E}^{+}(f,\bar{r})|$ decreases to 60 dB below its maximum value.

However, rather than sampling $\bar{E}^+(f,\bar{r})$ uniformly in the frequency range $0 - 10^9$ Hz, it is more desirable to compute $\bar{E}^+(f,\bar{r})$ relatively densely in the neighborhood of the aperture's resonant frequencies, where $\bar{E}^+(f,\bar{r})$ varies rapidly, and sparsely at the frequencies where response is smooth. Then, one can compute $\bar{E}^+(f,\bar{r})$ at the needed 1024 points by employing an interpolation scheme and the values of $\bar{E}^+(f,\bar{r})$ at the fewer (seventy in this example), nonuniformly-spaced points in the spectrum.

Fig. 31 shows the behavior of $e_y^{i-}(t,\bar{0})$ as a function of time and Fig. 32 shows the magnitude of its Fourier transform $|E_y^{i-}(f,\bar{0})|$. In Figs. 33 and 34 are found the frequency responses of the magnitude and real part of the electric field component E_y^+ evaluated at a point on the z axis 2 meters behind the aperture/screen. The time-domain electric field $e_y^+(t,\bar{r})$, calculated at $\bar{r} = 2\hat{z}$ by the procedure outlined above, is displayed in Fig. 35.

X. CONCLUSIONS

In this paper is provided a tutorial description of a number of analytical methods available for solving the problem of electromagnetic interaction and penetration through perforations in the walls of conducting bodies. Emphasis is placed on the development of an understanding of the various aspects of the formulation and solution of the aperture coupling problem, rather than on the numerical details of the procedures for extracting the solution.

Beginning with the simple case of an aperture in a planar screen that separates two homogeneous half spaces, the authors generalize the theory to the case of an aperture in an arbitrarily-shaped conducting surface. The pertinent equations for TE and TM illumination of infinite slots also are discussed and representative results for both of these problems are presented. Due to their importance in EMP studies, special attention is given to small apertures and to the determination of dipole moments that quantitatively describe the coupling through such apertures. Another important problem discussed in the paper is that of excitation of objects, e.g., wires and cables, located behind an aperture-perforated surface. It is shown that the energy scattered from the object back into the aperture can be significant and can strongly influence the aperture fields.

Although much of this paper is devoted to analyses in the frequency-domain, the procedure for constructing the time-domain behavior from knowledge of the frequency-domain solution is included for completeness. Data illustrating the transient response of an aperture to an EMP are given.

Space limitations preclude the inclusion of discussions of the numerical algorithms associated with the solution methods; instead, numerous publications available on this subject are cited for the benefit of the interested reader. A classified bibliography listing papers not cited by Bouwkamp [1] or Eggimann [2] is offered as a supplement to their extensive reviews of the aperture literature.

The authors earnestly hope that those who are concerned with the formidable task of designing practical, EMP-hardened systems find this paper beneficial as an aid to a better understanding of aperture theory and that they are able to utilize the data provided to guide their design procedures.

XI. ACKNOWLEDGMENT

The authors express their sincere appreciation to Miss Diane Arnett who so skillfully typed the manuscript without the benefit of the presence of two of the three authors.

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TABLE I

APERTURE POLARIZABILITIES*

Shape	α_e	$\alpha_{m\,xx}$	$\alpha_{m\,yy}$
Circle (Radius = R)	$\frac{2}{3} R^3$	$\frac{4}{3} R^3$	$\frac{4}{3} R^3$
Ellipse	$\frac{1}{3} \frac{\pi w^2 \ell}{E(\epsilon)}$	$\frac{1}{3} \frac{\pi \ell^3 \epsilon^2}{K(\epsilon)-E(\epsilon)}$	$\frac{1}{3} \frac{\pi \ell^3 \epsilon^2}{\left(\frac{\ell}{w}\right)^2 E(\epsilon) - K(\epsilon)}$
Narrow Ellipse ($w << \ell$)	$\frac{1}{3} \pi w^2 \ell$	$\frac{1}{3} \frac{\pi \ell^3}{\ln\left(\frac{4\ell}{w}\right) - 1}$	$\frac{1}{3} \pi w^2 \ell$

* See [1,10,19].

Notes: (1) $\bar{\bar{\alpha}}_m = \alpha_{m\,xx}\hat{x}\hat{x} + \alpha_{m\,yy}\hat{y}\hat{y}$ ($\bar{\bar{\alpha}}_m$ is diagonal for symmetric shapes given.)

(2) Ellipse Eccentricity $\epsilon = \sqrt{1 - \left(\frac{w}{\ell}\right)^2}$ (See Fig. 5.)

(3) K and E are the complete elliptic integrals of the first and second kind, respectively, as defined in [20].

TABLE II

GEOMETRIC PARAMETERS FOR PROBLEM OF WIRE BEHIND SLOTTED SCREEN

Figure	ℓ/λ	L/λ	x_c/λ	y_c/λ	z_c/λ
25	1/4	1/2	0	0	1/4
26	1/4	1/2	0	0	1/8
27	1/2	1/2	0	0	1/4
28	1/2	1	0	0	1/8
29	1/2	1	1/8	1/4	1/4
30	1	1	0	0	1/8

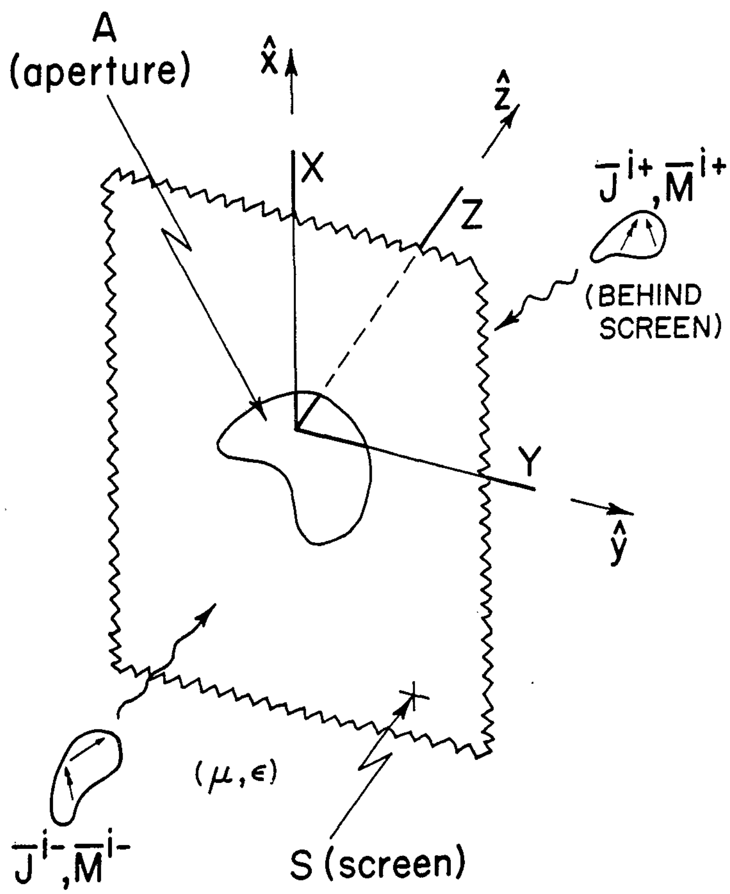
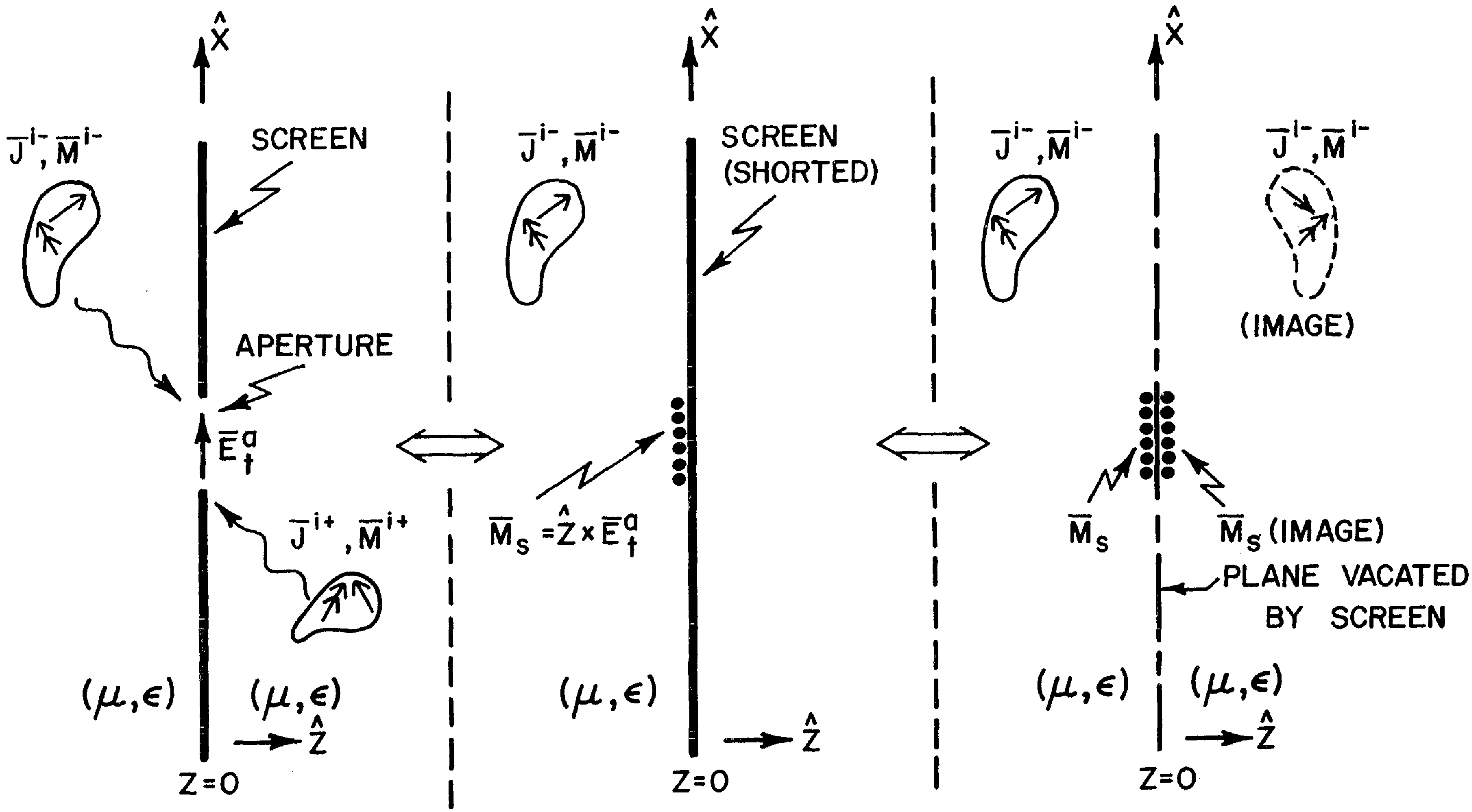


Fig. 1. Aperture in planar conducting screen of infinite extent.



(a)
ORIGINAL PROBLEM
(VALID EVERYWHERE)

(b)
EQUIVALENT PROBLEM
(VALID, Z<0)

(c)
EQUIVALENT PROBLEM
(VALID, Z<0)

Fig. 2. Left half space equivalences.

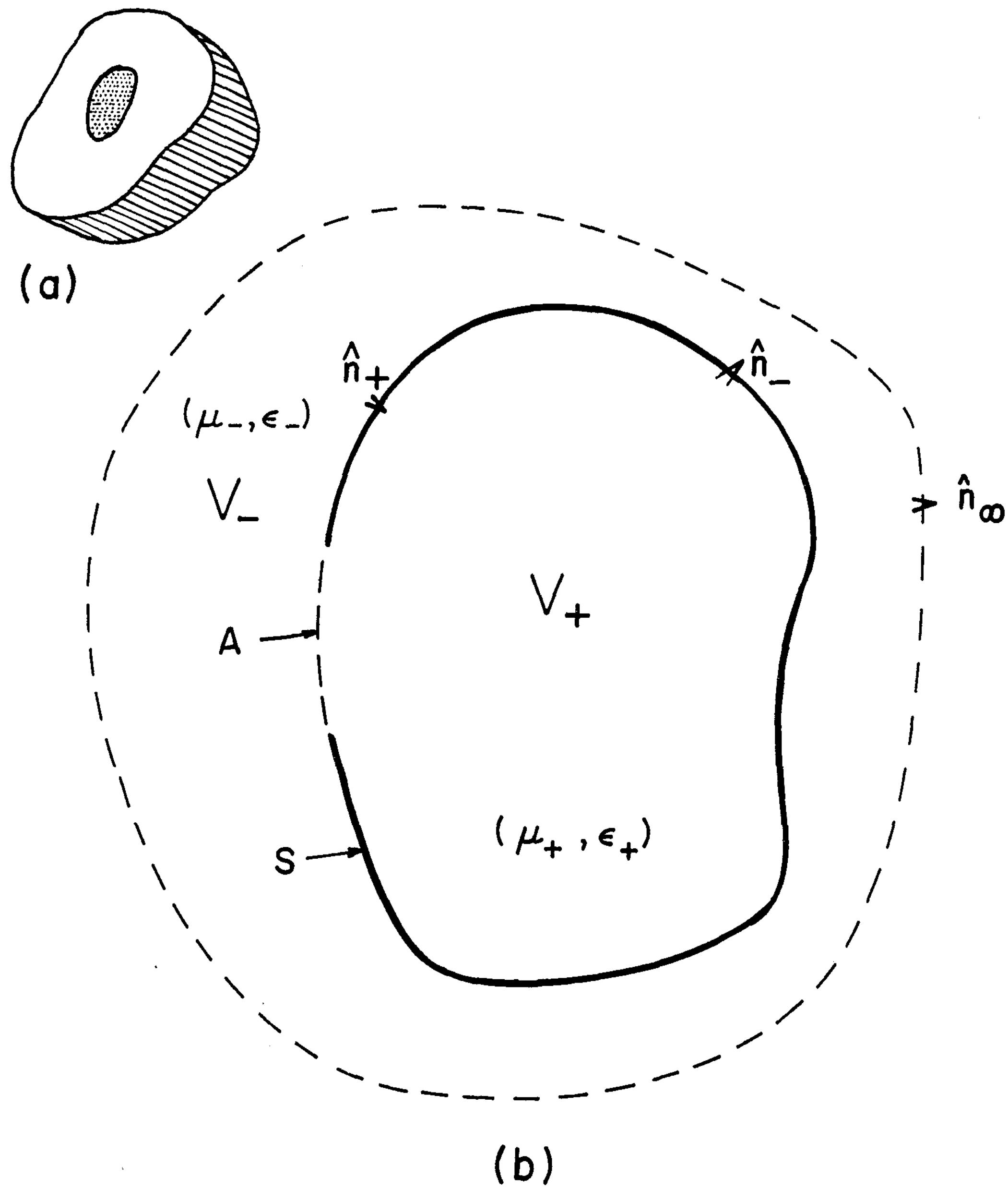


Fig. 3. (a) Aperture in a cavity. (b) Cross-sectional view of (a) where V_- and V_+ are exterior and interior regions, respectively; A is the aperture and S is the surface of the cavity.

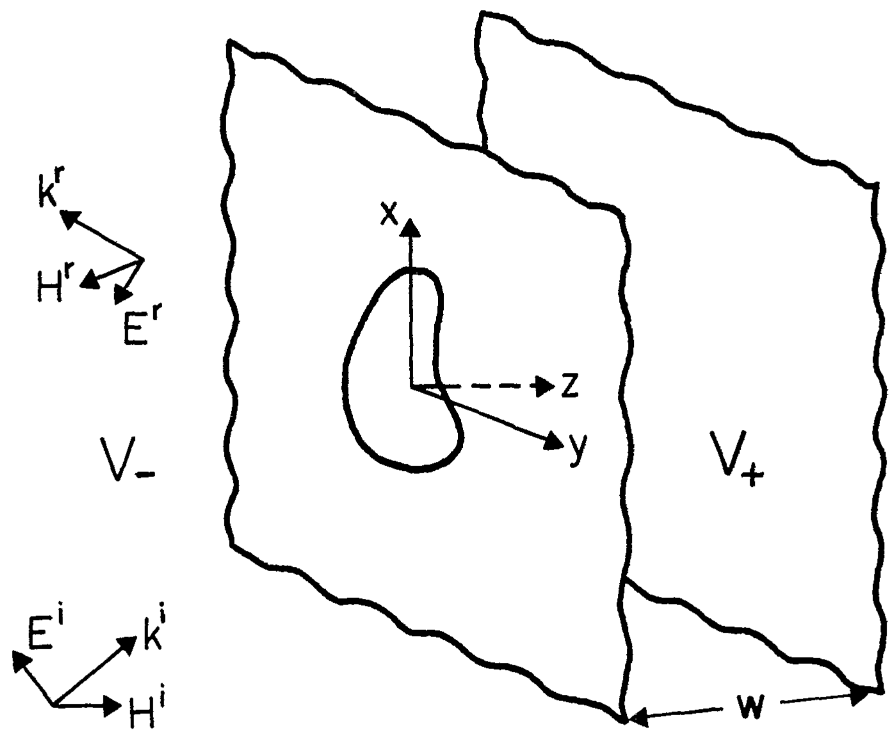


Fig. 4. Aperture in a perfectly conducting screen with a back plate illuminated by a plane wave.

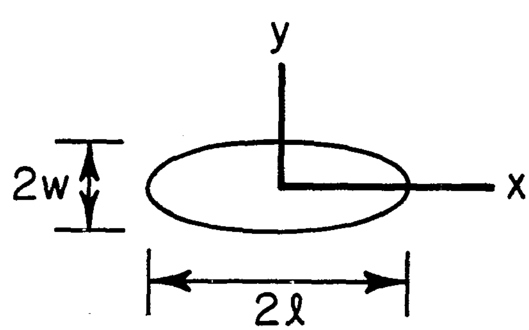
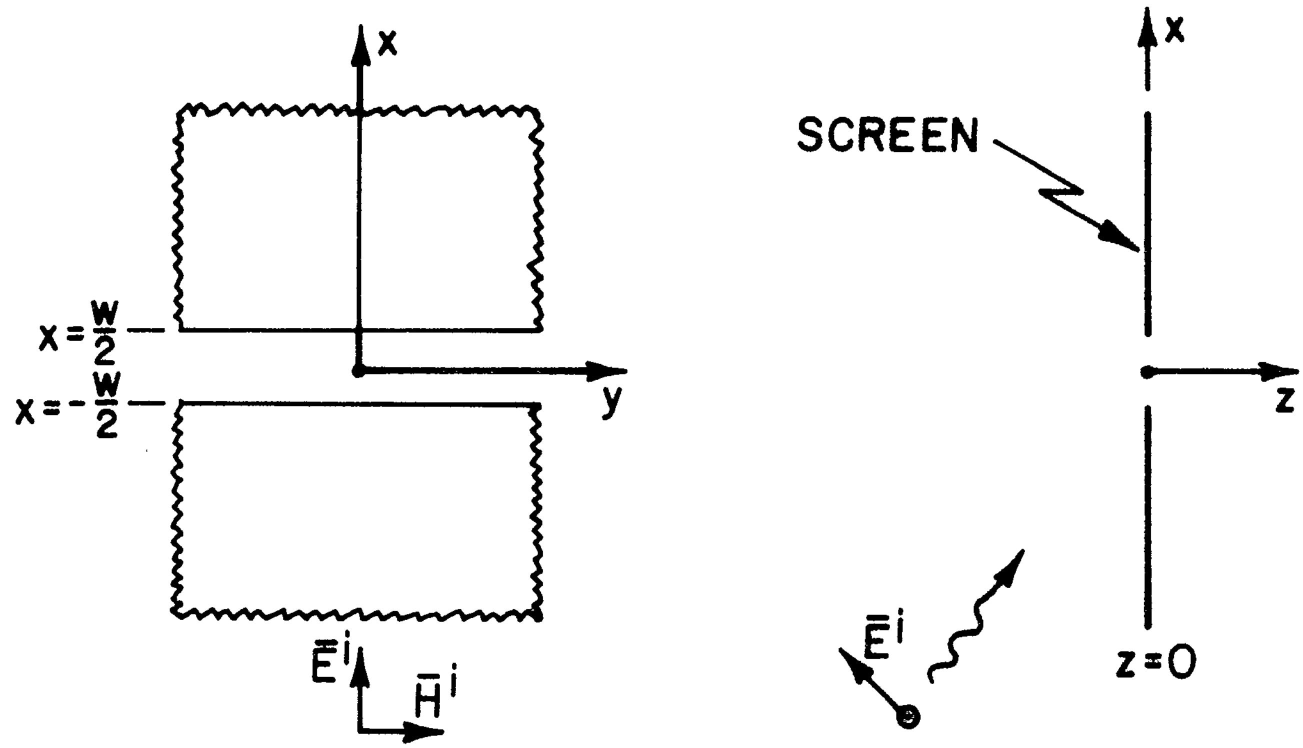
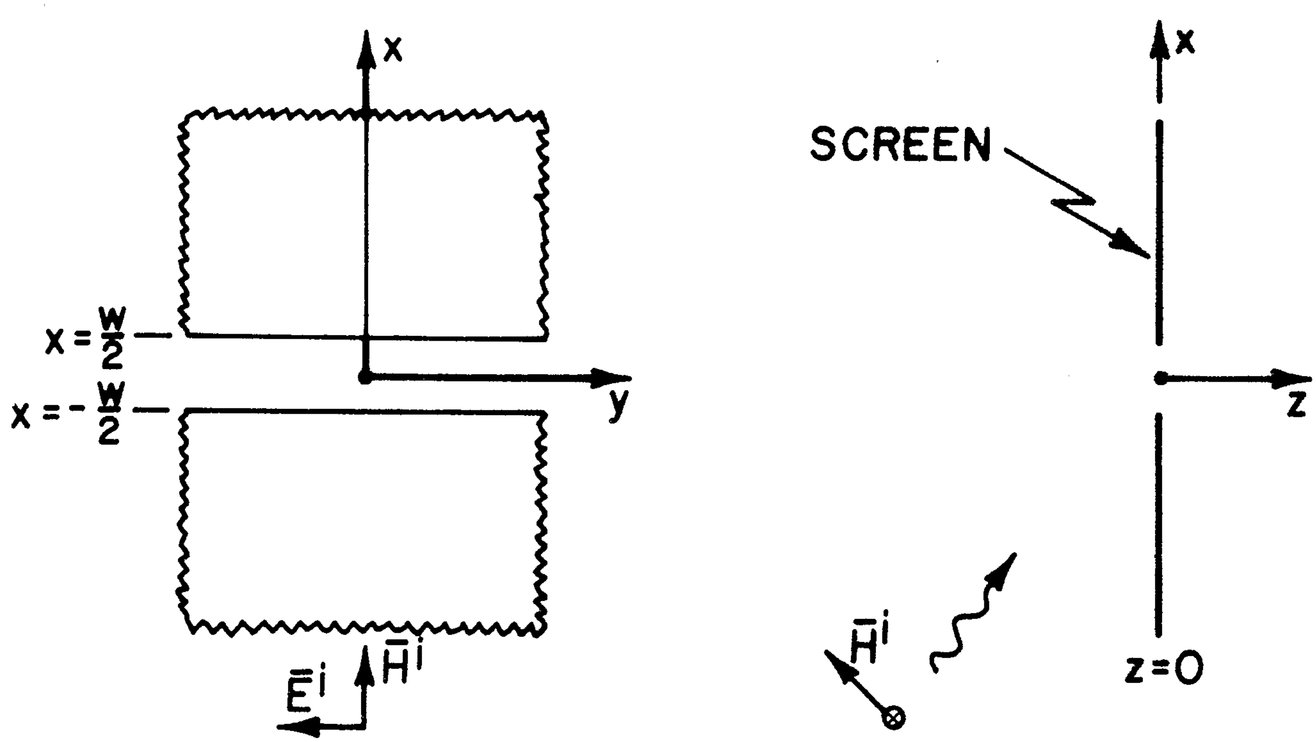


Fig. 5. Ellipse.



TE EXCITED SLOT



TM EXCITED SLOT

Fig. 6. Slotted screen (a) TE case and (b) TM case.

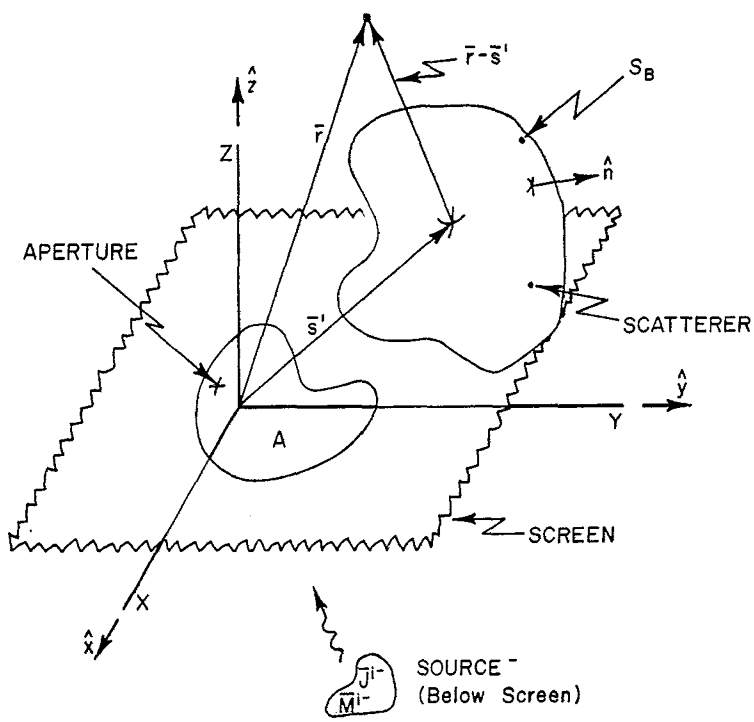


Fig. 7. Object (scatterer) above aperture-perforated screen.

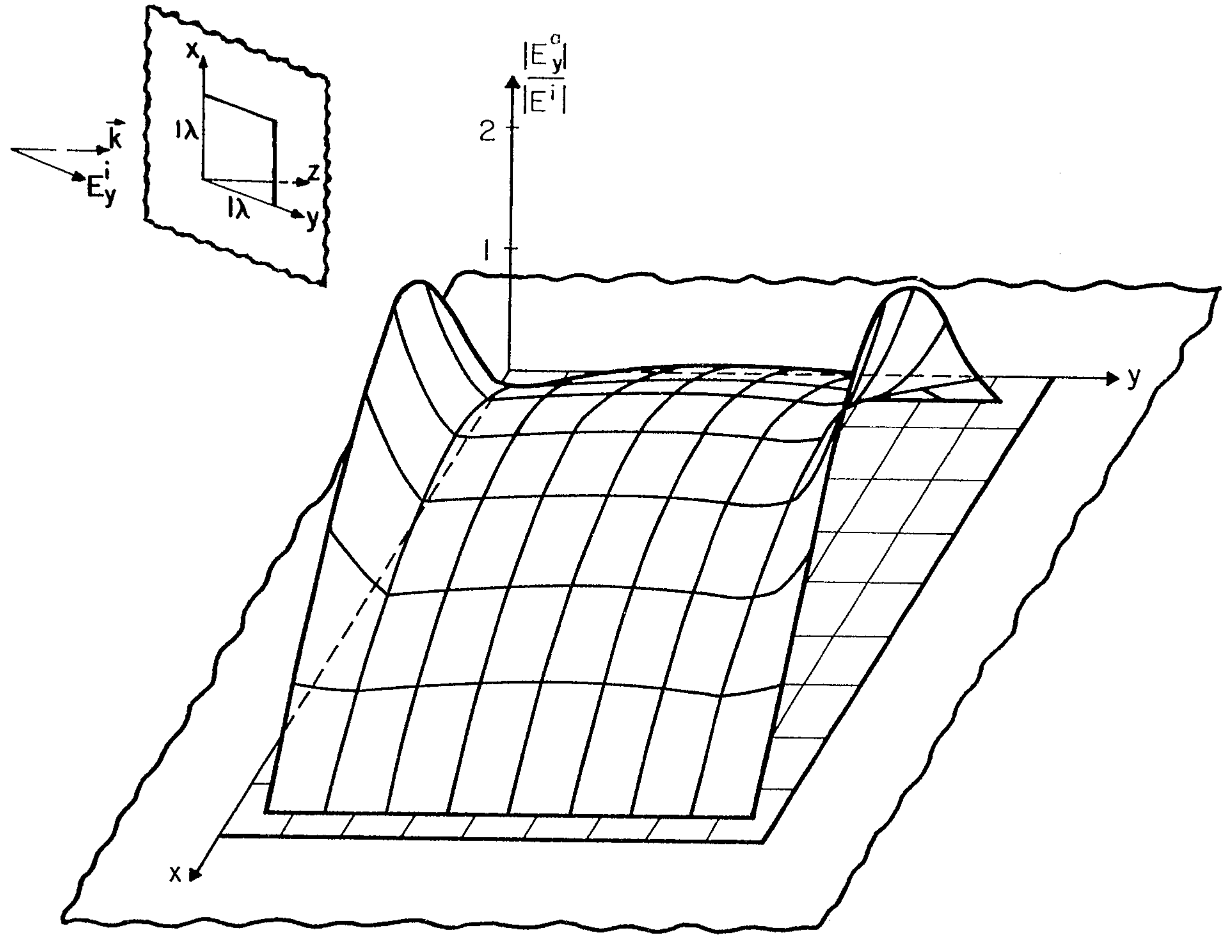


Fig. 8. Three-dimensional representation of the amplitude distribution of the E_y-field in the aperture.

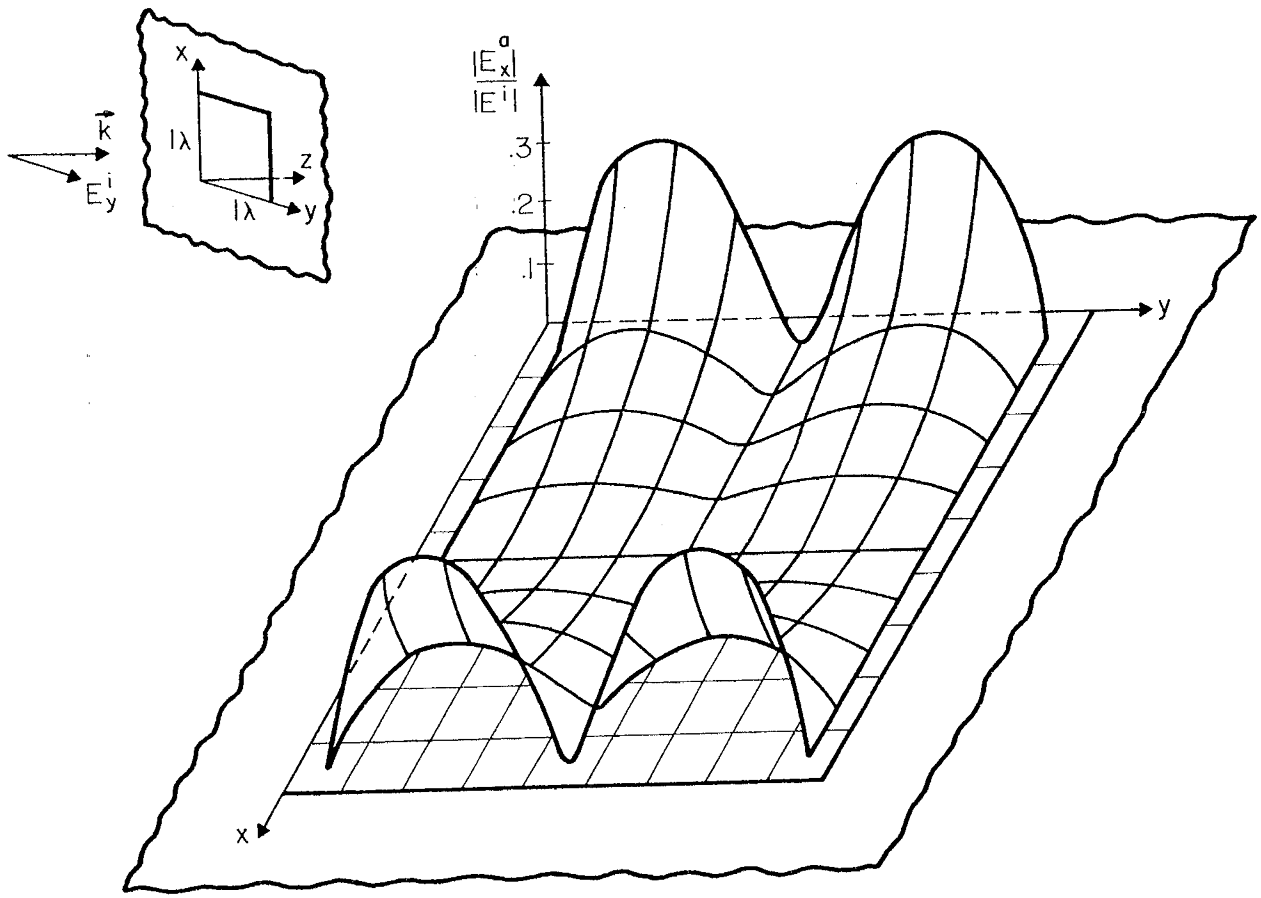


Fig. 9. Three-dimensional representation of the amplitude distribution of the E_x-field in the aperture.

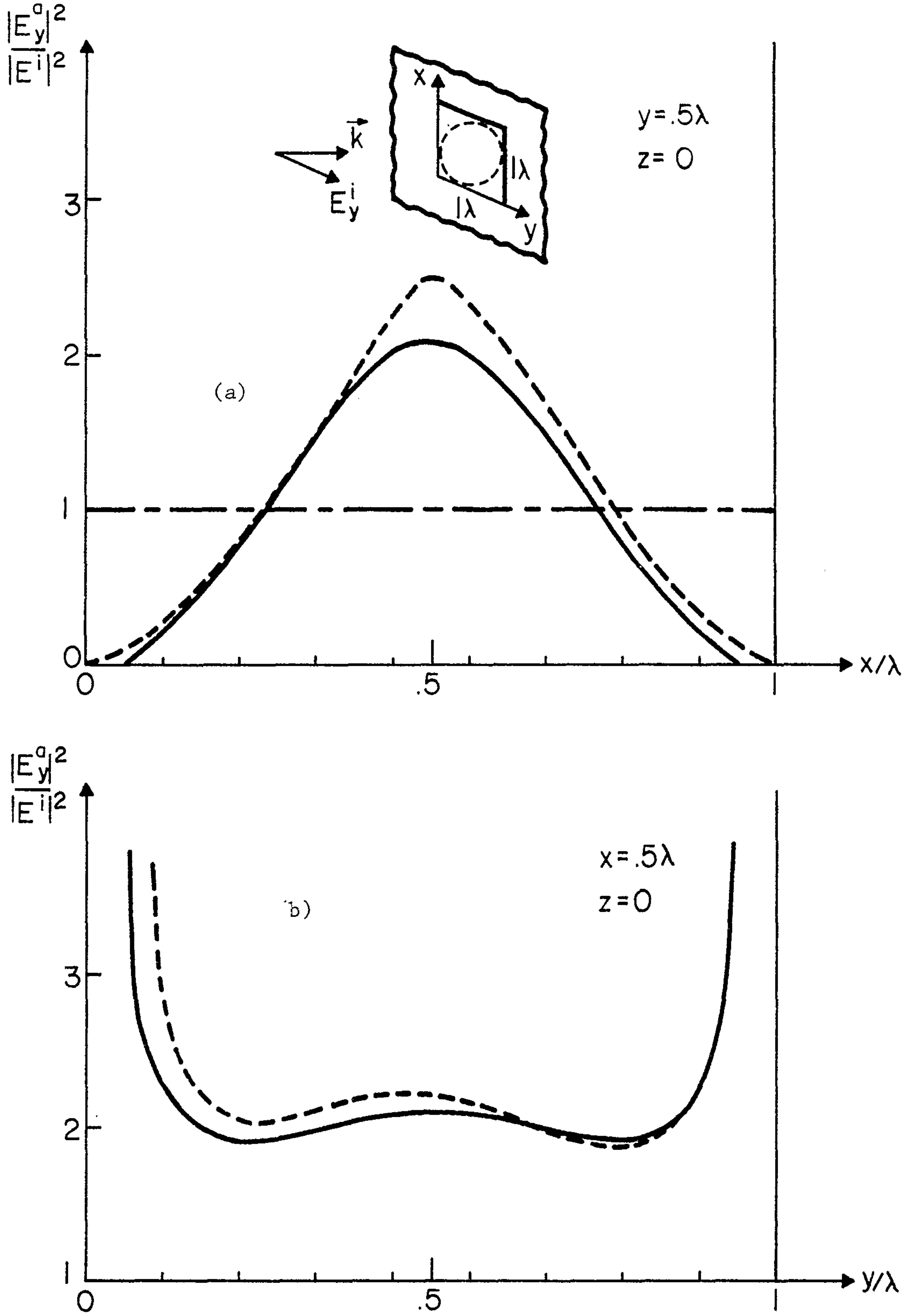


Fig. 10. Intensity distribution of the E_y-field sampled along the principal axes of a $1\lambda \times 1\lambda$ square aperture and a circular aperture of radius 1λ. (a) Intensity distribution sampled along a line parallel to the x-axis and passing through the center. (b) Intensity distribution sampled along a line parallel to the y-axis and passing through the center. Integral equation solution (——). Experimental results (- - -) from Robinson [31].

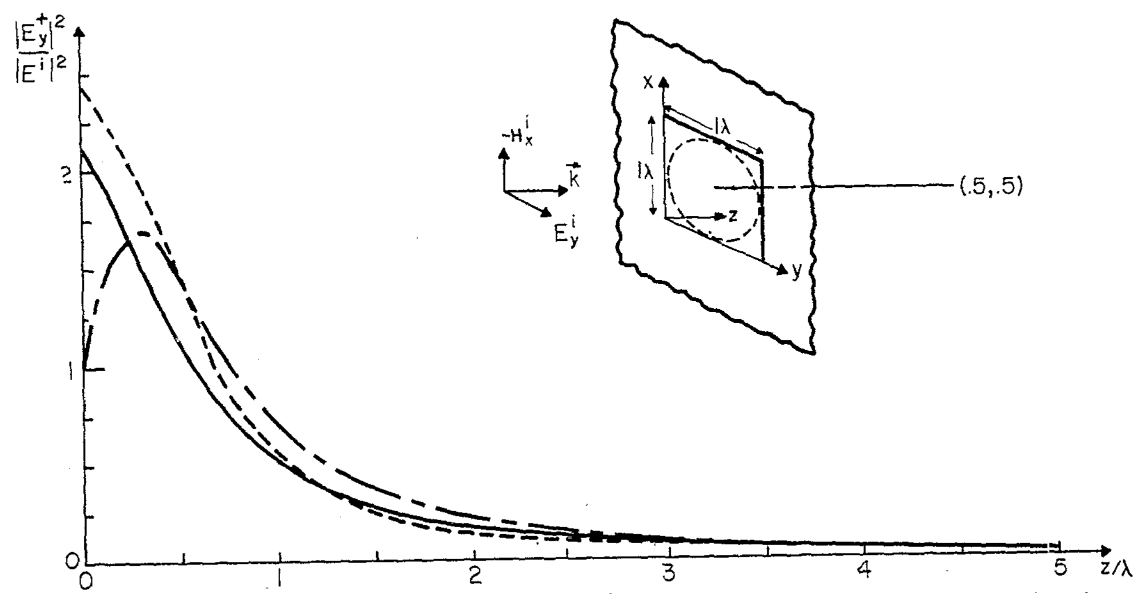
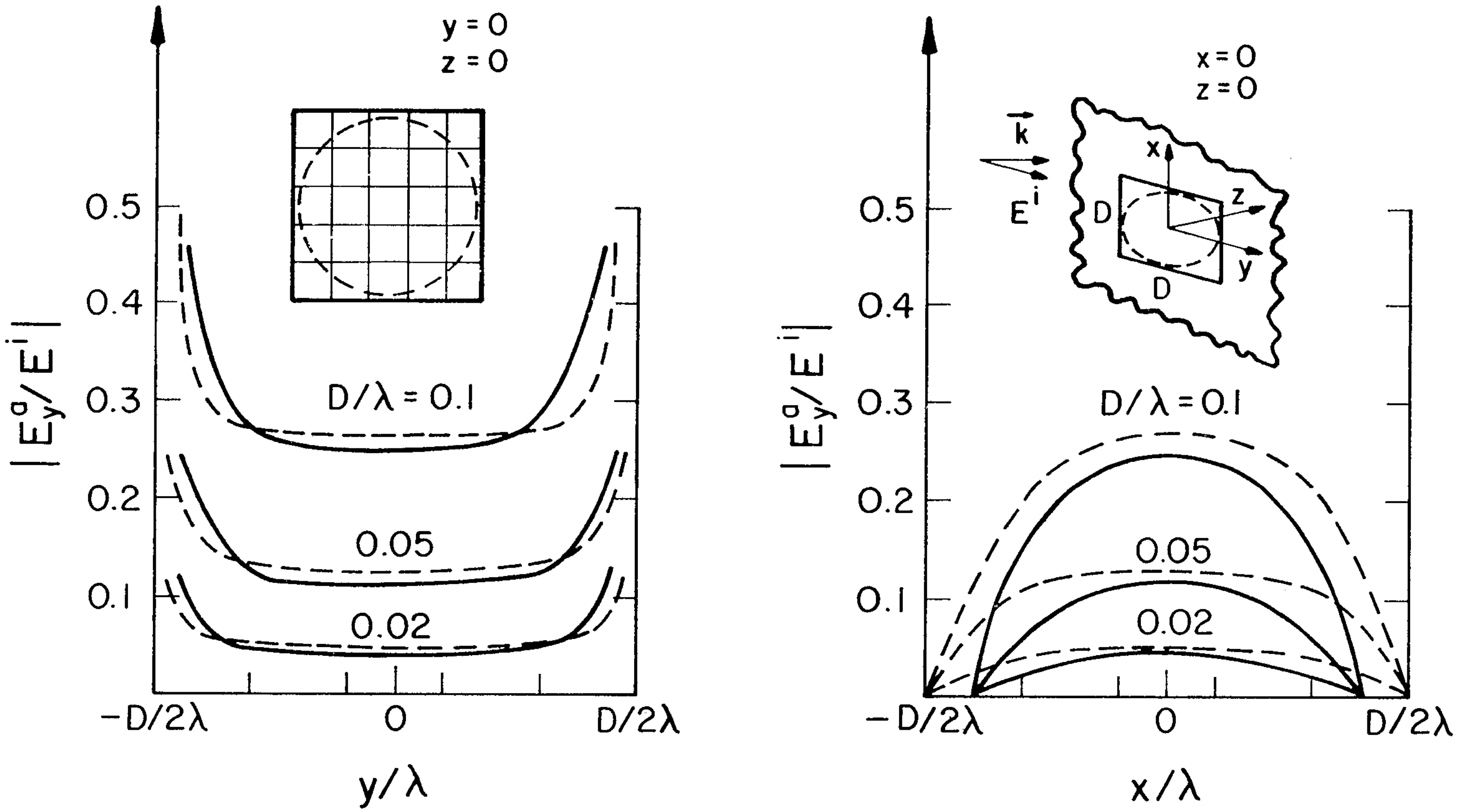


Fig. 11. Intensity distribution of E_y-field sampled along a line parallel to the z-axis and passing through the center of square and circular apertures. Integral equation (——) solution for square aperture. Kirchhoff approximation for square aperture (—— -). Experimental result (---) for circular aperture from Andrews [32].



(a) (b)

Fig. 12. E-field distribution in electrically small square and circular apertures. (a) E_y-field sampled along y-axis. (b) E_y-field sampled along x-axis. Integral equation (——) solution for square apertures. First-order (---), low frequency, from Eq. (45) for the circular apertures with diameter D.

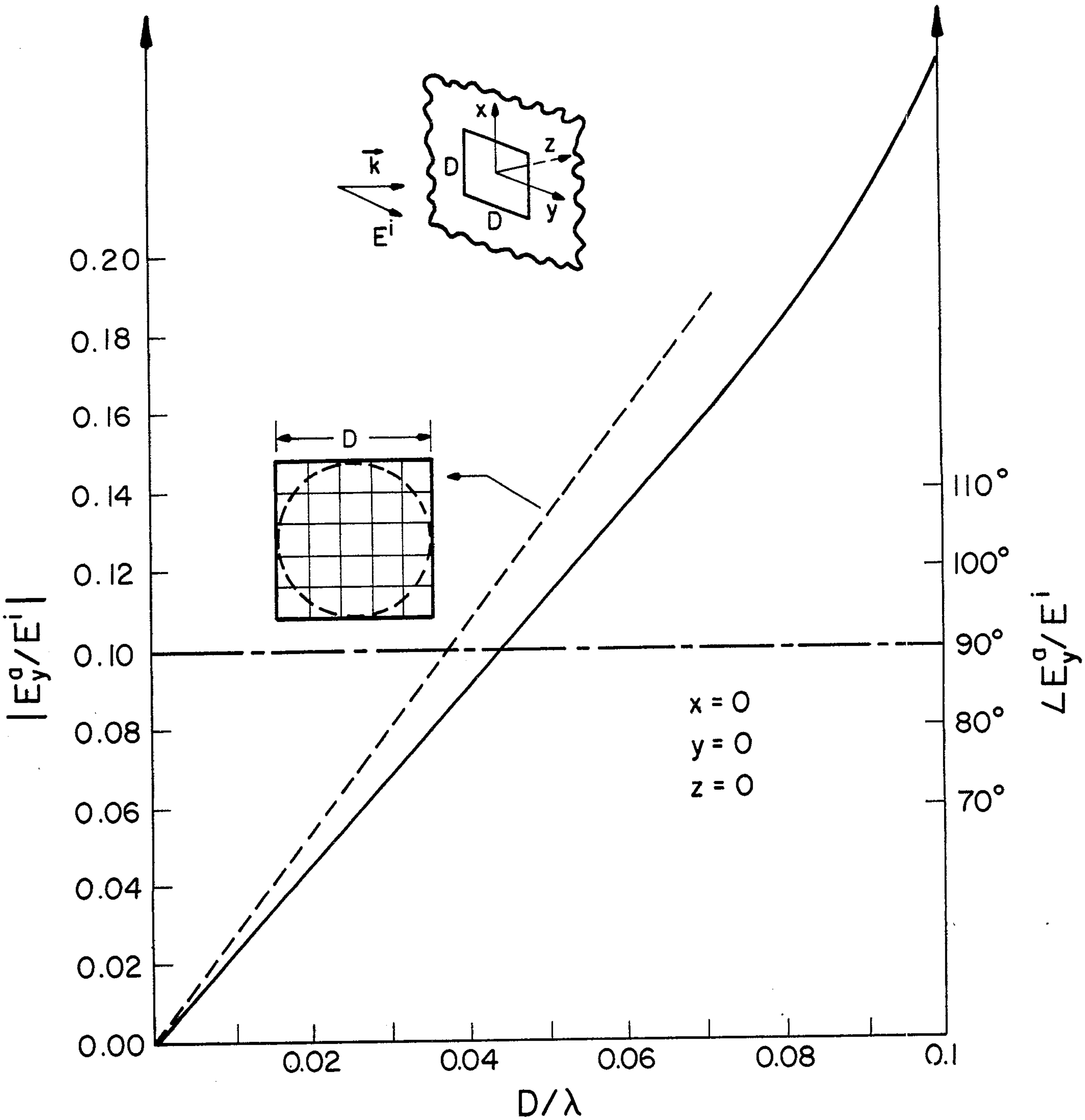


Fig. 13. E_y-field at the center of square and circular apertures.

Amplitude curves (——) obtained from integral equation solution for a square aperture. Amplitude curve (---) for the circular aperture from Eq. (45). Phase curve (—— -) for square and circular apertures.

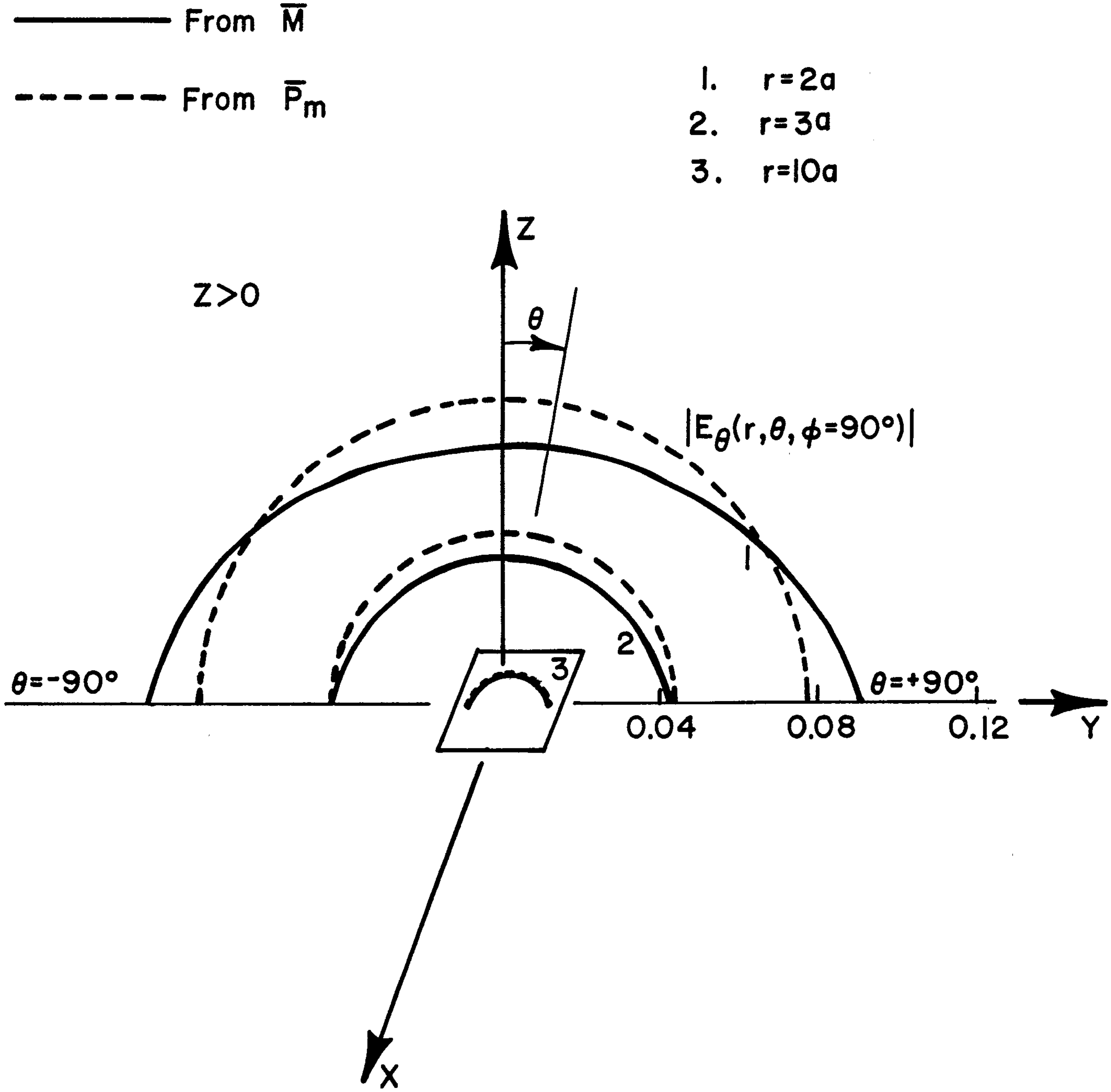


Fig. 14. Electric field on shadow side of square aperture ($2a = 2b = 0.15\lambda$, $E_y^{i-} = 1$ volt/meter, normal incidence).

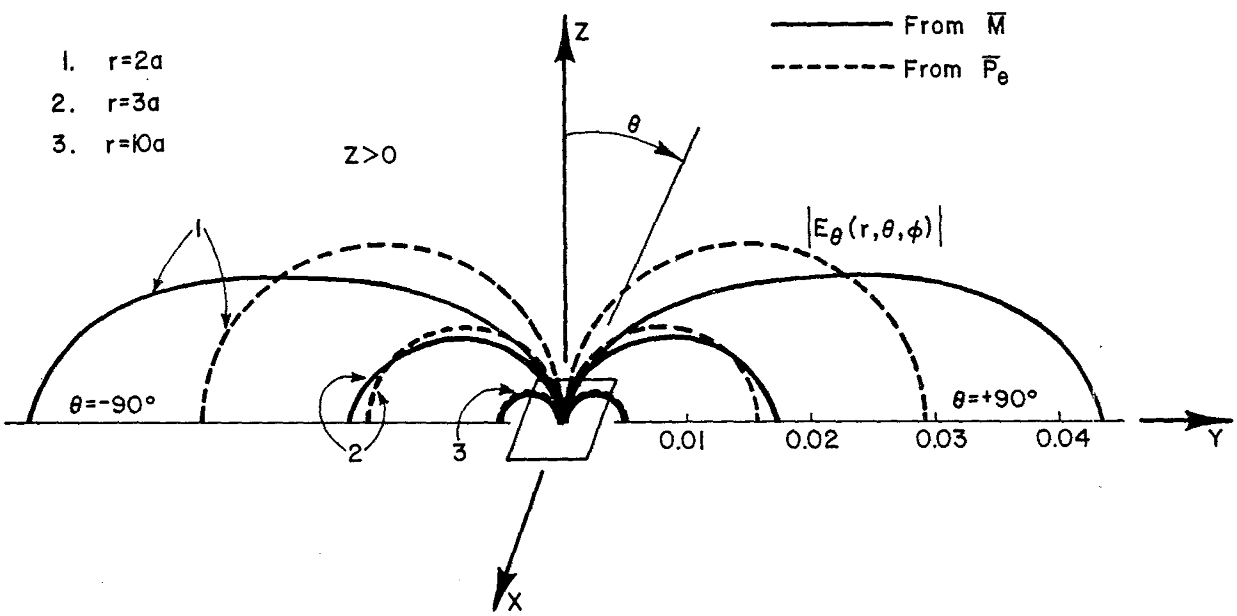


Fig. 15. Electric field on shadow side of square aperture ($2a = 2b = 0.15\lambda$, $E_y^{i-} = 1$ volt/meter, edge-on incidence).

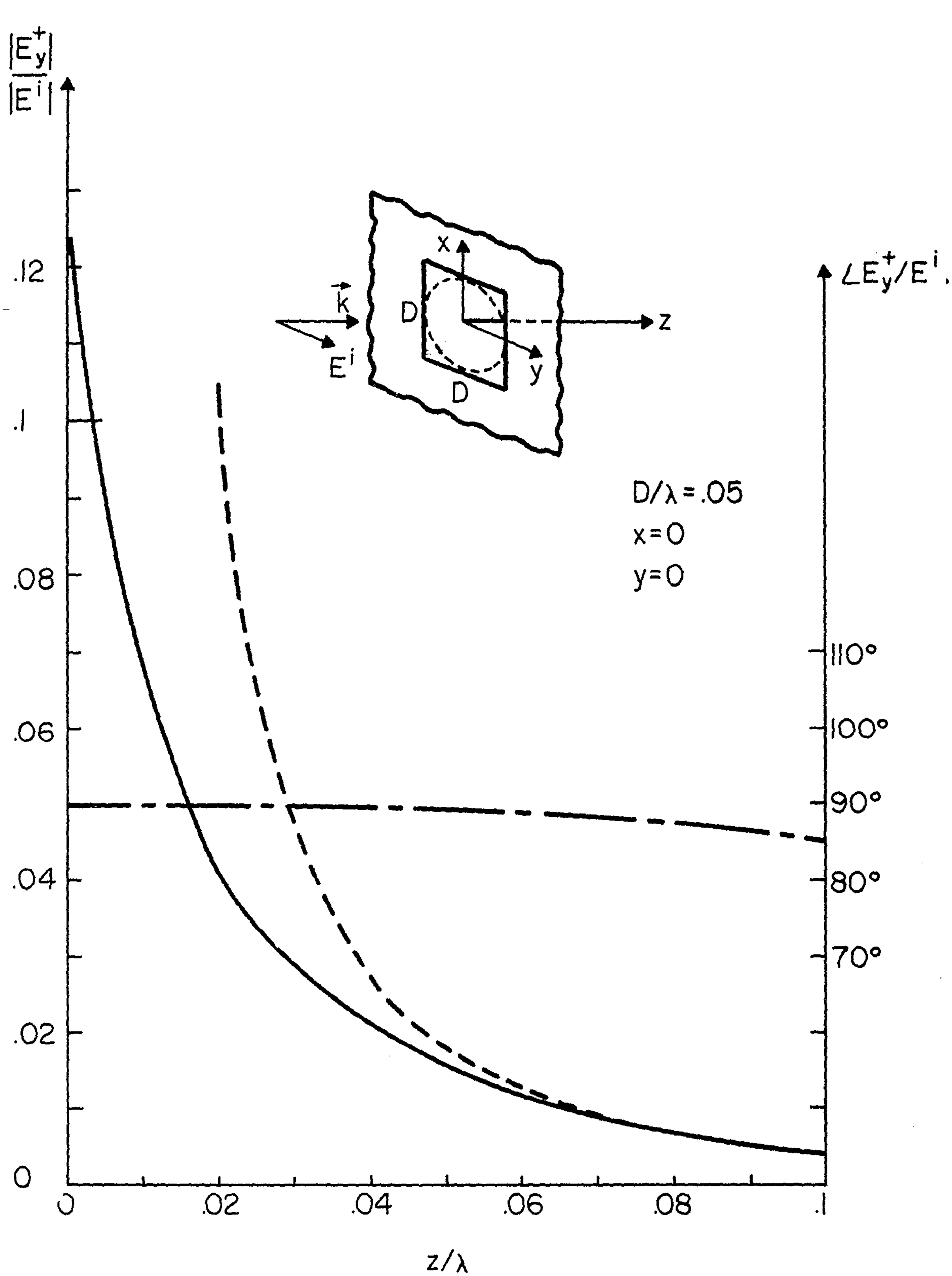


Fig. 16. E_y-field distribution sampled along the z-axis. Integral equation solution (——) for square aperture. Dipole moment results (---) for circular aperture. Phase curve (—— -) obtained from integral equation and dipole moment techniques.

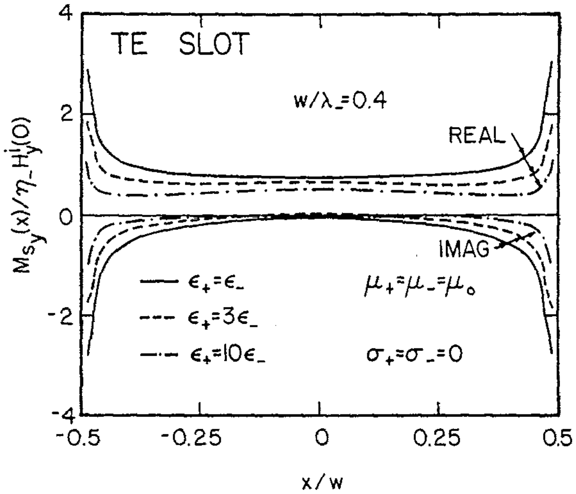


Fig. 17. TE equivalent magnetic current in 1.0-wavelength slot for different right half-space permittivities.

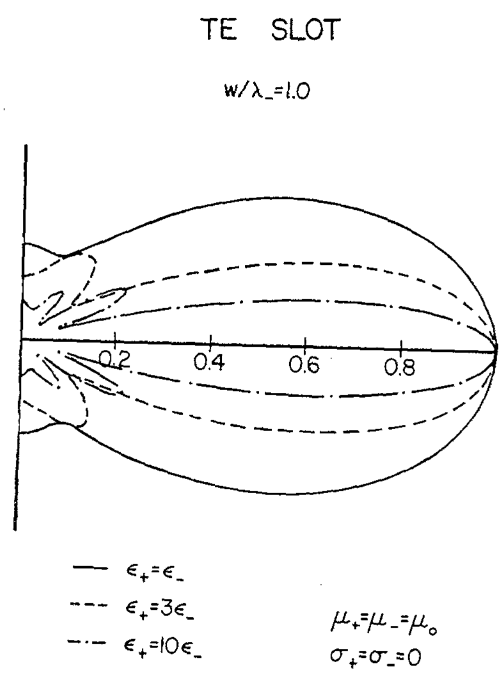


Fig. 18. Far magnetic field due to presence of TE-excited slot in screen for different right half-space permittivities.

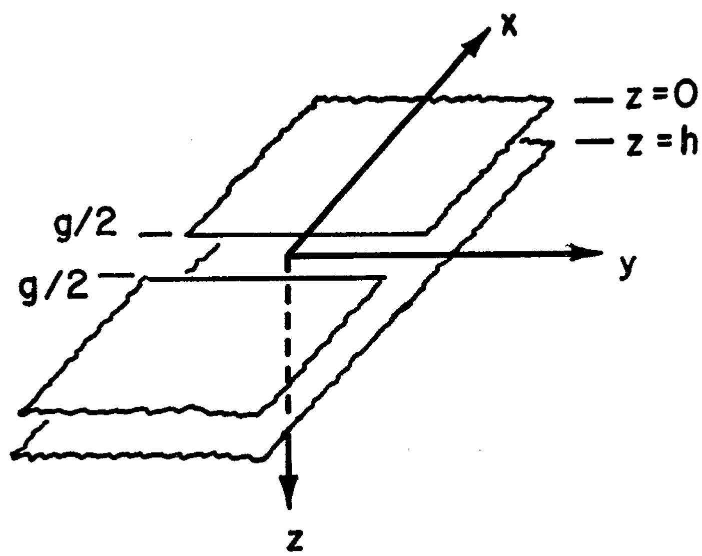


Fig. 19. Slotted parallel-plate waveguide.

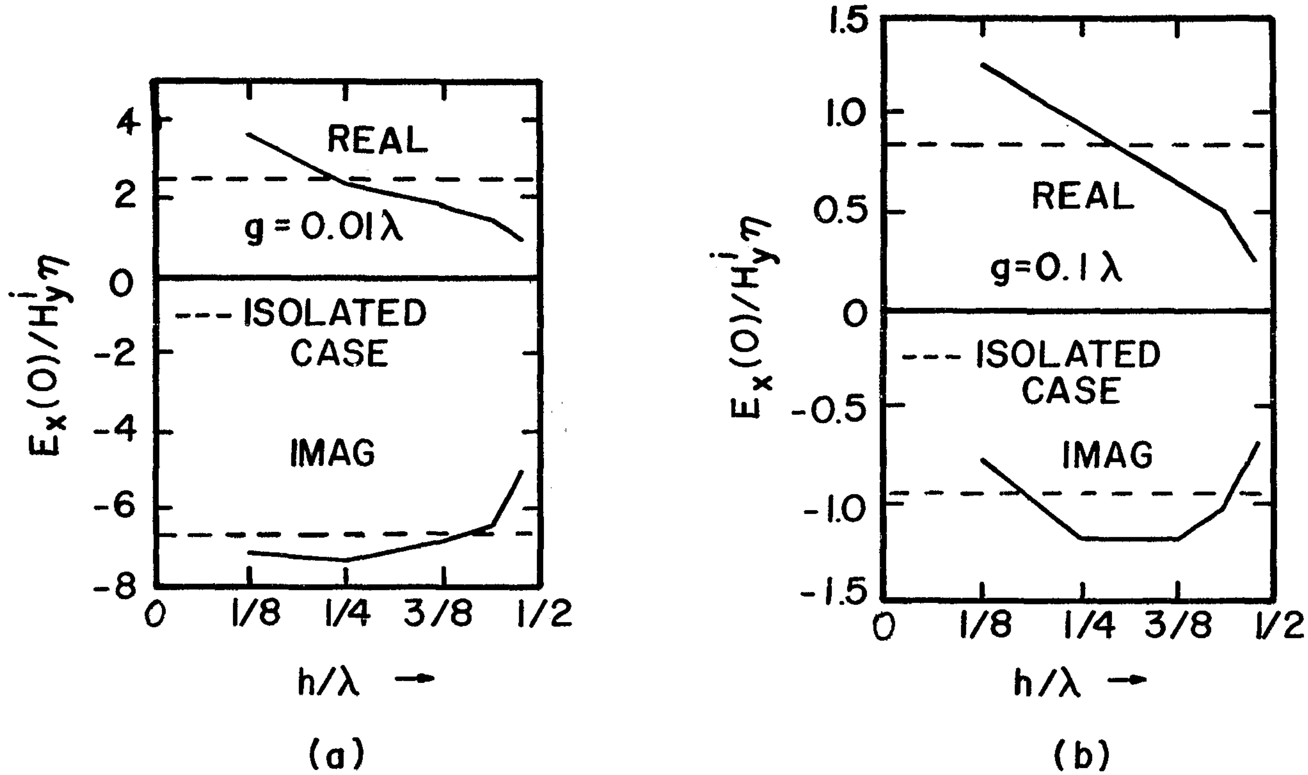


Fig. 20. Electric field E_x at center of slot in plate of parallel-plate waveguide as a function of plate separation h (normally incident, TE illumination).

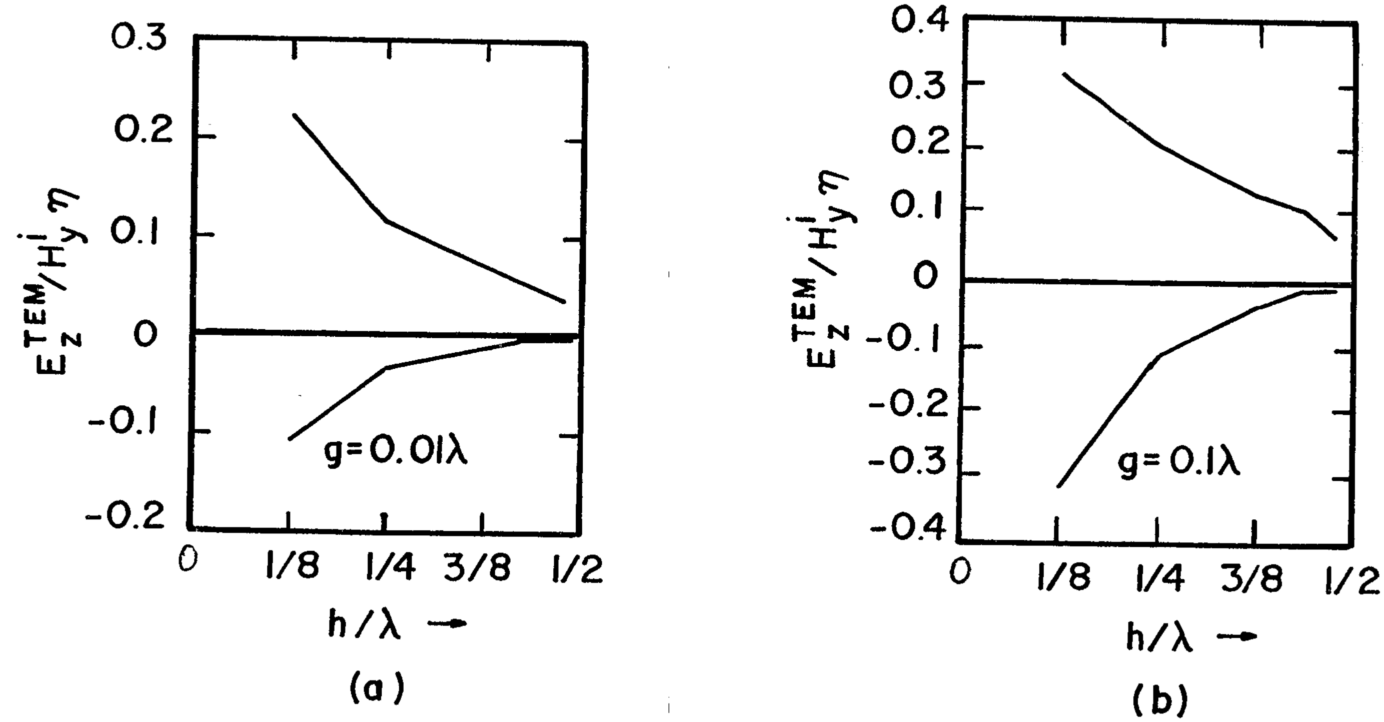


Fig. 21. TEM electric field in slotted parallel-plate guide due to normally incident TE illumination.

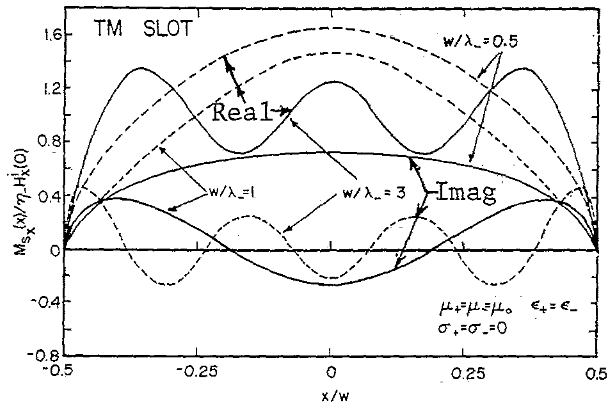


Fig. 22. TM equivalent magnetic current in 0.5-, 1.0-, and 3.0-wavelengths slots in screen separating half spaces of same media.

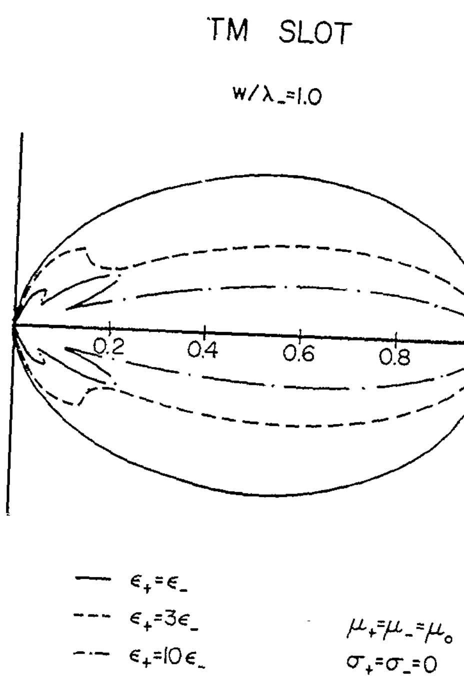


Fig. 23. Far electric field due to presence of TM-excited slot in screen for different right half-space permittivities.

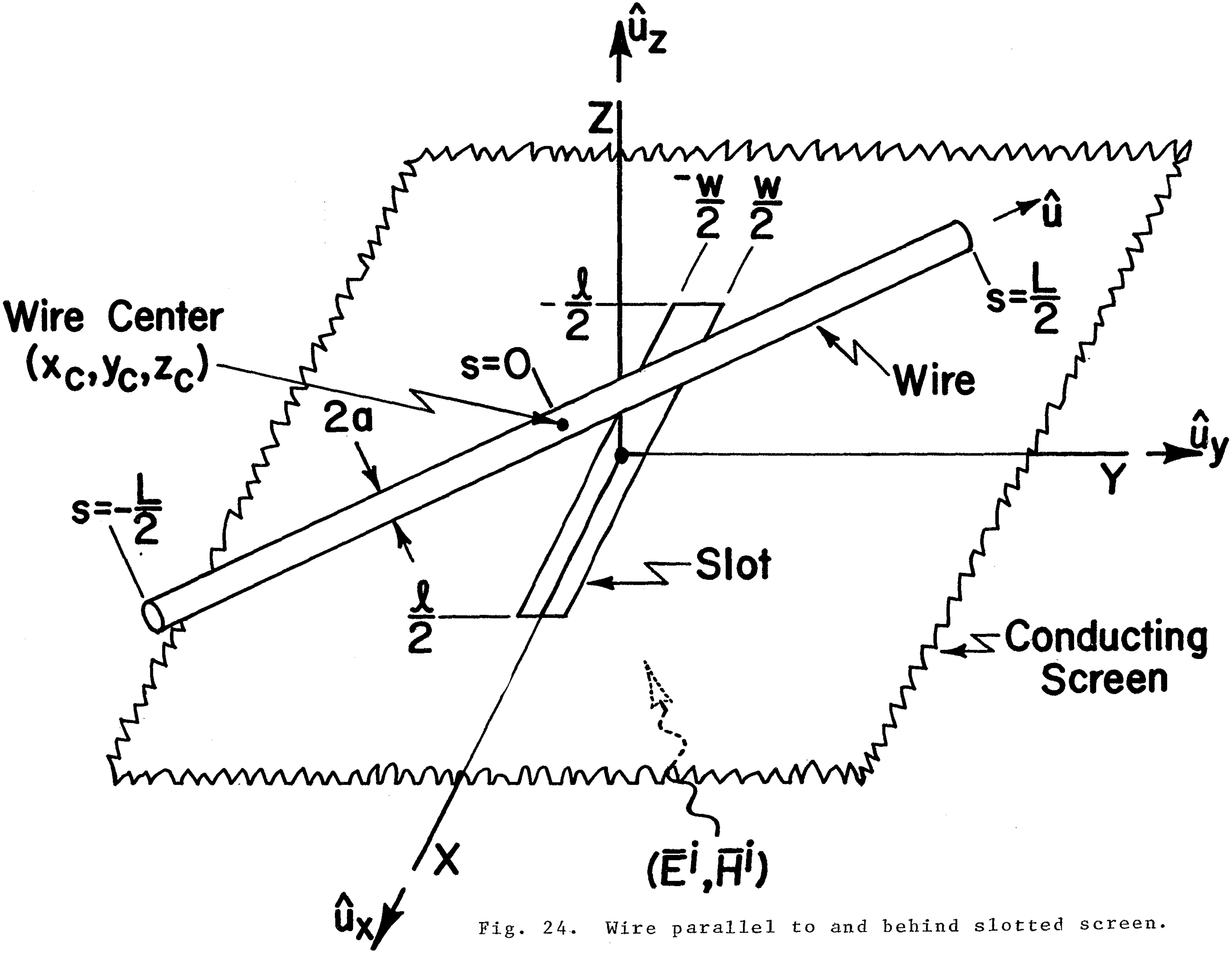


Fig. 24. Wire parallel to and behind slotted screen.

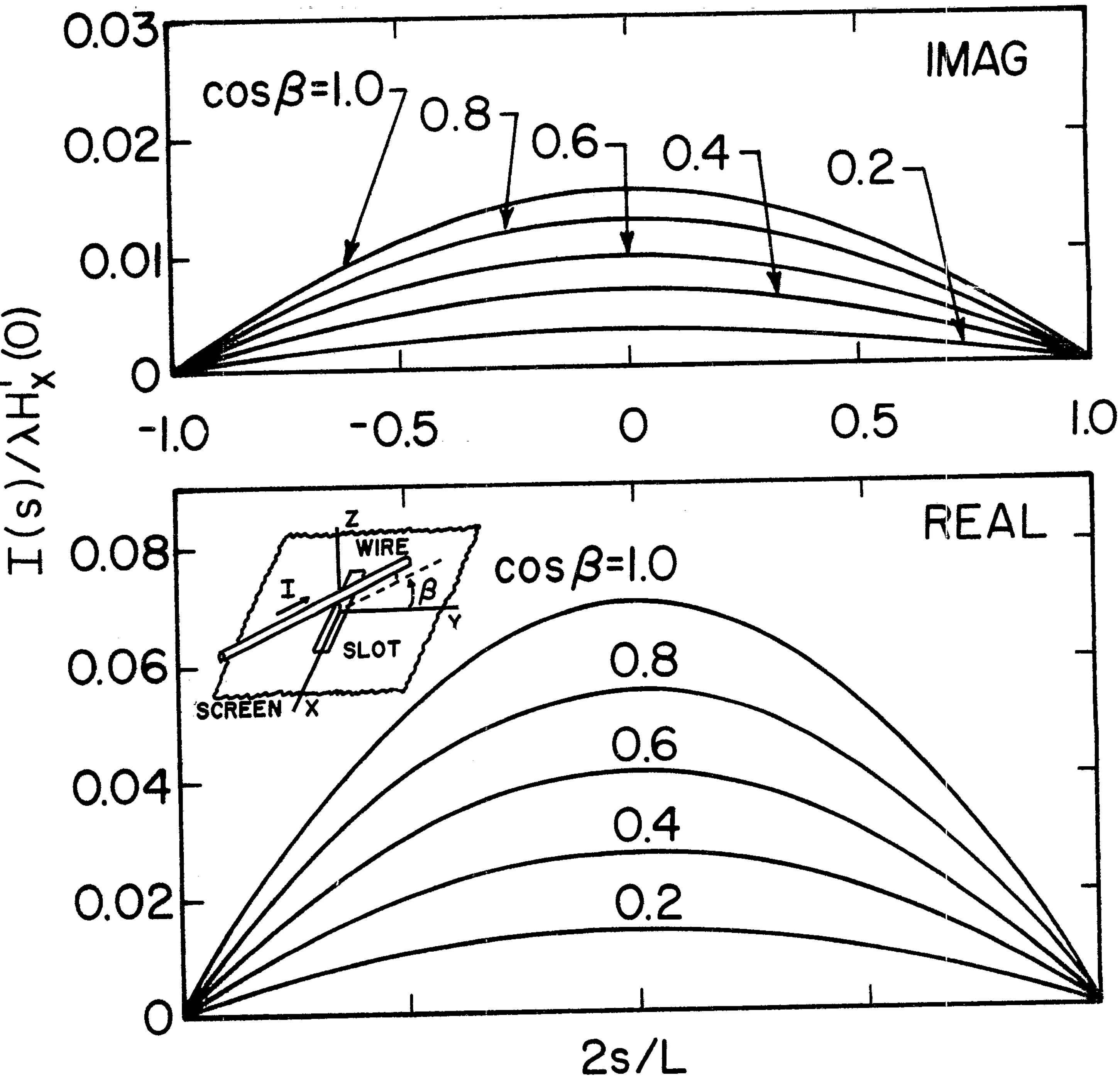


Fig. 25. Current on wire illuminated through slotted screen (w/λ = 0.05, ℓ/λ = 0.25; a/λ = 0.001, L/λ = 0.5; x_c/λ = 0, y_c/λ = 0, z_c/λ = 0.25; normal incidence).

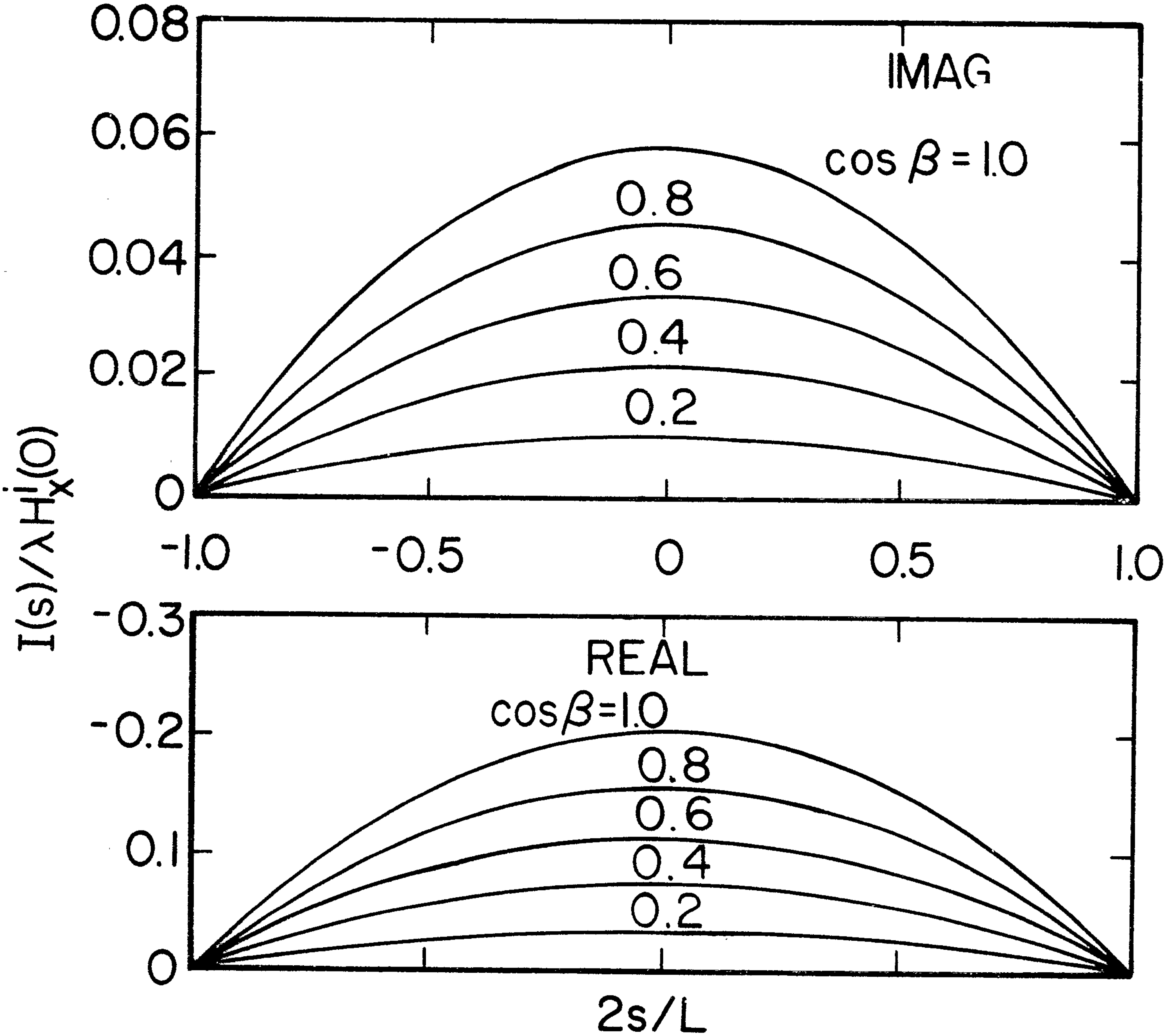


Fig. 26. Current on wire illuminated through slotted screen (w/λ = 0.05, ℓ/λ = 0.25; a/λ = 0.001, L/λ = 0.5; x_c/λ = 0, y_c/λ = 0, z_c/λ = 0.125; normal incidence).

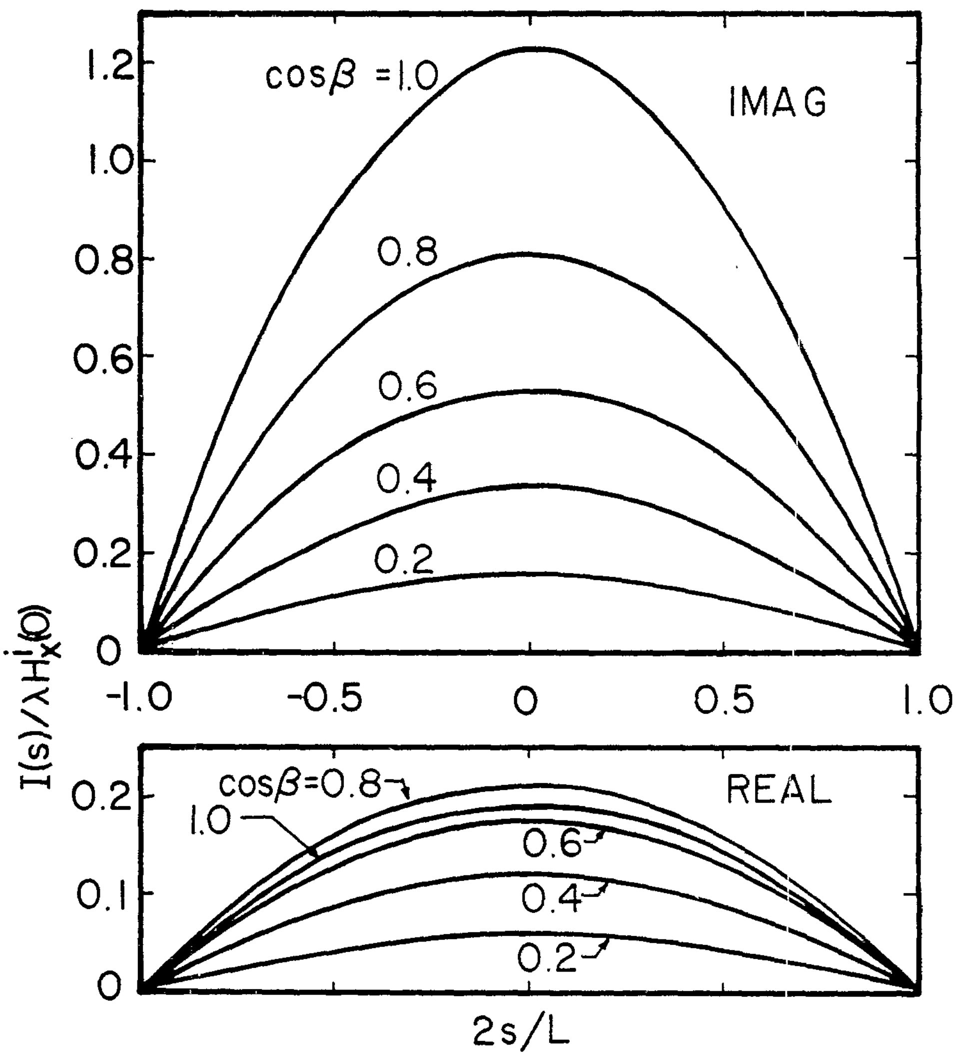


Fig. 27. Current on wire illuminated through slotted screen ($w/\lambda = 0.05$, $\ell/\lambda = 0.5$; $a/\lambda = 0.001$, $L/\lambda = 0.5$; $x_c/\lambda = 0$, $y_c/\lambda = 0$, $z_c/\lambda = 0.25$; normal incidence).

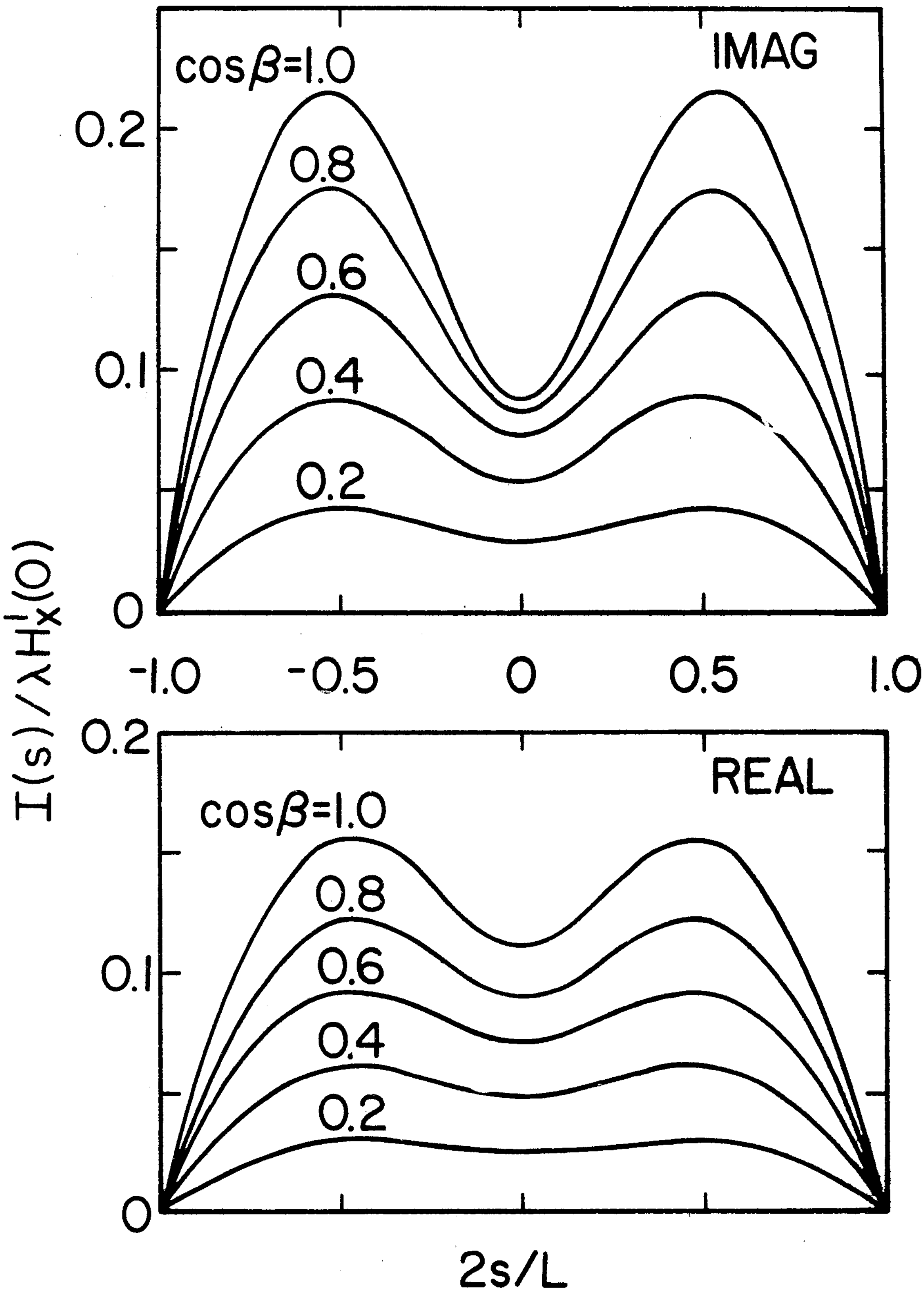


Fig. 28. Current on wire illuminated through slotted screen (w/λ = 0.05, ℓ/λ = 0.5; a/λ = 0.001, L/λ = 1.0; x_c/λ = 0, y_c/λ = 0, z_c/λ = 0.125; normal incidence).

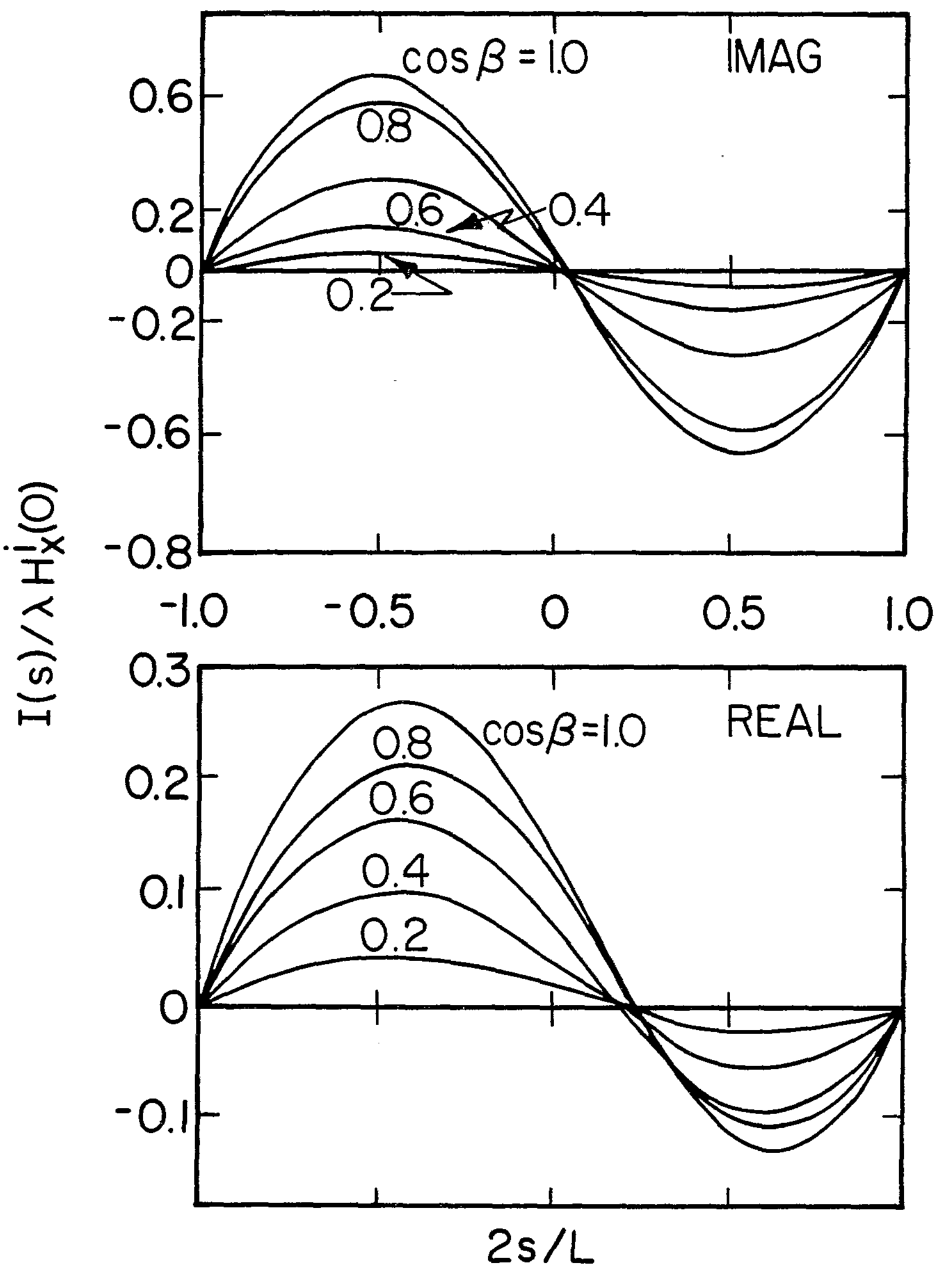


Fig. 29. Current on wire illuminated through slotted screen (w/λ = 0.05, ℓ/λ = 0.5; a/λ = 0.001, L/λ = 1.0; x_c/λ = 0.125, y_c/λ = 0.25, z_c/λ = 0.25; normal incidence).

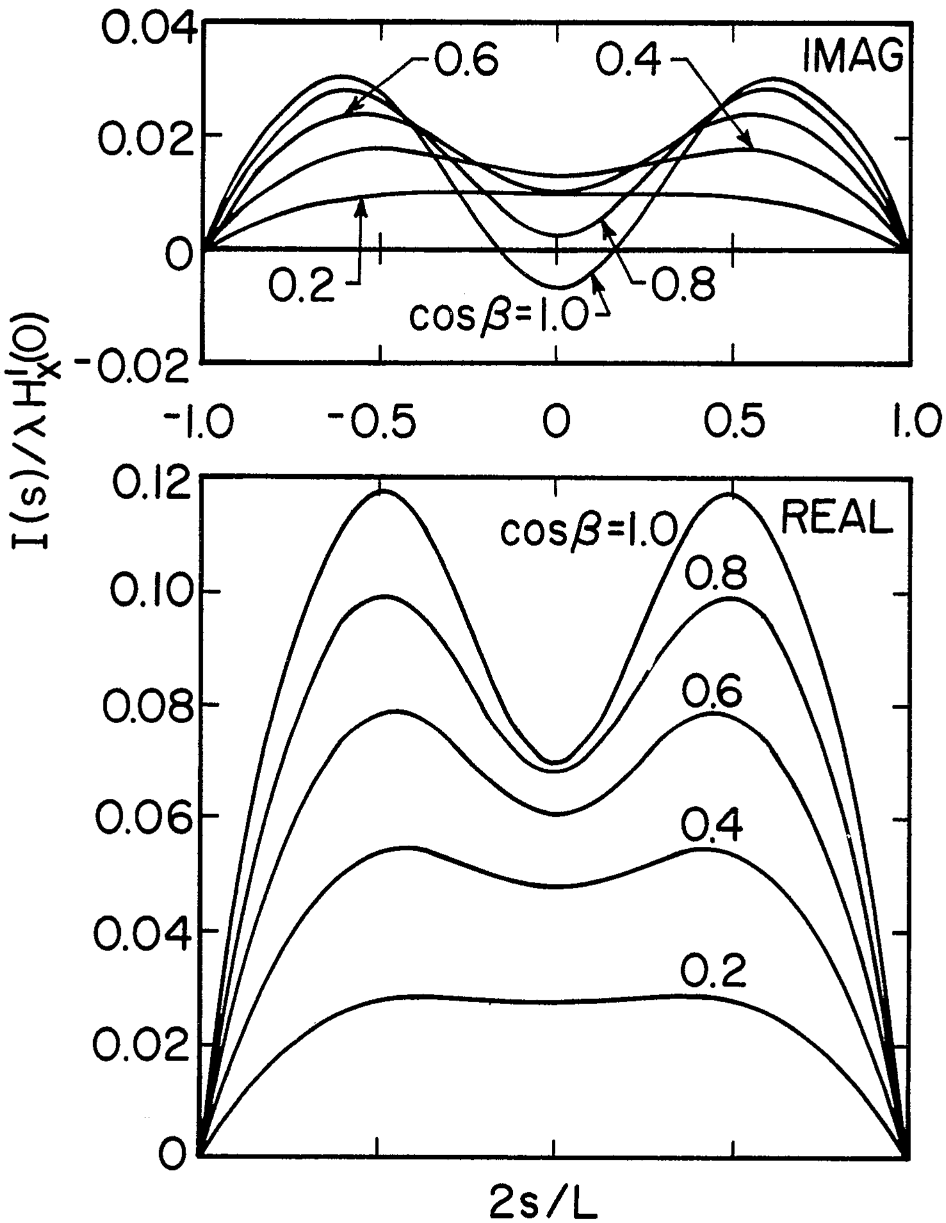


Fig. 30. Current on wire illuminated through slotted screen (w/λ = 0.05, ℓ/λ = 1.0; a/λ = 0.001, L/λ = 1.0; x_c/λ = 0, y_c/λ = 0, z_c/λ = 0.125; normal incidence).

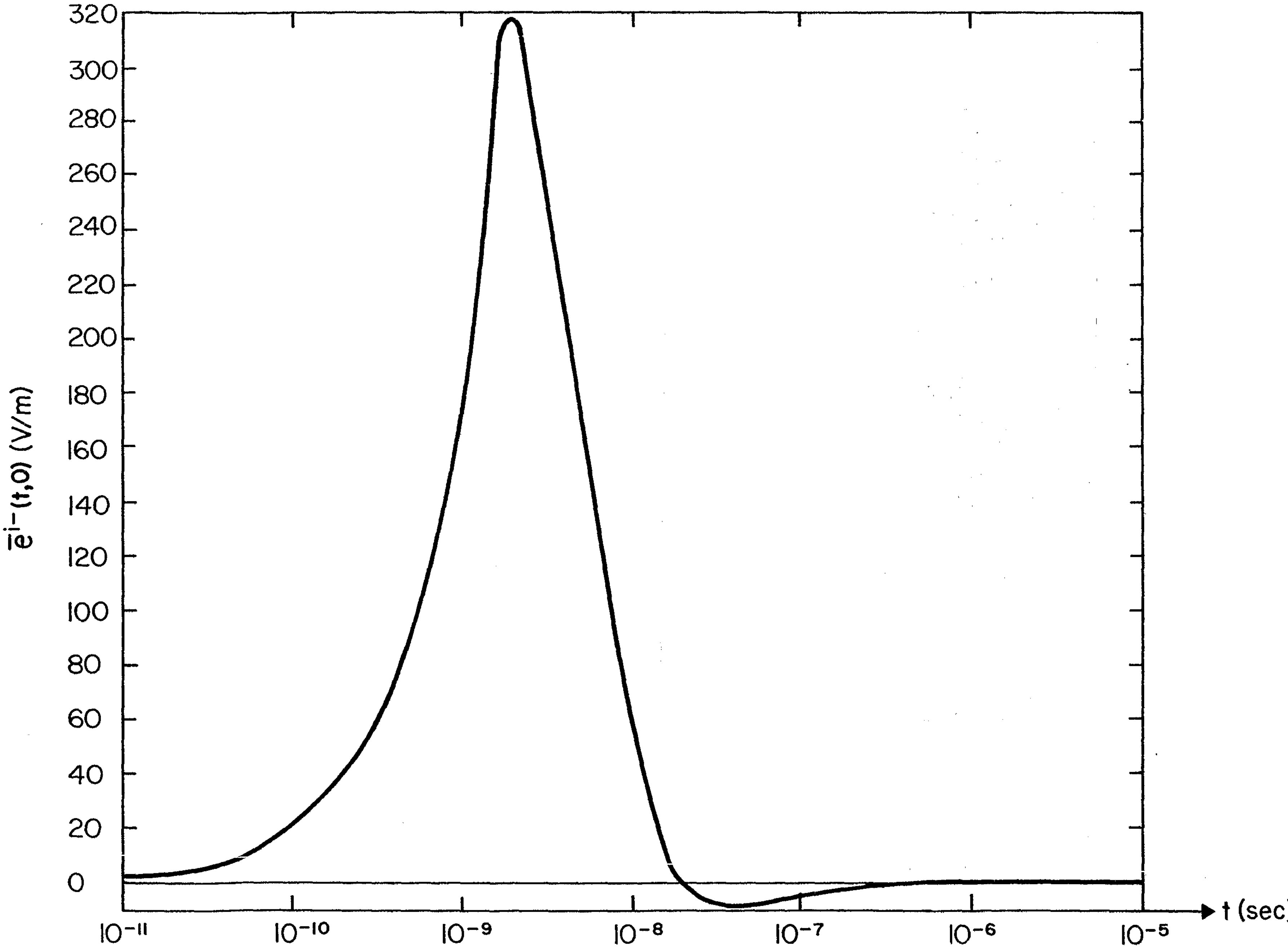


Fig. 31. Time domain plot of the incident pulse.

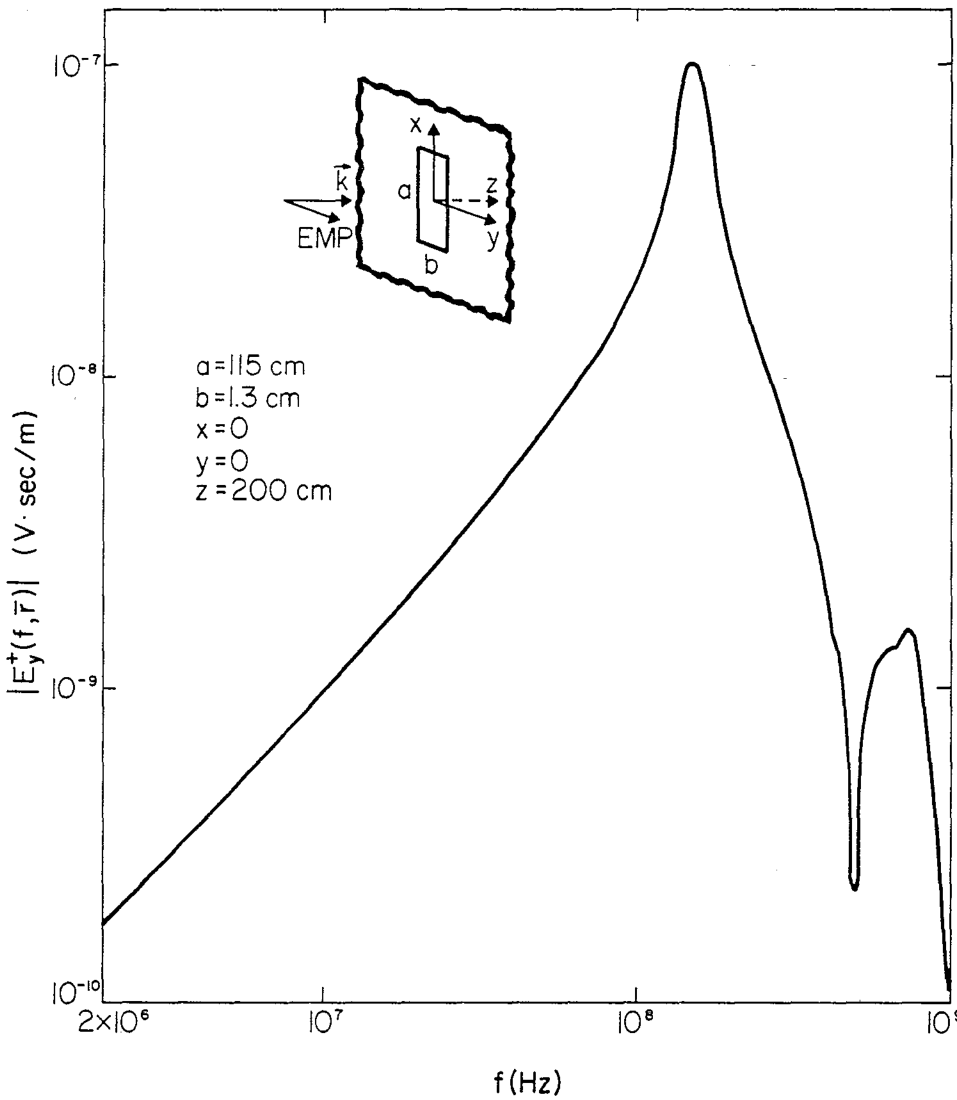


Fig. 33. Frequency domain behavior of $|E_y^+(f,\bar{r})|$ sampled at a point 2 meters behind the aperture on the z-axis.

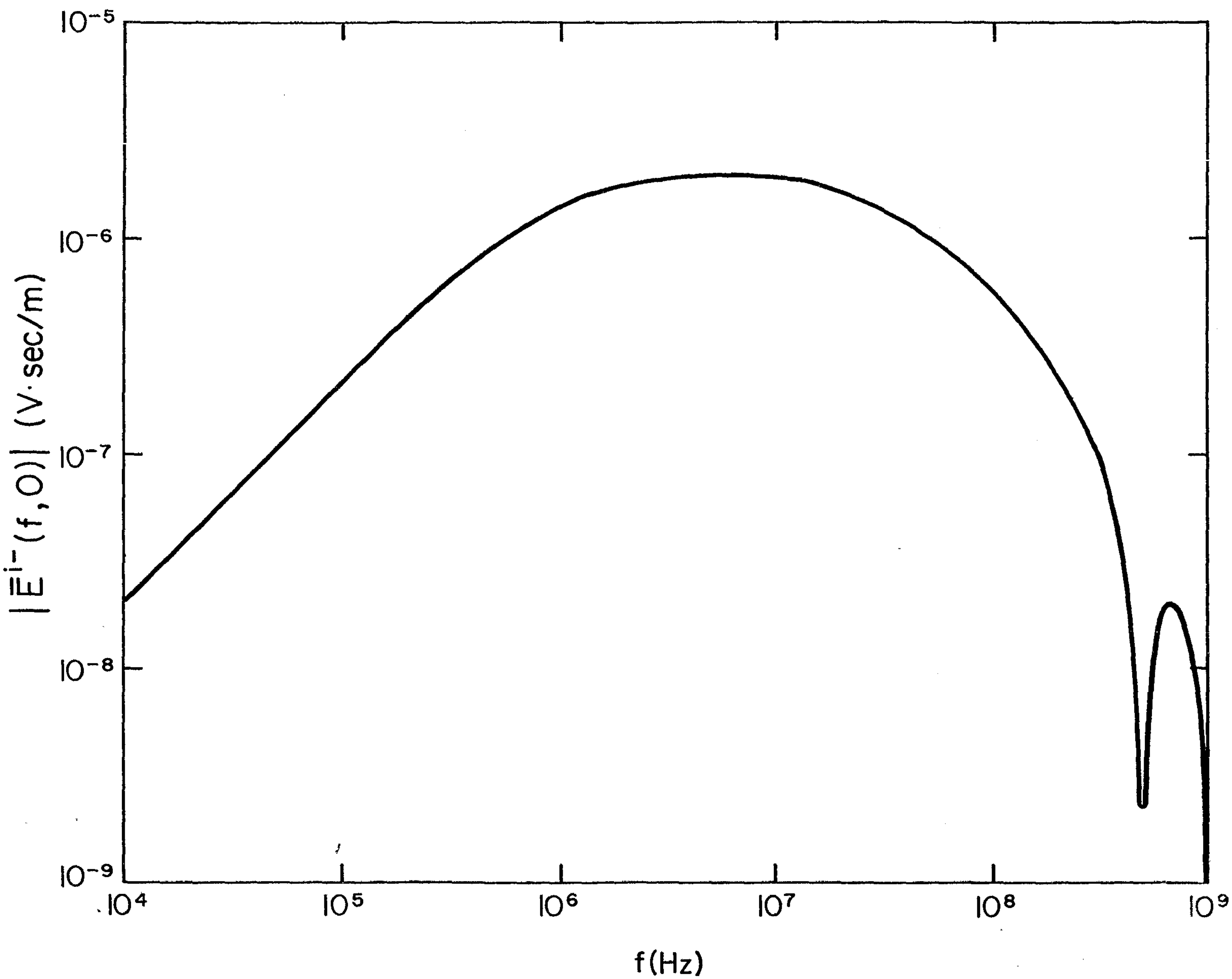


Fig. 32. Frequency domain behavior of the incident pulse.

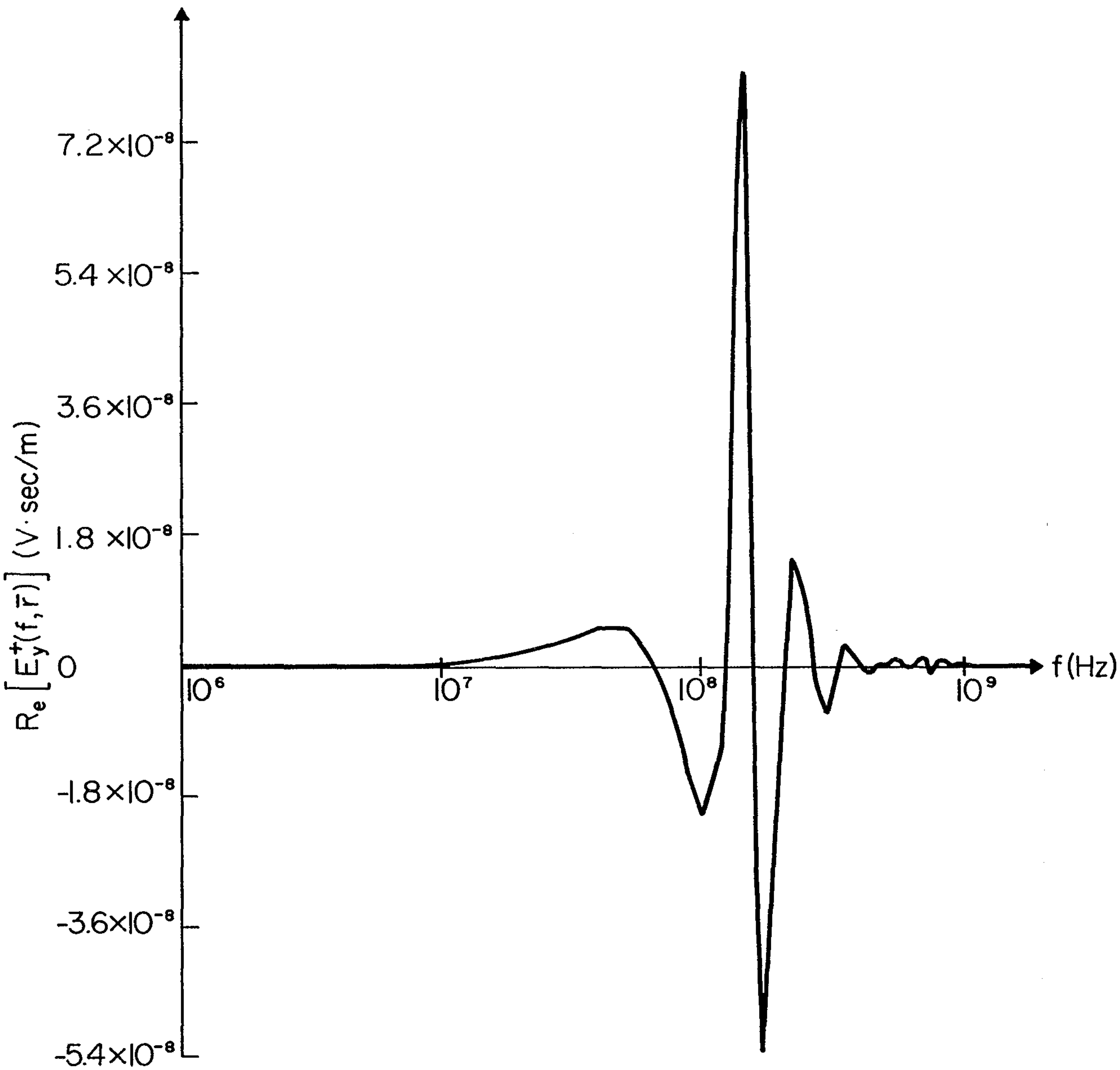


Fig. 34. Frequency domain behavior of $\mathrm{Re}[E_y^+(f,\bar{r})]$ sampled at a point 2 meters behind the aperture indicated in Fig. 33.

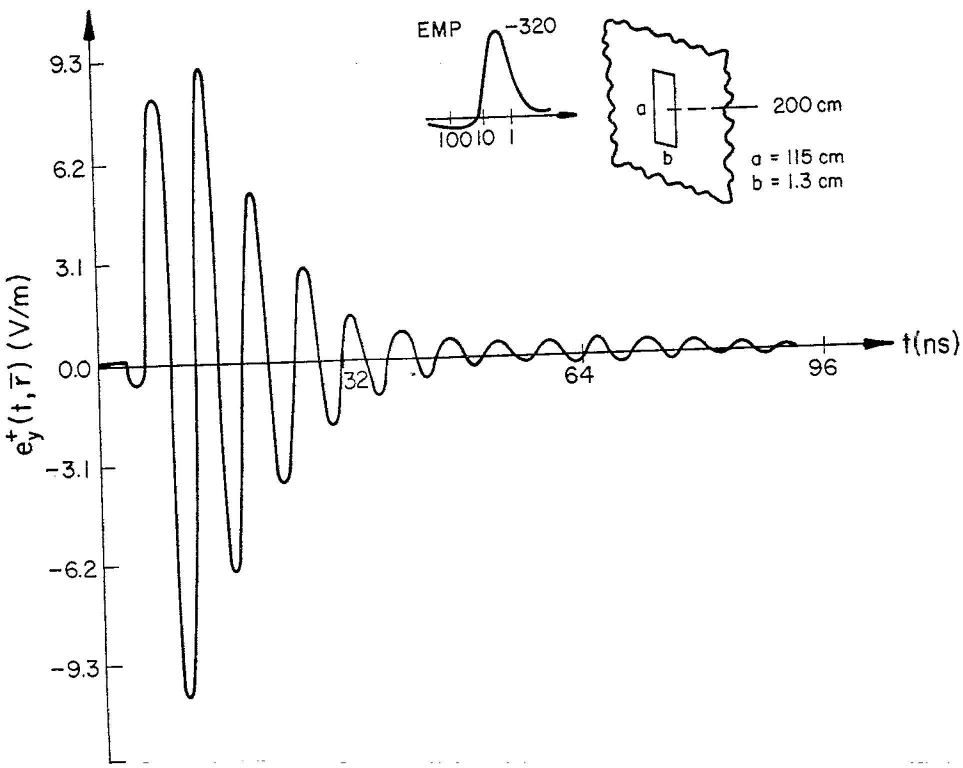


Fig. 35. Time domain behavior of the $e_y^+(t,\bar{r})$ field sampled at a point 2 meters behind a single aperture.

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